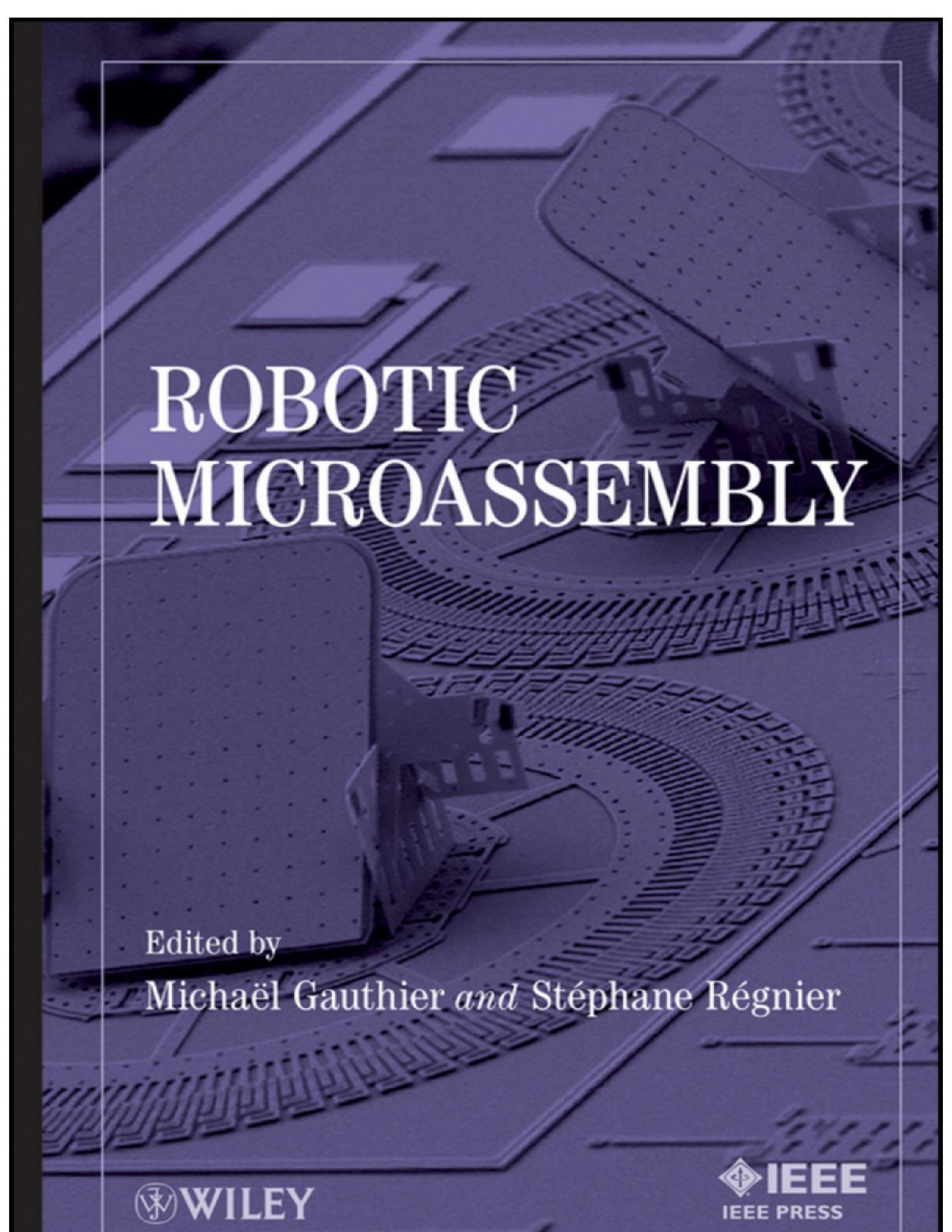
ROBOTIC
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Foreword

In 1995, two papers appeared at the 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS95) that helped precipitate the field of robotic assembly as a topic of research within the robotics community. One of the papers was written by Ron Fearing of the University of California, Berkeley, entitled “Survey of Sticking Effects for Micro-Parts,” and the other came out of Toshio Fukuda's group (first author Fumihito Arai) called “Micro Manipulation Based on Micro Physics.” Both papers did a wonderful job of illustrating the challenges and the opportunities of manipulating micrometer-size parts automatically, essentially defining the “mechanics of micromanipulation” and encouraging many of us, myself included, to pursue robotic microassembly as a topic of research.

However, one could argue that robotic microassembly got its start about 400 years ago with the invention of the optical microscope in Holland. Imagine being someone like Robert Hooke, one of the first to master the use of these instruments. In the 1650s, while he was at Oxford, Hooke began working with microscopes and discovered a hidden world of small insects and tiny creatures in addition to observing plant structures and various materials. In 1665 he published *Micrographia*, a book illustrating his observations, which is considered one of the greatest of his many considerable achievements. With the publication of *Micrographia*, others rapidly became aware of this secret world, and it was only natural for people to wonder how one could make similar things. Craftsmen learned that by using microscopic techniques, they could see smaller details that enabled them to make finer and finer mechanisms. Manual microassembly became an important industry in much of

Europe, where the watchmaking industry grew in places such as France, Switzerland, Germany, and England.

In the late 1940s, the invention of the transistor by Bardeen, Shockley, and Brattain at Bell Labs began another shift in micromanufacturing. Suddenly there was a newfound need to make really, really small things cheaply, driving Kilby and Noyce to the concept of the integrated circuit. Moore's law began, and batch fabrication, not serial assembly, was the obvious way to make small things cheaply, primarily out of silicon, of course. Then in 1982 Kurt Pedersen published his seminal paper “Silicon as a Mechanical Material,” a paper that is often cited as representing the beginning of the MEMS era (microelectromechanical systems). While the MEMS community abhorred assembly in the early days, the constraints that microfabrication processes placed on the materials with which microsystems could be made as well as their geometry were extremely limiting. What if we could actually assemble microsystems, instead of relying solely on top-down processes such as photolithography, thermal evaporation, and reactive ion etching? This question was being increasingly asked just as IROS began in Pittsburgh in August of 1995.

These historical trends are what motivate robotic microassembly. Though the field as it is currently defined has been highly active for almost 15 years, this book represents a pioneering achievement by creating, for the first time, a complete view of the field from the physics of micromanipulation, to microassembly, to microhandling in general. A first-class consortium of international authors has been assembled to provide a comprehensive, worldwide view. This effort, which helps further define the field of robotic microassembly, will undoubtedly spur researchers and industry to continue their quest to make small things cheaply.

Bradley Nelson

Zürich, Switzerland

May 2009

Preface

This book deals with the current methods developed around the world on robotic microassembly. It is dedicated to Master's and Ph.D. students, and also scientists and engineers involved in microrobotics and also in robotics. As robotic microassembly is a new way to manufacture microelectromechanical systems (MEMS), companies and research institutes involved in this domain will find in this book original methods that can be used to simulate, design, and build new generations of hybrid tridimensional microproducts.

Microproducts are usually divided into two categories by function of the manufactured process used. On the one hand, the standard fabrication using machining or molding is able to produce millimetric and submillimetric pieces (e.g., gears in watches). On the second hand, processes developed initially in microelectronics and based on photolithography have been extended to mechanical structures and are currently used to build MEMS (e.g., air bag sensors). In both cases, the resolution of the details built on the product could be around the micrometer or even less, but the global size of the pieces stays millimetric. The market of miniaturized products, which include always more functionalities in a smaller volume, is increasing very rapidly. In the future, the size of the piece should be reduced below 100 µm, and the microsystem should integrate a large variety of functions including mechanisms, electronic, and control, fluid or optic. It is the reason why a large number of research teams are currently focused on the topic of microassembly.

In MEMS microfabrication, hybridization of technologies is currently obtained using planar assembly (e.g., flip–chip process). However, this method is limited to planar products and does not enable out-of-plane assembly. The advent of a new generation of microsystems based on tridimensional

hybrid structures is directly linked with the ability to manufacture microsystems using advanced assembly processes. Two approaches are currently developed in microassembly: self-assembly and robotic microassembly.

Self-assembly consists in creating several minimums of potential energy with a physical field (i.e., electrostatic, capillarity). Microobjects thus need minimum energy and are directly positioned. Self-assembly is a natural process for molecular structures and many examples can be found in nature. These processes are massively parallel but the efficiency and the flexibility still stay low. On the other hand, manipulation robots can be used to assemble micro- or nanopieces. This robotic approach is classically divided into three steps: positioning, handling, and release. This approach is able to reach complex assembly with high flexibility. However, handling and especially release is sensibly disturbed by microscopic peculiarities (i.e., adhesion, deformations of the object, environment). This book focuses on this second approach called *robotic microassembly*.

Robotic assembly is usually carried out on robotic platforms that consist of a gripping device able to grasp the pieces; some sensing systems able to measure position and/or force; a robot able to position the gripper; and a controller to induce automatic movements and tasks. In the microscale, the same functions have to be considered. In *robotic microassembly*, the most critical phase is the gripping task, which depends on interactions between the manipulated object and the end-effector of the gripper. Indeed, at this scale, the behavior of the object is the function of the adhesion and surface forces (i.e., van der Waals, electrostatic forces, etc.), which are predominant compared to the volume effects (i.e., weight, inertia).

The design of robotic assembly platforms must be based on a good understanding and analysis of these adhesion

and surface forces, which is the objective of the first part of this book. The physical principles involved in the microscale is developed in order to present the expression of forces in several cases. As the environmental parameters (i.e., humidity, pH in a liquid) highly influence the adhesion and surface forces, a specific chapter is dedicated to the relationship between these forces and the environment. These forces induce specific behavior in microobjects that require specific handling strategies to be handled and assembled.

Based on the knowledge on predominant forces in the microscale, new microhandling methods and prototypes have been developed and are listed in the second part. A lot of handling strategies are presented and compared: hybrid handling strategies based on principles that combine self-assembly and robotic assembly; gripping and release principles in the air; and specific handling strategies dedicated to submerged microobjects. Based on this panorama, the reader will easily understand microscale difficulties and will find methods and information to design microhandling principles.

Even though, handling is a critical phase in microassembly, it is not enough to assemble microobjects. The design of the microobject itself and of the robot structure, which have both to be carefully studied, are presented in the third part. The microassembly requires to design and manufacture micropieces able to be connected together to ensure, at least, mechanical links. Today, electrical and fluid connections in the microscale remain a challenge. Moreover, the required mobility (in terms of number of degree of freedom, DoF) and the required repeatability and precision of the robot require a specific design, calibration, and characterization. This third part focuses on these crucial problems, which are keys to assembly micropieces.

We expect that this book, which proposes a complete overview of the state of the art in robotic assembly, will provide a better understanding of the microscale specificities and methods for robotic microassembly to students, engineers, and scientists. They will be able to apply the models and the methods on microproducts and contribute to the development of the robotic assembly in the microscale.

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Part I

MODELING OF THE MICROWORLD

Chapter 1

Microworld modeling in Vacuum and Gaseous Environments

Pierre, Lambert, and Stéphane Régnier

1.1 Introduction

1.1.1 Introduction on Microworld Modeling

This first part describes the physical models involved in the description of a micromanipulation task: adhesion, contact mechanics, surface forces, and scaling laws. The impact of surface roughness and liquid is discussed later on in Chapter 2.

The targeted readership of Chapters 1 and 2 is essentially composed of master's degree students and lecturers, Ph.D. students, and researchers to whom this contribution intends:

- To give the theoretical background as far as the physics and scaling laws for micromanipulation are concerned
- To propose design rules for micromanipulation tools and how to estimate the interaction force between a component and the related gripper or between a cantilever tip and a substrate

The goal of developing models may be questioned for many reasons:

- The task is huge and the forces dominating at the micro- and nanoscale can only be modeled very partially: for example, some of them cannot be modeled in a quantitative way (e.g., hydrogen bonds) suitable for robotics purposes, most of the proposed models are only valid at equilibrium (at least all the models based on the derivation of surface or potential energies).
- The parameters involved in the existing models are sometimes impossible to know, such as, for example, the electrical charge distribution on a dielectric oxide layer.
- Maybe as a consequence of the previous reason—that is, a full characterization is impossible—the micro- and nanoscale specifically suffer from a very large experimental dispersion, which makes the model refinements questionable. According to own experience, experimental results are difficult to keep within a few tens of a percent error interval. Yang and Lin (93) recently write that *the measurements usually show poor reproducibility*, suggesting that *the major causes of irreproducibility can be roughness and heterogeneity of the probe surface and sample*.

Nevertheless the use of—even basic—models helps the microrobotician to get into the nonintuitive physics dominating the microworld—mainly adhesion-related instabilities such as pull-in and pull-out—to give an explicating scheme of the experiments—what is the role of humidity? what is the influence of the coatings—to design at best grippers and tools on a comparative way—no matter the exact value of the force; but a geometries comparison leads to the best design. These advantages will be detailed later on.

Classical adhesion models (20, 41, 67) usually proposed to study adhesion in micromanipulation or atomic force microscopy (AFM) are based on the elastic deformation of

two antagonist solids (microcomponent/gripper in micromanipulation, cantilever tip/substrate in AFM). This part will introduce models that are now well known, but they will be introduced in the framework of microassembly. Modern models will refer to recent developments and/or recent papers. The theoretical background proposed in this part aims at detailing:

1. Every phenomenon leading to a force interaction: capillary forces, electrostatic forces (in both liquid and air environment, but restricted to conductive materials), van der Waals forces, and contact forces
2. The influence of surface science concepts such as topography, deformation, and wettability

These elements constitute a basis on which to model adhesion without using empirical global energy parameters such as surface energies.

When dealing with stiction and adhesion problems in micromanipulation, one is often referred to a list of many concepts (van der Waals interaction, capillary force, adhesion, pull-off), which sometimes can recover one another. Lambert and Régnier (53) have proposed to sort out these forces by making the distinction whether there is contact or not. When there is no physical contact between two solids, the forces in action are called distance or surface forces (according to the scientific literature in this domain (12, 22, 76), these latter are electrostatic, van der Waals, and capillary forces). When both solids contact one another, there is deformation and adhesion forces through the surfaces in contact. In this case, the authors considered contact forces and adhesion or pull-off forces. Electrostatic or capillary effects can be added, but van der Waals forces are not considered anymore because they are thought already involved in the pull-off term. The new idea conveyed in this part is to consider van der Waals, capillary,[1] and

electrostatic forces as parts into which the global pull-off force can be split.

Beside these contact or close to contact forces, it is also important to focus on other forces that affect the dynamics of small components. This description can only be done by considering the specificities of the working environment. In liquid environments, for example, we will consider viscous drag (Lenders et al. (58) have recently presented a design of microfeeder using these forces), electrostatic double-layer effects, and (di)electrophoresis. Very recently, a new focus has been found on the effect of gas bubbles in liquid media.

Additionally, we will try to address the question of mechanical contact from two points of view: what are the limits of the Hertz-based models (20, 41, 67) and what is the influence of a liquid environment on this contact?

1.1.2 Microworld Modeling for Van der Waals Forces and Contact Mechanics

The first chapter concerns vacuum or gaseous environments. First in Section 1.2 some well-accepted models are recalled, concerning van der Waals forces, elastic contact mechanics and the related adhesion models, and capillary force models at the submillimetric scale. Second, in Section 1.3 very recent published results are presented together with our own perspective: capillary condensation effects, the influence of surface roughness, and mechanical deformation on electrostatic forces.

Before going through these models, let us mention that many (attractive) effects contribute to adhesion. Based on Lee (57), we propose the schematic forces summary presented in Table 1.1.

Table 1.1 Forces Summary and Their Interaction Distances

Interaction Distance	Predominant Force
Up to infinite range	Gravity
>From a few nm up to 1 mm	Capillary forces
> 0.3 nm	Coulomb (electrostatic) forces
0.3 nm < separation distance < 100 nm	Lifshitz–van der Waals
< 0.3 nm	Molecular interactions
0.1–0.2 nm	Chemical interactions

Additional effects turn out to be also of importance: Let us cite the Casimir effect, which will not be detailed in this contribution. We refer to Klimchitskaya and Hostepanenko (45).

It seems, however, that capillary effect dominates all the microworld from a few nanometers up to tenths of millimeters van der Waals effects turn out to compete with capillary effect but only within the nano range up to a few tens of nanometers (we can consider the limit of the retardation effect as the limit, see later on). We therefore mainly focus on both effects together with the electrostatic adhesion, which comes from either the intense electrostatic fields coming from microrobotic actuation—they can be avoided using thermal actuation—or from the moderate effect of contact potentials.

1.2 Classical models

1.2.1 Van der Waals Forces

The so-called van der Waals forces are often cited in papers dealing with micromanipulation and microassembly, probably because the founding papers of these bibliography reviews (12, 22) present these forces next to the capillary and the electrostatic forces as being of the utmost importance in the sticking of microparts. Other authors (7)

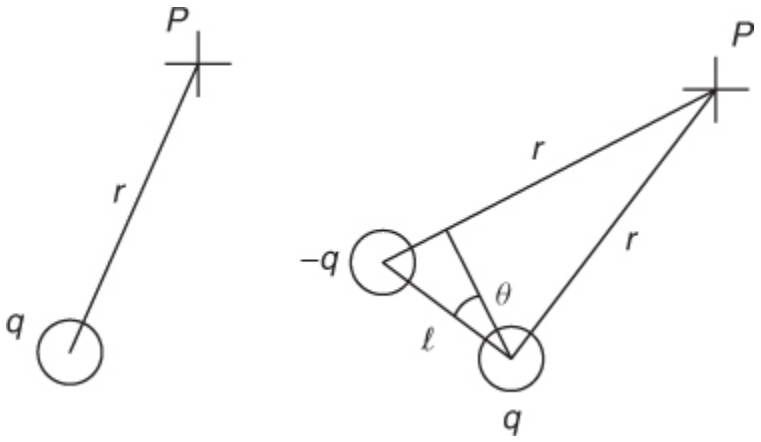
prefer to neglect these forces because they are of a smaller order. The reasons for this opposition do not seem to be clear, all the more so since some authors propose to use it as a suitable gripping principle (3, 23). The will to clarify this situation is a first reason to study van der Waals forces. A second reason lies in the fact that most force expressions used in the literature on microassembly are only approximations of simplified geometries (spheres and planes). If these approximations are sufficient for experimental case studies, the influence of more complex geometries (nonsymmetrical geometries) including roughness profiles should be studied for many applications. We propose to briefly present the physical underlying phenomena that explain these forces and to explain the way(s) they can be calculated. An overview of the approximations from the literature is proposed in the conclusion of this section.

A good and very didactic introduction to the subject can be found in Israelachvili (38), while a more exhaustive description of the van der Waals (VDW) forces has also been proposed (1, 26, 39). In order to explain, at least from a qualitative point of view, the power law describing the van der Waals interaction energy, let us start from the potential energy of an electric charge q (Eq. 1.1) and that of a permanent dipole p made of two charges q and $-q$ separated by a distance l (Eq. 1.2 states if $l << r$), in both cases in a point P at a separation distance r and in vacuum (see Fig. 1.1):

$$U(P) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \textbf{1.1}$$

$$U(P) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \quad \textbf{1.2}$$

Figure 1.1 Illustration of potentials of a charge and of a permanent dipole.



We see that the potential depends on the inverse of the first power of the separation distance in the case of a charge and on the inverse of the second power in that of a permanent dipole. If we now consider the interaction potential $w(r)$ of two permanent dipoles p_1 and p_2 separated by a distance r, it can be shown (89) that $w(r)$ also depends on the inverse of the third power of the separation distance:

1.3 $$w(r) \approx \frac{1}{4\pi\epsilon_0}\frac{p_1 p_2}{r^3}$$

We can now introduce the underlying idea to explain the van der Waals forces. Let us consider two molecules, separated by a distance r. If these two molecules are polar (which means that there is a permanent electric dipole inside the molecule due to the fact that the gravity center of the positive charges does not fit with that of the negative forces), their interaction energy can be described by Eq. 1.3. Actually, the van der Waals forces also act between totally neutral atoms and molecules such as helium, methane, and carbon dioxide. This is due to the fact that even in a nonpolar atom, the gravity center of the positive and negative charges are not instantaneously superposed, leading to an instantaneous dipole p_1, with a characteristic charge in the order of the electronic charge e and a separation distance of about one atom radius a_0 (note that this explanation was first applied by D. Tabor to the

interaction between two Bohr atoms, a_0 known as the first Bohr radius):

1.4 $p_1 \approx a_0 e$

If the considered molecules are polarizable, this instantaneous dipole will polarize the neighboring atom, and consequently produce a dipole p_2 given by

1.5 $p_2 \approx \alpha \frac{1}{4\pi\epsilon_0 r^3} a_0 e$

where α is the polarizability of the second atom, defined by

1.6 $\alpha \approx 4\pi\epsilon_0 a_0^3$

The two instantaneous dipoles p_1 and p_2 given by Eqs. 1.5 and 1.6 lead to an interaction potential described by Eq. 1.3:

1.7 $w(r) \approx \frac{1}{4\pi\epsilon_0}\frac{p_1 p_2}{r^3} \approx \frac{\alpha a_0^2 e^2}{(4\pi\epsilon_0)^2}\frac{1}{r^6} \div \frac{1}{r^6}$

This power law holds as far as the orientation (Keesom), the induction (Debye) and the dispersion (London) terms are concerned. Moreover, by assuming these interactions to be additive (= by assuming they do not depend on the surrounding molecules), these three terms can be regrouped:

1.8 $w(r) = \left(-\frac{K_K}{r^6}\right) + \left(-\frac{K_D}{r^6}\right) + \left(-\frac{K_L}{r^6}\right) = -\frac{K}{r^6}$

The so-called retardation effect occurs when the separation distance between the instantaneous dipole and the induced dipole increases over a cut-off length of the order of 5–10 nm: In this case, the traveling time of the electromagnetic wave from the instantaneous dipole and the induced dipole become bigger and, consequently, both dipoles lose their coherence, leading to an energy reduction. The decrease with the separation distance occurs faster and it is assumed that it can be described according to

1.9 $w(r) = -\frac{K_R}{r^7}$

The fast decrease of the van der Waals forces explains that they seem to be limited to the atomic domain.

Nevertheless, this decrease occurs more slowly when we consider the interaction between two macroscopic bodies (i.e., a body with a very large number of molecules, including bodies that have a size in the order of a few micrometers and that are consequently considered microcomponents when dealing with microassembly terminology). Therefore, it is not so obvious to choose whether these forces have to be dealt with or not.

Let us now have a look on the ways to compute the van der Waals interaction between two macroscopic bodies: The first one is known as the microscopic or Hamaker approach, and the second one is called the macroscopic or Lifshitz approach. From a strictly theoretical point of view, the van der Waals forces are nonadditive, nonisotropic, and retarded. However, London (60) proposed a straight and powerful way to establish the potential interaction by assuming a pairwise additivity of the interactions. Moreover, this approach does not consider the retardation effect. The results are therefore limited to separation distances between an upper limit of about 5–10 nm (because we neglect the retardation effect) and a lower limit of about one intermolecular distance [because Eq. 1.2 that $l << r$. This lower boundary is reinforced by the value of the equilibrium distance (about 0.1–0.2 nm) arising from the Lennard-Jones potential: for separation distances smaller than 0.1–0.2 nm, very strong repulsive forces occur that can no longer be neglected]. This lower limit is sometimes called the van der Waals radius (38). We should keep in mind that even with these restrictions, the results are not exactly correct for the interaction of solids and liquids because of the pairwise summation assumption. However, Israelachvili (39) and Russel et al. (78) consider that these approximations are useful in several applications. We will illustrate this method in what follows.

The Lifshitz method, also called macroscopic approach, consists in considering the two interacting objects as continuous media with a dielectric response to electromagnetic fields. The dispersion forces are then considered the mutual interaction of dipoles oscillating at a given frequency. When the separation distance becomes bigger than a cut-off length depending on this frequency and the light speed, the attraction tends to decrease because the propagation time becomes of the same order as the oscillation period of the dipoles, the field emitted by one dipole interacting with another dipole with a different phase. This effect has first been pointed out by Casimir and Polder (15) and computed by Lifhitz using the quantum field theory (59). Although this approach is of the greatest complexity, similar results can be obtained by using the Hamaker results, on the condition to replace the Hamaker constant by a pseudoconstant involving more parameters. This method is out of our scope, which is to roughly evaluate the importance of the van der Waals forces in microassembly and to investigate the influence of geometry, roughness, and orientation on the manipulation of microcomponents. We will therefore limit ourselves to the Hamaker method, despite its limitations. The interested reader will find further information about the Lifshitz approach in Adamson and Gast (1), Chapter VI, and in Israelachvili (39).

We present the Hamaker method to calculate the van der Waals forces in the case of the interaction between two spheres, a sphere and a infinite half-space, an infinite half-space limited by a smooth plane, and a rectangular box that has faces that are parallel or perpendicular to that plane. This last example is a good introduction for taking into account the influence of roughness. These results have been published in Lambert and Régnier (53).

In each case the Hamaker method consists in first determining the interaction potential W between two macroscopic objects [while $w(r)$ denotes the potential interaction between microscopic dipoles] and then in deriving it with respect to the separation distance D ($F = -dW/dD$).

1.2.1.1 Interaction Potential Between a Sphere and a Volume Element

The interaction potential $W_{(S,\ dV)}$ between a sphere S [center O, radius R, number density n_1 (m^3), volume element $d\Omega$] and a volume element dV (number density n_2) located at a distance D from the sphere is given by (see Fig. 1.2)

$$W_{(S,dV)} = -Kn_1n_2dV\int_{\Omega}\frac{1}{d^6}d\Omega \quad \textbf{1.10}$$

where d is the distance between dV and the volume element $d\Omega$ of S. Let us choose a spherical coordinate frame centered in O and a polar axis linking O and dV: Consequently, $d\Omega$ is located in the sphere by its distance r from O and the angle θ (the problem is symmetric as far as the azimutal angle ϕ is concerned). As a consequence, d is given by

$$d^2 = (D+R)^2 + r^2 + 2r(D+R)\cos\ \theta \quad \textbf{1.11}$$

and if we note $x = D + R$ the integral of Eq. 1.10 can be rewritten into

$$\int_{\Omega}\frac{1}{d^6}d\Omega = \int_0^{2\pi}d\phi\int_0^{\pi}d\theta\int_0^{R}\frac{r^2\ \sin\ \theta}{(r^2+x^2+2rx\ \cos\ \theta)^3}\,dr \quad \textbf{1.12}$$

$$= 2\pi\int_0^{R}dr\int_0^{\pi}\frac{r^2\ \sin\ \theta}{(r^2+x^2+2rx\ \cos\ \theta)^3}\,d\theta \quad \textbf{1.13}$$

The integral with respect to θ can be solved by assuming $\cos\theta = u$ (and thus $-\sin\theta\ d\theta = du$), leading to