Multigrid Finite Element Methods for Electromagnetic Field Modeling

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IEEE Antennas and Propagation Society, *Sponsor*

IEEE Press Series on Electromagnetic Wave Theory
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To Jie and Helen, for being there when no one else is.

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PREFACE

The central theme of this book is the development of robust preconditioners for the iterative solution of electromagnetic field boundary value problems (BVPs) discretized by means of finite methods. More precisely, the book provides a detailed presentation of our successful attempts to utilize concepts from multigrid and multilevel methods for the preconditioning of matrices resulting from the approximation of electromagnetic BVPs using finite elements.

Solution by iteration is the reluctant compromise of modern scientific computing. A brief review of the young history of numerical computation offers unequivocal support for Edward Teller's bleak conjecture that "*A state-of-the-art calculation requires 100 hours of CPU time on the state-of-the-art computer, independent of the decade.*" Hidden in this statement is recognition and praise of our innate drive to advance our understanding of the physical world and, eventually, through its predictive modeling, establish our dominance over it. Yet, the deeper our understanding becomes the higher the complexity of the mysteries we need to unravel. And this complexity pushes our computing technology to its current limits.

In our desire to stay one step ahead of this seemingly perpetual complexity barrier, iteration seems to be an invaluable ally. Loosely speaking an iterative process should be interpreted as the attempt to solve a given problem by trial and error. In the context of the theme of this book, the pertinent problem is the solution of the system of linear equations resulting from the discretization of the electromagnetic BVP using finite elements. A guess for the unknown vector is made and the error (residual) is calculated between the forcing vector and the vector resulting from the multiplication of the trial vector by the matrix. A convergent iterative process is one through which the residual is iteratively reduced below some a-priori defined threshold.

Thanks to continuing advances in iterative matrix-solving methods, we are now equipped with solid knowledge about the right ingredients for the construction of an effective iterative solver. Among them, the availability of a good preconditioner stands out. Effective preconditioning of the iteration matrix expedites convergence and can enhance solution accuracy. Multigrid and multilevel methods have been shown to be most effective as preconditioners. Their success stems from the fact that the convergence of standard relaxation techniques can be accelerated by utilizing different spatial samplings for the damping of different components in the decomposition of the error in terms of the eigenvectors of the iteration matrix. In the context of finite methods spatial sampling is most typically effected through the size of the element in the numerical grid or the order of the interpolating polynomial.

The construction of an effective multigrid/multilevel preconditioner requires the understanding of the properties of the eigenvectors and eigenvalues of the iteration matrix. This, in turn, calls for a thorough insight in the way the development of the finite element approximation of the boundary value problem maps the properties of the eigenpairs of the governing mathematical operator onto those of its finite element matrix approximation.

It was primarily due to lack of such insight and understanding that the application of multigrid/multilevel ideas to the development of effective preconditioners for finite element matrix approximations of vector electromagnetic BVPs was slow in coming. However, over the past ten years significant progress has been made toward filling this void. This book attempts to provide the reader with an elucidating description of the path that led us to today's state-of- the-art in multigrid/multilevel preconditioners for finite element-based iterative electromagnetic field solvers.

Chapter 1 begins with a brief overview of the finite element method and its application to the numerical solution of the electromagnetic (BVP). This overview is followed by a discussion of current and future challenges in finite element-based electromagnetic field modeling, which helps motivate the methodologies and algorithms presented in the following chapters.

Chapter 2 presents a complete and systematic development of spaces of functions suitable for the expansion of scalar potential fields, tangentially continuous vector fields, normally continuous vector fields, and scalar charge densities on the triangles and tetrahedra used for the discretization of the computational domain. The corresponding spaces for these four classes of quantities are often referred to as Whitney-0, Whitney-1, Whitney-2, and Whitney-3 forms. They are the natural choices for the representation of the scalar potential, the field intensities ($\vec{E}$ and $\vec{H}$), the flux densities ($\vec{D}$ and $\vec{B}$), and the scalar charge density, respectively. The reason for this is that their development is guided by their physical attributes, as described mathematically by the governing equations and associated boundary conditions. It is shown in Chapter 2 that explicit, hierarchical basis functions of arbitrary order can be developed for all four spaces. In addition, the mathematical relations of the four spaces are presented and discussed. The proposed basis functions can be used, either directly or after a further partial orthogonalization post-processing, to improve the condition number of the finite element matrix.

Chapter 3 is devoted to the development of the finite element formulations of various classes of electromagnetic BVPs. In particular, formulations pertinent to static, quasi-static, and dynamic problems are presented, along with their subsequent reduction into matrix equation statements of their discrete approximation using the Galerkin process. The emphasis of the chapter is on the appropriate choice of basis functions for the expansion of pertinent unknown field quantities and their sources, in a manner such that the requirements for solvability and uniqueness of the solution of the continuous problem are mapped correctly onto the finite element matrix approximation of the BVP. The chapter concludes with an investigation of the source of the so-called low-frequency numerical instability of integral equation-based approximations of electrodynamic problems, in the context of the

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