

# **Trigonometry Workbook**

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**by Mary Jane Sterling**

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## *About the Author*

Mary Jane Sterling is also the author of *Algebra For Dummies*, *Trigonometry For Dummies*, *Algebra Workbook For Dummies*, *Algebra I CliffStudySolver*, and *Algebra II CliffStudySolver* (all published by Wiley). She has taught at Bradley University in Peoria, Illinois, for over 25 years.





## *Dedication*

I would like to dedicate this book to my husband, Ted, for his good-natured patience and understanding during the tense times of this and other writing projects. I also dedicate this book to my three children — Jon, Jim, and Jane — who seem to get a kick out of having a mother who writes books about mathematics.

## *Author's Acknowledgments*

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## **Publisher's Acknowledgments**

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# Introduction

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**W**hat in the world is *trigonometry*? Well, for starters, trigonometry is *in* the world, *on* the world, and *above* the world — at least its uses are. Trigonometry started out as a practical way of finding out how far things are from one another when you can't measure them. Ancient mathematicians came up with a measure called an *angle*, and the rest is history.

So, what's my angle in this endeavor? (Pardon the pun.) I wanted to write this book because trigonometry just hasn't gotten enough attention lately. You can't do much navigating without trigonometry. You can't build bridges or skyscrapers without trigonometry. Why has it been neglected as of late? It hasn't been ignored as much as it just hasn't been the center of attention. And that's a shame.

Trigonometry is about angles, sure. You can't do anything without knowing what the different angle measures do to the different trig functions. But trigonometry is also about relationships — just like some of these new reality television shows. Did I get your attention? These relationships are nearly as exciting as those on TV where they decide who gets to stay and who gets to leave. The sine gets to stay and the cosecant has to leave when you know the identities and rules and apply them correctly. Trigonometry allows you to do some pretty neat things with equations and mathematical statements. It's got the power.

Another neat thing about trigonometry is the way it uses algebra. In fact, algebra is a huge part of trigonometry. Thinking back to my school days, I think I learned more about the finesse of algebra when doing those trig identities than I did in my algebra classes. It all fits together so nicely.

Whatever your plans are for trigonometry, you'll find the rules, the hints, the practice, and the support in this book. Have at it.

## About This Book

This book is intended to cement your understanding — to give you the confidence that you do, indeed, know about a particular aspect of trigonometry. In each section, you'll find brief explanations of the concept. If that isn't enough, refer to your copy of *Trigonometry For Dummies*, your textbook, or some other trig resource. With the examples I give, you'll probably be ready to try out the problems for yourself and move on from there. The exercises are carefully selected to incorporate the different possibilities that come with each topic — the effect of different kinds of angles or factoring or trig functions.

## Conventions Used in This Book

Reading any book involving mathematics can have an added challenge if you aren't familiar with the conventions being used. The following conventions are used throughout the text to make things consistent and easy to understand:

- ✔ New terms appear in *italic* and are closely followed by an easy-to-understand definition.
- ✔ **Bold** is used to highlight the action parts of numbered steps. **Bold** is also used on the answers to the example and practice problems to make them easily identifiable.
- ✔ Numbers are either written out as words or given in their numerical form — whichever seems to fit at the moment and cause the least amount of confusion.
- ✔ The *variables* (things that stand for some number or numbers — usually unknown at first) are usually represented by letters at the end of the alphabet such as  $x$ ,  $y$ , and  $z$ . The *constants* (numbers that never change) are usually represented by letters at the beginning of the alphabet such as  $a$ ,  $b$ , or  $c$ , and also by two big favorites,  $k$  or  $\pi$ . In any case, the variables and constants are *italicized* for your benefit.
- ✔ Angle measures are indicated with the word *degrees* or the symbol for a degree,  $^\circ$ , or the word *radians*. The radian measures are usually given as numbers or multiples of  $\pi$ . If the angle measure is unknown, I use the variable  $x$  or, sometimes, the Greek letter  $\Theta$ .
- ✔ I use the traditional symbols for the mathematical operations: addition,  $+$ ; subtraction,  $-$ ; multiplication,  $\times$  or sometimes just a *dot* between values; and division,  $\div$  or sometimes a slash,  $/$ .

## Foolish Assumptions

We all make foolish assumptions at times, and here are mine concerning you:

- ✔ You have a basic knowledge of algebra and can solve simple linear and quadratic equations. If this isn't true, you may want to brush up a bit with *Algebra For Dummies* or a textbook.
- ✔ You aren't afraid of fractions. FOF (fear of fractions) is a debilitating but completely curable malady. You just need to understand how they work — and don't work — and not let them throw you.
- ✔ You have a scientific calculator (one that does powers and roots) available so you can approximate the values of radical expressions and do computations that are too big or small for paper and pencil.
- ✔ You want to improve your skills in trigonometry, practice up on those topics that you're a little rusty at, or impress your son/daughter/boyfriend/girlfriend/boss/soul mate with your knowledge and skill in trigonometry.

## How This Book Is Organized

This book is organized into parts. Trigonometry divides up nicely into these groupings or parts with similar topics falling together. You can identify the part that you want to go to and cover as much if not all of the section before moving on.

### *Part I: Trying Out Trig: Starting at the Beginning*

The study of trigonometry starts with angles and their measures. This is what makes trigonometry so different from other mathematical topics — you get to see what angle

measures can do. For starters, I describe, pull apart, and inspect angles in triangles and circles. You get intimate with the circle and some of its features; think of it as becoming well-rounded. (Sorry, I just couldn't resist.)

One of the best things about trigonometry is how visual its topics are. You get to look at pictures of angles, triangles, circles, and sketches depicting practical applications. One of the visuals is the coordinate plane. You plot points, compute distances and slopes, determine midpoints, and write equations that represent circles. This is preparation for determining the values of the trig functions in terms of angles that are all over the place — angles that have positive or negative, very small or very large, degree or radian measures.

And, saving the best for last, I cover the right triangles. These triangles start you out in terms of the trig functions and are very user-friendly when doing practical applications.

## ***Part II: Trigonometric Functions***

The trig functions are unique. These six basic functions take a simple little angle measure, chew on it a bit, and spit out a number. How do they do that? That's what you find out in the chapters in this part. Each function has its own particular definition and inner workings. Each function has special things about it in terms of what angles it can accept and what numerical values it produces. You start with the right triangle to formulate these functions, and then you branch out into all the angles that can be formed going 'round and 'round the circle.

## ***Part III: Trigonometric Identities and Equations***

The trigonometric identities are those special equivalences that the six trig functions are involved with. These identities allow you to change from one function to another for your convenience, or just because you want to. You'll find out what the identities are and what to do with them. Sometimes they help make a complex expression much simpler. Sometimes they make an equation more manageable — and solvable. (Believe it or not, some people actually like to solve trig identities just for the pure pleasure of conquering the algebraic and trigonometric challenge they afford.)

In this part, I introduce you to the inverse trig functions. They undo what the original trig function did. These inverse functions are very helpful when solving trig equations — equations that use algebra to find out which angles make the statement true.

And, last but not least, you'll find the Law of Sines and Law of Cosines in this part. These two laws or equations describe some relationships between the angles and sides of a triangle — and then use these properties to find a missing angle measure or missing side of the triangle. They're most handy when you can't quite fit a right triangle into the situation.

## ***Part IV: Graphing the Trigonometric Functions***

The trig functions are all recognizable by their graphs — or, they will be by the time you finish with this part. The characteristics of the functions — in terms of what angle measures they accept and what values they spew out — are depicted graphically. Pictures are very helpful when you're trying to convince someone else or yourself what's going on.

The graphs of the trig functions are transformed in all the ways possible — shoved around the coordinate system, stretched out, squashed, and flipped. I describe all these possibilities

with symbols and algebra and with the actual graph. Even if you're graphing functions with a graphing calculator, you really need to know what's going on so you can either decipher what's on your calculator screen or tell if what you have is right or wrong.

## Part V: The Part of Tens

This is one of my favorite parts of this book. Here I was able to introduce some information that just didn't fit in the other parts — stuff I wanted to show you and couldn't have otherwise. You'll find some identities that fit special situations and all have a connection with the minus sign. You'll find everything you've always wanted to know about a circle but were afraid to ask. And, finally, I explore and lay bare for all to see the relationships between the angles and sides of a triangle.

## Icons Used in This Book

To make this book easier to read and simpler to use, I include some icons that can help you find and fathom key ideas and information.



You'll find one or more examples with each section in this book. These are designed to cover the techniques and properties of the topic at hand. They get you started on doing the practice problems that follow. The solutions at the end of each chapter provide even more detail on how to solve those problems.



This icon appears when I'm thinking, "Oh, it would help if I could mention that. . . ." These situations occur when there's a particularly confusing or special or complicated step in a problem. I use this icon when I want to point out something to save you time and frustration.



Sometimes, when you're in the thick of things, recalling a particular rule or process that can ease your way is difficult. I use this icon when I'm mentioning something you'll want to try to remember, or when I'm reminding you of something I've covered already.



Do you remember the old *Star Trek* series in which the computer would say, "Warning, warning!" and alert Commander Kirk and the others? Think of this icon as being an alert to watch out for Klingons or any other nasty, tricky, or troublesome situation.

## Where to Go from Here

Where do you start? You can start anywhere you want. As with all *For Dummies* books, the design is with you in mind. You won't spoil the ending by doing those exercises, first. You can open to a random page or, more likely, look in the table of contents or index for that topic that's been bugging you. You don't have to start at the beginning and slog your way through. All through the book, I reference preceding and later chapters that either offer more explanation or a place for further discovery.

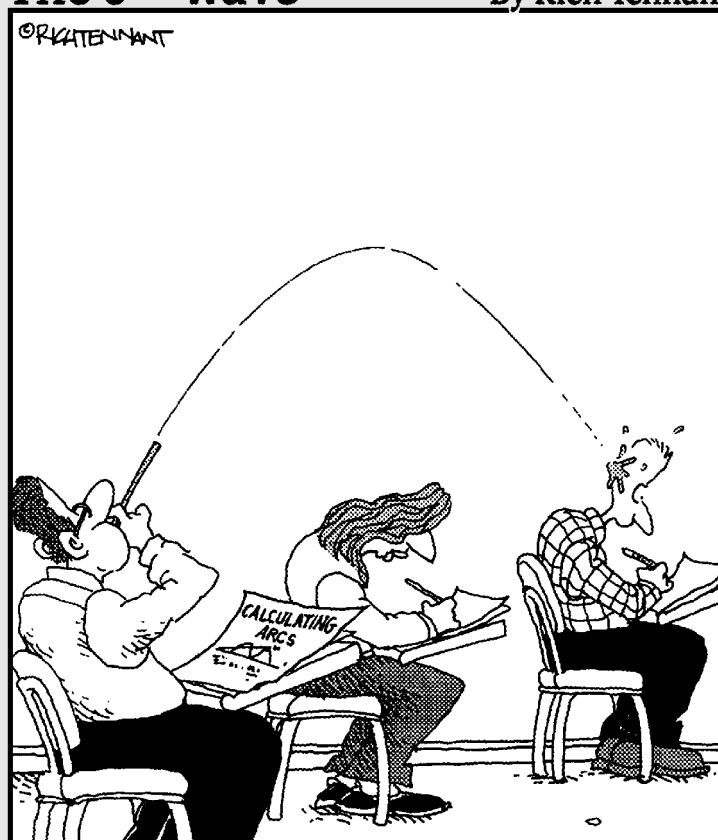
There's a great companion book to this workbook called, just by coincidence, *Trigonometry For Dummies*. It has more detail on the topics in this workbook, if you want to delve further into a topic or get something clarified.

# Part I

## Trying Out Trig: Starting at the Beginning

The 5th Wave

By Rich Tennant



## *In this part . . .*

**I** could've called this part FBI: Fabulous Basic Information. Here you find all you need to know about angles, triangles, and circles. You get to relate degrees to radians and back again. All the basics are here for you to start with, refer back to, or ignore — it's your choice.



## Chapter 1

# Tackling Technical Trig

### *In This Chapter*

- ▶ Acquainting yourself with angles
- ▶ Identifying angles in triangles
- ▶ Taking apart circles

**A**ngles are what trigonometry is all about. This is where it all started, way back when. Early astronomers needed a measure to tell something meaningful about the sun and moon and stars and their relationship between man standing on the earth or how they were positioned in relation to one another. Angles are the input values for the trig functions.

This chapter gives you background on how angles are measured, how they are named, and how they relate to one another in two familiar figures, including the triangle and circle. A lot of this material is terminology. The words describe things very specific, but this is a good thing, because they're consistent in trigonometry and other mathematics.

## *Getting Angles Labeled by Size*

An angle is formed where two *rays* (straight objects with an endpoint that go on forever in one direction) have a common endpoint. This endpoint is called the *vertex*. An angle can also be formed when two segments or lines intersect. But, technically, even if it's formed by two segments, those two segments can be extended into rays to describe the angle. Angle measure is sort of *how far apart* the two sides are. The measurement system is unique to these shapes.

Angles can be classified by their size. The measures given here are all in terms of degrees. *Radian measures* (measures of angles that use multiples of  $\pi$  and relationships to the circumference) are covered in Chapter 4, so you can refer to that chapter when needed.

- ✔ **Acute angle:** An angle measuring less than 90 degrees.
- ✔ **Right angle:** An angle measuring exactly 90 degrees; the two sides are perpendicular.
- ✔ **Obtuse angle:** An angle measuring greater than 90 degrees and less than 180 degrees
- ✔ **Straight angle:** An angle measuring exactly 180 degrees.



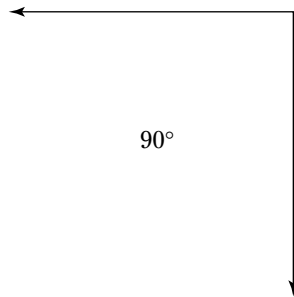
**Q.** Is an angle measuring 47 degrees acute, right, obtuse, or straight?

**A.** An angle measuring 47 degrees is acute.

**Q.** Is an angle measuring 163 degrees acute, right, obtuse, or straight?

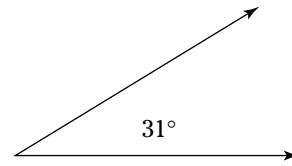
**A.** An angle measuring 163 degrees is obtuse.

1. What type of angle is shown in the figure?



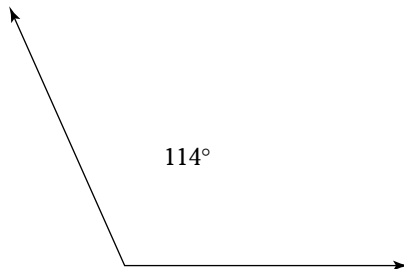
*Solve It*

2. What type of angle is shown in the figure?



*Solve It*

3. What type of angle is shown in the figure?



*Solve It*

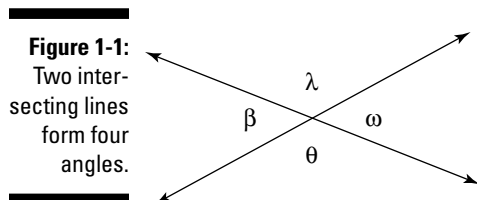
4. What type of angle is shown in the figure?



*Solve It*

## Naming Angles Where Lines Intersect

When two lines cross one another, four angles are formed, and there's something special about the pairs of angles that can be identified there. Look at Figure 1-1. The two lines have intersected, and I've named the angles by putting Greek letters inside them to identify the angles.



The angles that are opposite one another, when two lines intersect, are called *vertical angles*. The special thing they have in common, besides the lines they share, is that their measures are the same, too. There are two pairs of vertical angles in Figure 1-1. Angles  $\beta$  and  $\omega$  are vertical. So are angles  $\lambda$  and  $\theta$ .

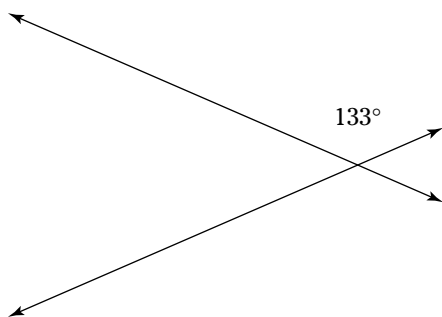
The other special angles that are formed are pairs of *supplementary angles*. Two angles are supplementary when their sum is 180 degrees. The supplementary angles in Figure 1-1 are those that lie along the same straight line with a shared ray between them. The pairs of supplementary angles are:  $\lambda$  and  $\omega$ ,  $\omega$  and  $\theta$ ,  $\theta$  and  $\beta$ , and  $\beta$  and  $\lambda$ .



**Q.** If one angle in a pair of supplementary angles measures 80 degrees, what does the other angle measure?

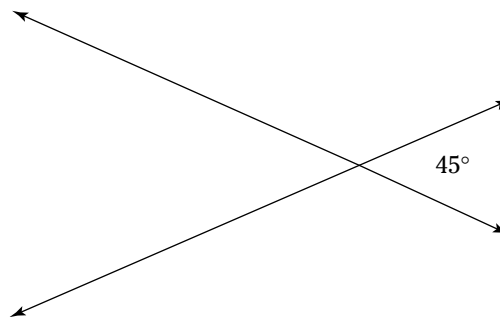
**A.** The other measures  $180 - 80 = 100$  degrees.

**5.** Give the measure of the angles that are *supplementary* to the angle shown in the figure.



**Solve It**

**6.** Give the measure of the angle that is *vertical* to the angle shown in the figure.



**Solve It**

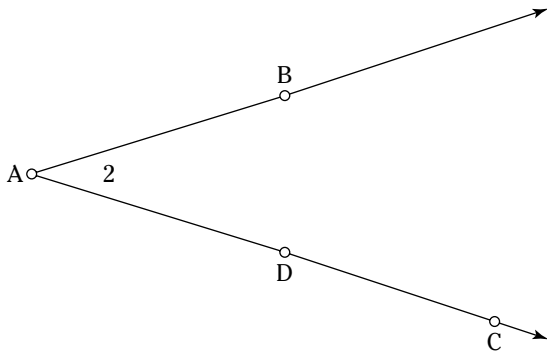
## Writing Angle Names Correctly

An angle can be identified in several different ways:

- ✔ Use the letter labeling the point that's the *vertex* of the angle. Points are labeled with capital letters.
- ✔ Use three letters that label points — one on one ray of the angle, then the vertex, and the last on the other ray.
- ✔ Use a letter or number in the inside of the angle. Usually, the letters used are Greek or lowercase.



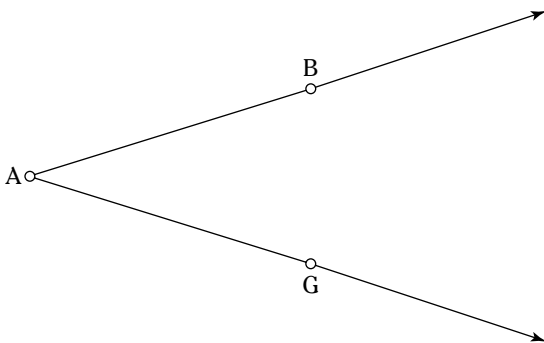
**Q.** Give all the different names that can be used to identify the angle shown in the figure.



**A.** The names for this angle are:

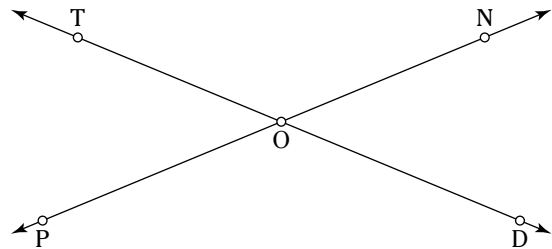
- Angle  $A$  (just using the label for the vertex)
- Angle  $BAD$  (using  $B$  on the top ray, the vertex, and  $D$  on the bottom ray)
- Angle  $BAC$  (using  $B$  on the top ray, the vertex, and  $C$  on the bottom ray)
- Angle  $DAB$  (using  $D$  on the bottom ray, the vertex, and  $B$  on the top ray)
- Angle  $CAB$  (using  $C$  on the bottom ray, the vertex, and  $B$  on the top ray)
- Angle  $2$  (using the number inside the angle)

**7.** Find all the names for the angle shown in the figure.



**Solve It**

**8.** Find all the names for the angle that's vertical to the angle  $POT$  in the figure.



**Solve It**

## Finding Missing Angle Measures in Triangles

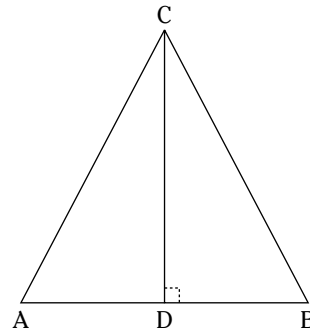
Triangles are probably one of the most familiar forms in geometry and trigonometry. They're studied and restudied and gone over for the minutest of details. One thing that stands out, is always true, and is often used, is the fact that the sum of the measures of the angles of any triangle is 180 degrees. It's always that sum — never more, never less. This is a good thing. It allows you to find missing measures — when they go missing — for angles in a triangle.



**Q.** If the measures of two of the angles of a triangle are 16 degrees and 47 degrees, what is the measure of the third angle?

**A.** To solve this, add  $16 + 47 = 63$ . Then subtract  $180 - 63 = 117$  degrees.

**Q.** An *equilateral triangle* has three equal sides and three equal angles. If you draw a segment from the vertex of an equilateral triangle perpendicular to the opposite side, then what are the measures of the angles in the two new triangles formed? Look at the figure to help you visualize this.



**A.** Because the triangle is equilateral, the angles must each be 60 degrees, because  $3 \times 60 = 180$ . That means that angles *A* and *B* are each 60 degrees. If the segment *CD* is perpendicular to the bottom of the triangle, *AB*, then angle *ADC* and angle *BDC* must each measure 90 degrees. What about the two top angles? Because angle *A* is 60 degrees and angle *ADC* is 90 degrees, and because  $60 + 90 = 150$ , that leaves  $180 - 150 = 30$  degrees for angle *ACD*. The same goes for angle *BCD*.

9. Triangle  $SIR$  is isosceles. An *isosceles triangle* has two sides that are equal; the angles opposite those sides are also equal. If the vertex angle,  $I$ , measures 140 degrees, what do the other two angles measure?

**Solve It**

10. A triangle has angles that measure  $n$  degrees,  $n + 20$  degrees, and  $3n - 15$  degrees. What are their measures?

**Solve It**

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## Determining Angle Measures along Lines and outside Triangles

Angles can be all over the place and arbitrary, or they can behave and be predictable. Two of the situations in the predictable category are those where a transversal cuts through two parallel lines (a *transversal* is another line cutting through both lines), and where a side of a triangle is extended to form an exterior angle.

When a transversal cuts through two parallel lines, the acute angles formed are all equal and the obtuse angles formed are all equal (unless the transversal is perpendicular to the line — in that case, they're all right angles). In Figure 1-2, on the left, you can see how creating acute and obtuse angles comes about. Also, the acute and obtuse angles are supplementary to one another.

An *exterior angle* of a triangle is an angle that's formed when one side of the triangle is extended. The exterior angle is supplementary to the interior angle it's adjacent to. Also, the exterior angle's measure is equal to the sum of the two nonadjacent interior angles.