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Sixth Edition

FINANCIAL RISK MANAGER HANDBOOK

FRM® PART I / PART II

Philippe Jorion

GARP | Global Association of Risk Professionals

Financial Risk Manager Handbook Plus Test Bank

Sixth Edition

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Contents

Preface

The *Financial Risk Manager Handbook Plus Test Bank* provides the core body of knowledge for financial risk managers. Risk management has rapidly evolved over the past decade and has become an indispensable function in many institutions.

This *Handbook* was originally written to provide support for candidates taking the FRM examination administered by GARP. As such, it reviews a wide variety of practical topics in a consistent and systematic fashion. It covers quantitative methods, major financial products, as well as market, credit, operational, and integrated risk management. It also discusses investment risk management issues essential for risk professionals.

This edition has been thoroughly updated to reflect recent developments in financial markets and changes in the structure of the FRM program. The book is now structured to correspond to the two levels of the FRM exams. All of the chapters have been updated to account for recent developments in financial markets and regulations. In particular, current issues are integrated in the second part of the book. New chapters have been added, including chapters that deal with advanced univariate and multivariate models, as well as advanced option models. Finally, this *Handbook* incorporates the latest questions from the FRM examinations.

Modern risk management systems cut across the entire organization. This breadth is reflected in the subjects covered in this *Handbook*. The *Handbook* was designed to be self-contained, but only for readers who already have some exposure to financial markets. To reap maximum benefit from this book, readers ideally should have taken the equivalent of an MBA-level class on investments.

Finally, I want to acknowledge the help received in writing this *Handbook*. In particular, I would like to thank the numerous readers who shared comments on previous editions. Any comment or suggestion for improvement will be welcome. This feedback will help us to maintain the high quality of the FRM designation.

Philippe Jorion
October 2010

About the Author

Philippe Jorion is a Professor of Finance at the Paul Merage School of Business at the University of California at Irvine. He has also been a professor at Columbia University, Northwestern University, the University of Chicago, and the University of British Columbia. In addition, he taught the risk management class in the Master of Financial Engineering programs at the University of California at Berkeley and University of California at Los Angeles. He holds an M.B.A. and a Ph.D. from the University of Chicago and a degree in engineering from the University of Brussels.

Dr. Jorion is also a managing director at Pacific Alternative Asset Management Company (PAAMCO), a global fund of hedge funds with approximately $10 billion under management. PAAMCO is one of the few funds of funds to require position-level transparency from all invested hedge funds. This information is used to provide various measures of portfolio risk as well as to develop tools that help investors to understand the drivers of the funds' alpha and to detect style drift.

Dr. Jorion is the author of more than 100 publications directed to academics and practitioners on the topics of risk management and international finance. He has also written a number of books, including *Big Bets Gone Bad: Derivatives and Bankruptcy in Orange County*, the first account of the largest municipal failure in U.S. history, and *Value at Risk: The New Benchmark for Managing Financial Risk*, which is aimed at finance practitioners and has become an industry standard.

Philippe Jorion is a frequent speaker at academic and professional conferences. He is on the editorial board of a number of finance journals and was editor in chief of the *Journal of Risk*.

About GARP

Founded in 1996, the Global Association of Risk Professionals (GARP) is the leading not-for-profit association for world-class financial risk certification, education, and training, with close to 100,000 members representing 167 countries. With deep expertise and a strong reputation, GARP sets global standards and creates risk management programs valued worldwide. All GARP programs are developed with input from experts around the world to ensure that concepts and content reflect globally accepted practices.

GARP is dedicated to advancing the risk profession. For more information about GARP, please visit www.garp.com.

FINANCIAL RISK MANAGER (FRM®) CERTIFICATION

The benchmark FRM designation is the globally accepted risk management certification for financial risk professionals. The FRM objectively measures competency in the risk management profession based on globally accepted standards. With a compound annual growth rate of 25% over the past seven years, the FRM program has experienced significant growth in every financial center around the world. Now 16,000+ individuals hold the FRM designation in over 90 countries. In addition, organizations with five or more FRM registrants grew from 105 in 2003 to 424 in 2008, further demonstrating the FRM program's global acceptance.

The FRM Continuing Professional Education (CPE) program, offered exclusively for certified FRM holders, provides the perspective and framework needed to further develop competencies in the ever-evolving field of risk management.

For more information about the FRM program, please visit www.garp.com/frmexam.

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The ICBRR allows individuals to expand their knowledge and understanding of the various risks, regulations, and supervisory requirements banks must face in today's economy, with emphasis on the Basel II Accord. This certificate is ideal for employees who are not professional risk managers but who have a strong need to understand risk concepts. The ICBRR program is designed for employees in nonrisk departments such as internal audit, accounting, information technology

(IT), legal, compliance, and sales, acknowledging that everyone in the organization is a risk manager!

Energy Risk Professional Program

The Energy Risk Professional (ERP®) program is designed to measure a candidate's knowledge of the major energy markets and gauge their ability to manage the physical and financial risks inherent in the complex world of energy. This program is valuable for anyone working in or servicing the energy field, requiring an understanding of the physical and financial markets, how they interrelate, and the risks involved.

GARP DIGITAL LIBRARY

As the world's largest digital library dedicated to financial risk management, the GARP Digital Library (GDL) is the hub for risk management education and research material. The library's unique iReadings™ allow users to download individual chapters of books, saving both time and money. There are over 1,000 readings available from 12 different publishers. The GDL collection offers readings to meet the needs of anyone interested in risk management.

For more information, please visit www.garpdigitallibrary.org.

GARP EVENTS AND NETWORKING

GARP hosts major conventions throughout the world, where risk professionals come together to share knowledge, network, and learn from leading experts in the field. Conventions are bookended with interactive workshops that provide practical insights and case studies presented by the industry's leading practitioners.

GARP regional chapters provide an opportunity for financial risk professionals to network and share new trends and discoveries in risk management. Each of our 52 chapters holds several meetings each year, in some locations more often, focusing on issues of importance to the risk management community, either globally or locally.

Introduction

GARP's formal mission is to be the leading professional association for financial risk managers, managed by and for its members and dedicated to the advancement of the risk profession through education, training, and the promotion of best practices globally. As a part of delivering on that mission, GARP has again teamed with Philippe Jorion to produce the sixth edition of the *Financial Risk Manager Handbook Plus Test Bank*.

The *Handbook* follows GARP's FRM Committee's published FRM Study Guide, which sets forth primary topics and subtopics covered in the FRM exam. The topics are selected by the FRM Committee as being representative of the theories and concepts utilized by risk management professionals as they address current issues.

Over the years the Study Guide has taken on an importance far exceeding its initial intent of providing guidance for FRM candidates. The Study Guide is now being used by universities, educators, and executives around the world to develop graduate-level business and finance courses, as a reference list for purchasing new readings for personal and professional libraries, as an objective outline to assess an employee's or job applicant's risk management qualifications, and as guidance on the important trends currently affecting the financial risk management profession.

Given the expanded and dramatically growing recognition of the financial risk management profession globally, the *Handbook* has similarly assumed a natural and advanced role beyond its original purpose. It has now become the primary reference manual for risk professionals, academicians, and executives around the world. Professional risk managers must be well versed in a wide variety of risk-related concepts and theories, and must also keep themselves up-to-date with a rapidly changing marketplace. The *Handbook* is designed to allow them to do just that. It provides a financial risk management practitioner with the latest thinking and approaches to financial risk-related issues. It also provides coverage of advanced topics with questions and tutorials to enhance the reader's learning experience.

This sixth edition of the *Handbook* includes revised coverage of the primary topic areas covered by the FRM examination. Importantly, this edition also includes the latest lessons from the recent credit crisis, as well as new and more recent sample FRM questions.

The *Handbook* continues to keep pace with the dynamic financial risk profession while simultaneously offering serious risk professionals an excellent and cost-effective tool to keep abreast of the latest issues affecting the global risk management community.

Developing credibility and global acceptance for a professional certification program is a lengthy and complicated process. When GARP first administered its FRM exam in 1997, the concept of a professional risk manager and a global certification relating to that person's skill set was more theory than reality. That has now completely changed, as the number of current FRM holders exceeds 16,000.

The FRM is now the benchmark for a financial risk manager anywhere around the world. Professional risk managers having earned the FRM credential are globally recognized as having achieved a level of professional competency and a demonstrated ability to dynamically measure and manage financial risk in a real-world setting in accordance with global standards.

GARP is proud to continue to make this *Handbook* available to financial risk professionals around the world. Philippe Jorion, a preeminent risk professional, has again compiled an exceptional reference book. Supplemented by an interactive Test Bank, this *Handbook* is a requirement for any risk professional's library.

The Test Bank is a preparatory review for anyone studying for the FRM exam and for risk professionals interested in self-study to review and improve their knowledge of market, credit, and operational risk management. The Test Bank contains hundreds of multiple-choice questions from the 2007, 2008, and 2009 FRM exams, with answers and solutions provided. The Test Bank can be downloaded following the instructions on the FRM® Test Bank Download page at the end of this book.

Global Association of Risk Professionals
October 2010

PART

One

Foundations of Risk Management

CHAPTER 1

Risk Management

Financial **risk management** is the process by which financial risks are identified, assessed, measured, and managed in order to create economic value.

Some risks can be measured reasonably well. For those, risk can be quantified using statistical tools to generate a probability distribution of profits and losses. Other risks are not amenable to formal measurement but are nonetheless important. The function of the risk manager is to evaluate financial risks using both quantitative tools and judgment.

As financial markets have expanded over recent decades, the risk management function has become more important. Risk can never be entirely avoided. More generally, the goal is not to minimize risk; it is to take smart risks.

Risk that can be measured can be managed better. Investors assume risk only because they expect to be compensated for it in the form of higher returns. To decide how to balance risk against return, however, requires risk measurement.

Centralized risk management tools such as value at risk (VAR) were developed in the early 1990s. They combine two main ideas. The first is that risk should be measured at the top level of the institution or the portfolio. This idea is not new. It was developed by Harry Markowitz (1952), who emphasized the importance of measuring risk in a total portfolio context.[1] A centralized risk measure properly accounts for hedging and diversification effects. It also reflects the fact that equity is a common capital buffer to absorb all risks. The second idea is that risk should be measured on a forward-looking basis, using the current positions.

This chapter gives an overview of the foundations of risk management. Section 1.1 provides an introduction to the risk measurement process, using an illustration. Next, Section 1.2 discusses how to evaluate the quality of risk management processes. Section 1.3 then turns to the integration of risk measurement with business decisions, which is a portfolio construction problem. These portfolio decisions can be aggregated across investors, leading to asset pricing theories that can be used as yardsticks for performance evaluation and for judging risk management and are covered in Section 1.4. Finally, Section 1.5 discusses how risk management can add economic value.

FRM Exam Part 1 topic. In addition to the topics described in this chapter, FRM candidates should also read the GARP Code of Conduct.

[1] Harry Markowitz, "Portfolio Selection," *Journal of Finance* 7 (1952): 77–91.

1.1 RISK MEASUREMENT

1.1.1 Example

The first step in risk management is the measurement of risk. To illustrate, consider a portfolio with $100 million invested in U.S. equities. Presumably, the investor undertook the position because of an expectation for profit, or investment growth. This portfolio is also risky, however.

The key issue is whether the expected profit for this portfolio warrants the assumed risk. Thus a trade-off is involved, as in most economic problems. To help answer this question, the risk manager should construct the distribution of potential profits and losses on this investment. This shows how much the portfolio can lose, thus enabling the investor to make investment decisions.

Define ΔP as the profit or loss for the portfolio over a fixed horizon, say the coming month. This must be measured in a *risk currency*, such as the dollar. This is also the product of the initial investment value P and the future rate of return R_P. The latter is a random variable, which should be described using its **probability density function**. Using historical data over a long period, for example, the risk manager produces Figure 1.1.

This graph is based on the actual distribution of total returns on the S&P 500 index since 1925. The line is a smoothed histogram and does not assume a simplified model such as the normal distribution.

The vertical axis represents the frequency, or probability, of a gain or loss of a size indicated on the horizontal axis. The entire area under the curve covers all of the possible realizations, so should add up to a total probability of 1.

Most of the weight is in the center of the distribution. This shows that it is most likely that the return will be small, whether positive or negative. The tails have less weight, indicating that large returns are less likely. This is a typical characteristic of returns on financial assets. So far, this pattern resembles the bell-shaped curve for a normal distribution.

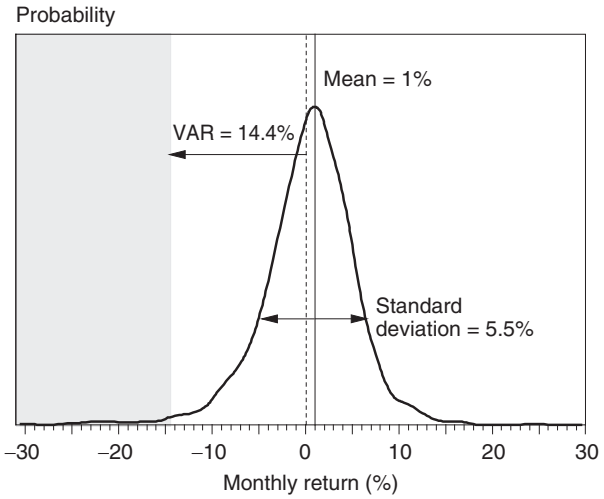

FIGURE 1.1 Distribution of Monthly Returns on U.S. Stocks

On the downside, however, there is a substantial probability of losing 10% or more in a month. This cumulative probability is 3%, meaning that in a repeated sample with 100 months, we should expect to lose 10% or more for a total of three months. This risk is worse than predicted by a normal distribution.

If this risk is too large for the investor, then some money should be allocated to cash. Of course, this comes at the expense of lower expected returns.

The distribution can be characterized in several ways. The entire shape is most informative because it could reveal a greater propensity to large losses than to gains. The distribution could be described by just a few summary statistics, keeping in mind that this is an oversimplification. Other chapters offer formal definitions of these statistics.

- The *mean*, or average return, which is approximately 1% per month. Define this as $\mu(R_P)$, or μ_P in short, or even μ when there is no other asset.
- The *standard deviation*, which is approximately 5.5%. This is often called volatility and is a measure of dispersion around the mean. Define this as σ. This is the square root of the portfolio variance, σ^2.
- The *value at risk* (VAR), which is the cutoff point such that there is a low probability of a greater loss. This is also the percentile of the distribution. Using a 99% confidence level, for example, we find a VAR of 14.4%.

1.1.2 Absolute versus Relative Risk

So far, we have assumed that risk is measured by the dispersion of dollar returns, or in absolute terms. In some cases, however, risk should be measured relative to some **benchmark**. For example, the performance of an active manager is compared to that of an index such as the S&P 500 index for U.S. equities. Alternatively, an investor may have future liabilities, in which case the benchmark is an index of the present value of liabilities. An investor may also want to measure returns after accounting for the effect of inflation. In all of these cases, the investor is concerned with **relative risk**.

- **Absolute risk** is measured in terms of shortfall relative to the initial value of the investment, or perhaps an investment in cash. Using the standard deviation as the risk measure, absolute risk in dollar terms is

$$\sigma(\Delta P) = \sigma(\Delta P/P) \times P = \sigma(R_P) \times P \tag{1.1}$$

- **Relative risk** is measured relative to a benchmark index B. The deviation is $e = R_P - R_B$, which is also known as the **tracking error**. In dollar terms, this is $e \times P$. The risk is

$$\sigma(e)P = [\sigma(R_P - R_B)] \times P = \omega \times P \tag{1.2}$$

where ω is called **tracking error volatility** (TEV).

To compare these two approaches, take the case of an active equity portfolio manager who is given the task of beating a benchmark. In the first year, the active portfolio returns −6% but the benchmark drops by −10%. So, the excess return is positive: $e = -6\% - (-10\%) = 4\%$. In relative terms, the portfolio has done well even though the absolute performance is negative. In the second year, the portfolio returns +6%, which is good using absolute measures, but not so good if the benchmark goes up by +10%.

EXAMPLE 1.1: ABSOLUTE AND RELATIVE RISK

An investment manager is given the task of beating a benchmark. Hence the risk should be measured in terms of

a. Loss relative to the initial investment
b. Loss relative to the expected portfolio value
c. Loss relative to the benchmark
d. Loss attributed to the benchmark

1.2 EVALUATION OF THE RISK MEASUREMENT PROCESS

A major function of the risk measurement process is to estimate the distribution of future profits and losses. The first part of this assignment is easy. The scale of the dollar returns should be proportional to the initial investment. In other words, given the distribution in Figure 1.1, an investment of $100 million should have a standard deviation of $\sigma(\Delta P) = \$100 \times 5.5\% = \$5.5$ million. Scaling the current position by a factor of 2 should increase this risk to $11 million.

The second part of the assignment, which consists of constructing the distribution of future rates of return, is much harder. In Figure 1.1, we have taken the historical distribution and assumed that this provides a good representation of future risks. Because we have a long history of returns over many different cycles, this is a reasonable approach.

This is not always the case, however. The return may have been constant over its recent history. This does not mean that it could not change in the future. For example, the price of gold was fixed to $35 per ounce from 1934 to 1967 by the U.S. government. As a result, using a historical distribution over the 30 years ending in 1967 would have shown no risk. Instead, gold prices started to fluctuate wildly thereafter. By 2008, gold prices had reached $1,000. Thus, the responsibility of the risk manager is to judge whether the history is directly relevant.

How do we evaluate the quality of a risk measurement process? The occurrence of a large loss does not mean that risk management has failed. This could be simply due to bad luck. An investment in stocks would have lost 17% in October 2008. While this is a grievous loss, Figure 1.1 shows that it was not inconceivable. For

example, the stock market lost 30% in September 1931 and 22% on October 19, 1987, before recovering. So, the risk manager could have done a perfect job of forecasting the distribution of returns. How can we tell whether this loss is due to bad luck or a flaw in the risk model?

1.2.1 Known Knowns

To help answer this question, it is useful to classify risks into various categories, which we can call (1) known knowns, (2) known unknowns, and (3) unknown unknowns.[2] The first category consists of risks that are properly identified and measured, as in the example of the position in stocks. Losses can still occur due to a combination of bad luck and portfolio decisions.

Such losses, however, should not happen too often. Suppose that VAR at the 99% level of confidence is reported as 14.4%. Under these conditions, a string of consecutive losses of 15% or more several months in a row should be highly unusual. If this were to happen, it would be an indication of a flawed model. A later chapter will show how backtesting can be used to detect flaws in risk measurement systems.

1.2.2 Known Unknowns

The second category, called known unknowns, includes model weaknesses that are known or should be known to exist but are not properly measured by risk managers. For example, the risk manager could have ignored important known risk factors. Second, the distribution of risk factors, including volatilities and correlations, could be measured inaccurately. Third, the mapping process, which consists of replacing positions with exposures on the risk factors, could be incorrect. This is typically called **model risk**. Such risks can be evaluated using stress tests, which shock financial variables or models beyond typical ranges.

As an example, consider the $19 billion loss suffered by UBS in 2007 alone from positions in structured credit securities backed by subprime and Alt-A mortgage-backed loans.[3] UBS had invested in top-rated tranches that the bank thought were perfectly safe (yet yielded high returns). As a result, it had accumulated a position of $90 billion in exposures to these securities, compared to $41 billion in book equity. The bank reported that its risk measurement process relied on simplified models based on a recent period of positive growth in housing prices. As in the example of gold, the recent history gave a biased view of the true risks. In addition, UBS's risk managers overrelied on ratings provided by the credit rating agencies. Because risk management gave little indication of the downside

[2] Philippe Jorion, "Risk Management Lessons from the Credit Crisis," *European Financial Management* 15 (2009): 923–933.

[3] See UBS, *Shareholder Report on UBS's Write-Downs* (Zurich: UBS, 2008). Loans can be classified into prime, Alt-A, and subprime, in order of decreasing credit quality. Subprime loans are loans made to consumers with low credit scores (typically below 640 out of a possible maximum of 850). Alt-A loans, short for Alternative A-paper, are the next category (typically with credit scores below 680 or for loans lacking full documentation). Subprime and Alt-A mortgage loans are expected to have higher credit risk than other (prime) loans.

risk of these investments, these losses can be viewed as a failure of risk management. Even so, the UBS report indicates that the growth strategy undertaken by top management was a "contributing factor to the buildup of UBS's subprime positions which subsequently incurred losses." In other words, top management was largely responsible for the losses.

Another form of known unknown is **liquidity risk**. Most risk models assume that the position can be liquidated over the selected horizon. In practice, this depends on a number of factors. First is the intrinsic liquidity of the asset. Treasury bills, for instance, are much more liquid than high-yield bonds. They trade at a lower spread and with less market impact. Second is the size of the position. This is especially a problem when the position is very large relative to normal trading activity, which would require accepting a large price drop to execute the trade.

1.2.3 Unknown Unknowns

The risks in the last category tend to be the difficult ones. They represent events totally outside the scope of most scenarios. Examples include regulatory risks such as the sudden restrictions on short sales, which can play havoc with hedging strategies, or structural changes such as the conversion of investment banks to commercial banks, which accelerated the deleveraging of the industry. Indeed, a 2010 survey reports that the top concern of risk managers is "government changing the rules."[4]

Similarly, it is difficult to account fully for counterparty risk. It is not enough to know your counterparty; you need to know your counterparty's counterparties, too. In other words, there are network externalities. Understanding the full consequences of Lehman's failure, for example, would have required information on the entire topology of the financial network.[5] Because no individual firm has access to this information, this contagion risk cannot be measured directly.

Similarly, some form of liquidity risk is very difficult to assess. This involves the activity and positions of similar traders, which are generally unknown. In illiquid markets, a forced sale will be much more expensive if a number of similar portfolios are sold at the same time.

This category is sometimes called Knightian **uncertainty**, a form of risk that is immeasurable. Financial institutions cannot possibly carry enough capital to withstand massive counterparty failures, or systemic risk. In such situations, the central bank or the government becomes effectively the risk manager of last resort.

1.2.4 Risk Management Failures

More generally, the role of risk management involves several tasks:

- Identifying all risks faced by the firm
- Assessing and monitoring those risks

[4] *Risk Governance: A Benchmarking Survey* (New York: Capital Markets Risk Advisors, 2010).
[5] A. Haldane, *Why Banks Failed the Stress Test* (London: Bank of England, 2009).

- Managing those risks if given the authority to do so
- Communicating these risks to the decision makers

A large loss is not necessarily an indication of a risk management failure. It could have been within the scope of known knowns and properly communicated to the firm, in which case it reflects bad luck. After all, the objective of risk management is not to prevent losses.

Otherwise, risk management can fail if any of these tasks has not been met. Some risks could go unrecognized. Mismeasurement of risk can occur due to model risk, due to liquidity risk, or if distributions are not adequately measured. Risk limits could not have been enforced. Finally, risk management fails when it does not communicate risks effectively.

EXAMPLE 1.2: FRM EXAM 2009—QUESTION 1-11

Based on the risk assessment of the CRO, Bank United's CEO decided to make a large investment in a levered portfolio of CDOs. The CRO had estimated that the portfolio had a 1% chance of losing $1 billion or more over one year, a loss that would make the bank insolvent. At the end of the first year the portfolio has lost $2 billion and the bank was closed by regulators.

Which of the following statements is correct?

a. The outcome demonstrates a risk management failure because the bank did not eliminate the possibility of financial distress.
b. The outcome demonstrates a risk management failure because the fact that an extremely unlikely outcome occurred means that the probability of the outcome was poorly estimated.
c. The outcome demonstrates a risk management failure because the CRO failed to go to regulators to stop the shutdown.
d. Based on the information provided, one cannot determine whether it was a risk management failure.

1.3 PORTFOLIO CONSTRUCTION

1.3.1 Comparing Multiple Assets

We now turn to the portfolio construction process, which involves combining expected return and risk. Assume that another choice is to invest in long-term U.S. government bonds.

Over the same period, the monthly average return of this other asset class was 0.47%. This is half that of equities. The monthly standard deviation was 2.3%,

TABLE 1.1 Risk and Expected Return on Two Assets

	Average	Volatility	Correlation
Equities	11.2%	19.2%	
Long-term bonds	5.6%	8.1%	0.13

again lower than for equities. To make the numbers more intuitive, monthly returns have been converted to annualized terms, as shown in Table 1.1.

Here, our investor is faced with a typical trade-off, which is to choose between these two alternatives. Neither dominates the other, as shown in Figure 1.2.

This graph describes a simple investment decision. More generally, it also represents more complex business decisions that involve risk. For instance, a bank must decide how much leverage to assume, as defined by the amount of assets divided by the amount of equity on its balance sheet. The horizontal axis could then represent the bank's credit rating. On one hand, higher leverage involves higher risk and accordingly a lower credit rating. In Figure 1.2, this corresponds to a move to the right. On the other hand, higher leverage means that the expected return to equity should be higher. This is because the amount of equity on the balance sheet is lower, implying that profits will be distributed to a smaller equity base. In Figure 1.2, this corresponds to a move up. Again, we observe a trade-off between higher risk and higher return. Without risk measures, deciding where to invest would be difficult.

1.3.2 Risk-Adjusted Performance Measurement

The next question is how the performance can be adjusted for risk in a single measure. The same methods apply to past performance, using historical averages, or prospective performance, using projected numbers.

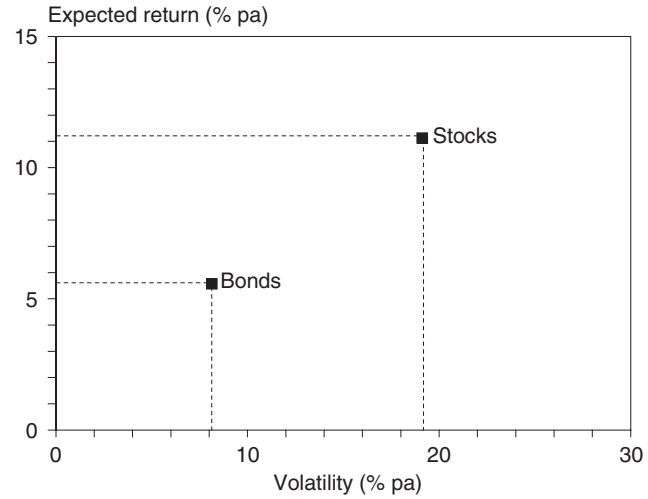

FIGURE 1.2 Comparing Risk and Expected Return

The simplest metric is the **Sharpe ratio** (SR), which is the ratio of the average rate of return, $\mu(R_P)$, in excess of the risk-free rate R_F, to the absolute risk:

$$\text{SR} = \frac{[\mu(R_P) - R_F]}{\sigma(R_P)} \tag{1.3}$$

The Sharpe ratio focuses on total risk measured in absolute terms. This approach can be extended to include VAR in the denominator instead of the volatility of returns.

Figure 1.3 compares the SR for the two investment choices. Assume that we have a risk-free asset, cash, with a return of 3%. The SR is the slope of the line from cash to each asset. This line represents a portfolio mix between cash and each asset. In this case, stocks have a higher SR than bonds. This means that a mix of cash and stocks could be chosen with the same volatility as bonds but with higher returns.

This can be extended to relative risk measures. The **information ratio** (IR) is the ratio of the average rate of return of portfolio P in excess of the benchmark B to the TEV:

$$\text{IR} = \frac{[\mu(R_P) - \mu(R_B)]}{\sigma(R_P - R_B)} \tag{1.4}$$

Table 1.2 presents an illustration. The risk-free interest rate is $R_F = 3\%$ and the portfolio average return is -6%, with volatility of 25%. Hence, the Sharpe ratio of the portfolio is $\text{SR} = [(-6\%) - (3\%)]/25\% = -0.36$. Because this is negative, the absolute performance is poor.

Assume now that the benchmark returned -10% over the same period and that the tracking error volatility was 8%. Hence, the information ratio is $\text{IR} = [(-6\%) - (-10\%)]/8\% = 0.50$, which is positive. The relative performance is good even though the absolute performance is poor.

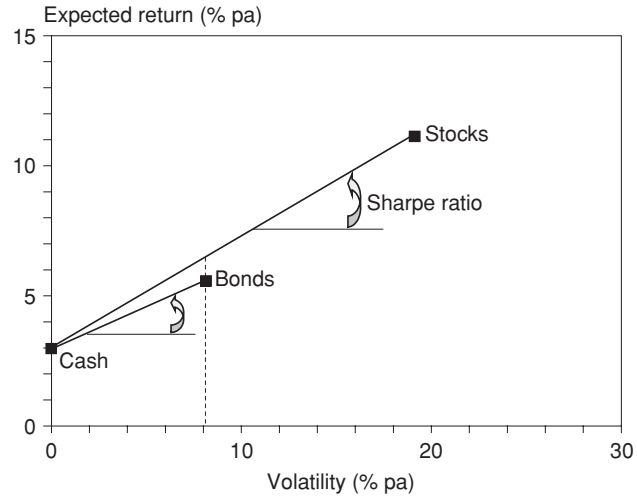

FIGURE 1.3 Comparing Sharpe Ratios

TABLE 1.2 Absolute and Relative Performance

	Average	Volatility	Performance
Cash	3%	0%	
Portfolio P	−6%	25%	$SR = -0.36$
Benchmark B	−10%	20%	$SR = -0.65$
Deviation e	4%	8%	$IR = 0.50$

The tracking error volatility can be derived from the volatilities σ_P and σ_B of the portfolio and the benchmark as well as their correlation ρ. Chapter 2 shows that the variance of a sum of random variables can be expressed in terms of the sum of the individual variances plus twice a covariance term. In terms of difference, the variance is

$$\omega^2 = \sigma_P^2 - 2\rho\sigma_P\sigma_B + \sigma_B^2 \tag{1.5}$$

In this case, if $\sigma_P = 25\%$, $\sigma_B = 20\%$, and $\rho = 0.961$, we have $\omega^2 = 25\%^2 - 2 \times 0.961 \times 25\% \times 20\% + 20\%^2 = 0.0064$, giving $\omega = 8\%$.

The IR has become commonly used to compare active managers in the same peer group. It is a pure measure of active management skill that is scaled for active risk. Consider, for example, two managers. Manager A has TEV of 2% per annum and excess return of 1%. Manager B has TEV of 6% per annum and excess return of 2%. Manager A has lower excess return but a higher information ratio, $1/2 = 0.50$, vs. $2/6 = 0.33$. As a result, it has better management skills. For example, Manager A could be asked to amplify its tracking error by a factor of 3, which would lead to an excess return of 3%, thus beating Manager B with the same level of tracking error of 6%. An information ratio of 0.50 is typical of the performance of the top 25th percentile of money managers and is considered "good."[6]

One of the drawbacks of the information ratio is that the TEV does not adjust for average returns. For instance, a portfolio could be systematically above its benchmark by 0.10% per month. In this case, the tracking error has an average of 0.10% and a standard deviation close to zero. This leads to a very high information ratio, which is not realistic if the active risk cannot be scaled easily.

1.3.3 Mixing Assets

The analysis has so far considered a discrete choice to invest in either asset. More generally, a portfolio can be divided between the two assets. Define w_i as the weight placed on asset i. With full investment, we must have $\sum_{i=1}^{N} w_i = 1$, where N is the total number of assets. In other words, the portfolio weights must sum to 1.

[6] Grinold, Richard and Ronald Kahn, *Active Portfolio Management* (New York: McGraw-Hill, 2000).