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# MODEL-BASED SIGNAL PROCESSING

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James V. Candy

Lawrence Livermore National Laboratory  
University of California  
Santa Barbara, CA



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Praise the Lord, Jesus Christ, Our Savior! In times of great need  
and distress—He comforts us!



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# CONTENTS

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<b>Preface</b>	<b>xv</b>
<b>Acknowledgments</b>	<b>xxi</b>
<b>1. Introduction</b>	<b>1</b>
1.1 Background / 1	
1.2 Signal Estimation / 5	
1.3 Model-Based Processing Example / 7	
1.4 Model-Based Signal Processing Concepts / 11	
1.5 Notation and Terminology / 16	
1.6 Summary / 16	
<i>MATLAB</i> <sup>®</sup> Notes / 16	
References / 17	
Problems / 17	
<b>2. Discrete Random Signals and Systems</b>	<b>21</b>
2.1 Introduction / 21	
2.2 Deterministic Signals and Systems / 21	
2.3 Spectral Representation of Discrete Signals / 24	
2.3.1 Discrete Systems / 26	
2.3.2 Frequency Response of Discrete Systems / 29	
2.4 Discrete Random Signals / 32	
2.4.1 Motivation / 32	
2.4.2 Random Signals / 36	
2.5 Spectral Representation of Random Signals / 44	

- 2.6 Discrete Systems with Random Inputs / 57
- 2.7 *ARMAX* (*AR*, *ARX*, *MA*, *ARMA*) Models / 60
- 2.8 Lattice Models / 71
- 2.9 Exponential (Harmonic) Models / 79
- 2.10 Spatiotemporal Wave Models / 83
  - 2.10.1 Plane Waves / 83
  - 2.10.2 Spherical Waves / 87
  - 2.10.3 Spatiotemporal Wave Model / 89
- 2.11 State-Space Models / 92
  - 2.11.1 Continuous State-Space Models / 92
  - 2.11.2 Discrete State-Space Models / 98
  - 2.11.3 Discrete Systems Theory / 102
  - 2.11.4 Gauss-Markov (State-Space) Models / 105
  - 2.11.5 Innovations (State-Space) Models / 111
- 2.12 State-Space, *ARMAX* (*AR*, *MA*, *ARMA*, Lattice) Equivalence Models / 112
- 2.13 State-Space and Wave Model Equivalence / 120
- 2.14 Summary / 124
  - MATLAB* Notes / 124
  - References / 125
  - Problems / 127

### 3. Estimation Theory

**135**

- 3.1 Introduction / 135
  - 3.1.1 Estimator Properties / 136
  - 3.1.2 Estimator Performance / 137
- 3.2 Minimum Variance (*MV*) Estimation / 139
  - 3.2.1 Maximum a Posteriori (*MAP*) Estimation / 142
  - 3.2.2 Maximum Likelihood (*ML*) Estimation / 143
- 3.3 Least-Squares (*LS*) Estimation / 147
  - 3.3.1 Batch Least Squares / 147
  - 3.3.2 *LS*: A Geometric Perspective / 150
  - 3.3.3 Recursive Least Squares / 156
- 3.4 Optimal Signal Estimation / 160
- 3.5 Summary / 167
  - MATLAB* Notes / 167
  - References / 167
  - Problems / 168

<b>4. AR, MA, ARMAX, Lattice, Exponential, Wave Model-Based Processors</b>	<b>175</b>
4.1 Introduction / 175	
4.2 AR (All-Pole) MBP / 176	
4.2.1 Levinson-Durbin Recursion / 179	
4.2.2 Toeplitz Matrices for AR Model-Based Processors / 185	
4.2.3 Model-Based AR Spectral Estimation / 187	
4.3 MA (All-Zero) MBP / 191	
4.3.1 Levinson-Wiggins-Robinson (LWR) Recursion / 193	
4.3.2 Optimal Deconvolution / 198	
4.3.3 Optimal Time Delay Estimation / 201	
4.4 Lattice MBP / 207	
4.5 ARMAX (Pole-Zero) MBP / 213	
4.6 Order Estimation for MBP / 220	
4.7 Case Study: Electromagnetic Signal Processing / 227	
4.8 Exponential (Harmonic) MBP / 238	
4.8.1 Exponential MBP / 240	
4.8.2 SVD Exponential MBP / 247	
4.8.3 Harmonic MBP / 250	
4.9 Wave MBP / 262	
4.10 Summary / 271	
MATLAB Notes / 272	
References / 272	
Problems / 275	
<b>5. Linear State-Space Model-Based Processors</b>	<b>281</b>
5.1 State-Space MBP (Kalman Filter) / 281	
5.2 Innovations Approach to the MBP / 284	
5.3 Innovations Sequence of the MBP / 291	
5.4 Bayesian Approach to the MBP / 295	
5.5 Tuned MBP / 299	
5.6 Tuning and Model Mismatch in the MBP / 308	
5.6.1 Tuning with State-Space MBP Parameters / 308	
5.6.2 Model Mismatch Performance in the State-Space MBP / 312	
5.7 MBP Design Methodology / 318	
5.8 MBP Extensions / 327	
5.8.1 Model-Based Processor: Prediction-Form / 327	
5.8.2 Model-Based Processor: Colored Noise / 329	
5.8.3 Model-Based Processor: Bias Correction / 335	

- 5.9 *MBP* Identifier / 338
- 5.10 *MBP* Deconvolver / 342
- 5.11 Steady-State *MBP* Design / 345
  - 5.11.1 Steady-State *MBP* / 345
  - 5.11.2 Steady-State *MBP* and the Wiener Filter / 349
- 5.12 Case Study: *MBP* Design for a Storage Tank / 351
- 5.13 Summary / 358
  - MATLAB* Notes / 358
  - References / 359
  - Problems / 361

## **6. Nonlinear State-Space Model-Based Processors 367**

- 6.1 Linearized *MBP* (Kalman Filter) / 367
- 6.2 Extended *MBP* (Extended Kalman Filter) / 377
- 6.3 Iterated-Extended *MBP* (Iterated-Extended Kalman Filter) / 385
- 6.4 Unscented *MBP* (Kalman Filter) / 392
  - 6.4.1 Unscented Transformations / 393
  - 6.4.2 Unscented Processor / 397
- 6.5 Case Study: 2D-Tracking Problem / 404
- 6.6 Summary / 411
  - MATLAB* Notes / 411
  - References / 412
  - Problems / 413

## **7. Adaptive *AR*, *MA*, *ARMAX*, Exponential Model-Based Processors 419**

- 7.1 Introduction / 419
- 7.2 Adaption Algorithms / 420
- 7.3 All-Zero Adaptive *MBP* / 423
  - 7.3.1 Stochastic Gradient Adaptive Processor / 424
  - 7.3.2 Instantaneous Gradient *LMS* Adaptive Processor / 430
  - 7.3.3 Normalized *LMS* Adaptive Processor / 433
  - 7.3.4 Recursive Least-Squares (*RLS*) Adaptive Processor / 436
- 7.4 Pole-Zero Adaptive *MBP* / 443
  - 7.4.1 IIR Adaptive *MBP* / 443
  - 7.4.2 All-Pole Adaptive Predictor / 445
- 7.5 Lattice Adaptive *MBP* / 451
  - 7.5.1 All-Pole Adaptive Lattice *MBP* / 451
  - 7.5.2 Joint Adaptive Lattice Processor / 458

- 7.6 Adaptive *MBP* Applications / 460
  - 7.6.1 Adaptive Noise Canceler *MBP* / 460
  - 7.6.2 Adaptive D-Step Predictor *MBP* / 465
  - 7.6.3 Adaptive Harmonic *MBP* / 469
  - 7.6.4 Adaptive Time-Frequency *MBP* / 473
- 7.7 Case Study: Plasma Pulse Estimation Using *MBP* / 475
- 7.8 Summary / 481
  - MATLAB* Notes / 481
  - References / 481
  - Problems / 483

## **8. Adaptive State-Space Model-Based Processors 489**

- 8.1 State-Space Adaption Algorithms / 489
- 8.2 Adaptive Linear State-Space *MBP* / 491
- 8.3 Adaptive Innovations State-Space *MBP* / 495
  - 8.3.1 Innovations Model / 495
  - 8.3.2 RPE Approach Using the Innovations Model / 500
- 8.4 Adaptive Covariance State-Space *MBP* / 507
- 8.5 Adaptive Nonlinear State-Space *MBP* / 512
- 8.6 Case Study: *AMBP* for Ocean Acoustic Sound Speed Inversion / 522
  - 8.6.1 State-Space Forward Propagator / 522
  - 8.6.2 Sound-Speed Estimation: *AMBP* Development / 526
  - 8.6.3 Experimental Data Results / 528
- 8.7 Summary / 531
  - MATLAB* Notes / 531
  - References / 532
  - Problems / 533

## **9. Applied Physics-Based Processors 539**

- 9.1 *MBP* for Reentry Vehicle Tracking / 539
  - 9.1.1 RV Simplified Dynamics / 540
  - 9.1.2 Signal Processing Model / 542
  - 9.1.3 Processing of RV Signatures / 546
  - 9.1.4 Flight Data Processing / 556
  - 9.1.5 Summary / 559
- 9.2 *MBP* for Laser Ultrasonic Inspections / 561
  - 9.2.1 Laser Ultrasonic Propagation Modeling / 562
  - 9.2.2 Model-Based Laser Ultrasonic Processing / 563
  - 9.2.3 Laser Ultrasonics Experiment / 567

- 9.2.4 Summary / 570
- 9.3 *MBP* for Structural Failure Detection / 571
  - 9.3.1 Structural Dynamics Model / 572
  - 9.3.2 Model-Based Condition Monitor / 574
  - 9.3.3 Model-Based Monitor Design / 577
  - 9.3.4 *MBP* Vibrations Application / 577
  - 9.3.5 Summary / 583
- 9.4 *MBP* for Passive Sonar Direction-of-Arrival and Range Estimation / 583
  - 9.4.1 Model-Based Adaptive Array Processing for Passive Sonar Applications / 584
  - 9.4.2 Model-Based Adaptive Processing Application to Synthesized Sonar Data / 587
  - 9.4.3 Model-Based Ranging / 590
  - 9.4.4 Summary / 594
- 9.5 *MBP* for Passive Localization in a Shallow Ocean / 594
  - 9.5.1 Ocean Acoustic Forward Propagator / 595
  - 9.5.2 *AMBP* for Localization / 599
  - 9.5.3 *AMBP* Application to Experimental Data / 603
  - 9.5.4 Summary / 607
- 9.6 *MBP* for Dispersive Waves / 607
  - 9.6.1 Background / 608
  - 9.6.2 Dispersive State-Space Propagator / 609
  - 9.6.3 Dispersive Model-Based Processor / 612
  - 9.6.4 Internal Wave Processor / 614
  - 9.6.5 Summary / 621
- 9.7 *MBP* for Groundwater Flow / 621
  - 9.7.1 Groundwater Flow Model / 621
  - 9.7.2 *AMBP* Design / 625
  - 9.7.3 Summary / 627
- 9.8 Summary / 627
  - References / 628

## **Appendix A Probability and Statistics Overview**

**631**

- A.1 Probability Theory / 631
- A.2 Gaussian Random Vectors / 637
- A.3 Uncorrelated Transformation: Gaussian Random Vectors / 638
  - References / 639

**Appendix B SEQUENTIAL MBP and UD-FACTORIZATION 641**

- B.1 Sequential *MBP* / 641
- B.2 *UD*-Factorization Algorithm for *MBP* / 644
- References / 646

**Appendix C SSPACK\_PC: AN INTERACTIVE MODEL-BASED PROCESSING SOFTWARE PACKAGE 647**

- C.1 Introduction / 647
- C.2 Supervisor / 648
- C.3 Preprocessor / 649
- C.4 Postprocessor / 650
- C.5 Algorithms / 650
- C.6 Availability / 653
- References / 653

**Index 655**



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# PREFACE

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This text develops the “model-based approach” to signal processing for a variety of useful model-sets, including what has become popularly termed “physics-based” models. It presents a unique viewpoint of signal processing from the model-based perspective. Although designed primarily as a graduate text, it will prove useful to practicing signal processing professionals and scientists, since a wide variety of case studies are included to demonstrate the applicability of the model-based approach to real-world problems. The prerequisite for such a text is a melding of undergraduate work in linear algebra, random processes, linear systems, and digital signal processing. It is somewhat unique in the sense that many texts cover some of its topics in piecemeal fashion. The underlying model-based approach of this text is uniformly developed and followed throughout in the algorithms, examples, applications, and case studies. It is the model-based theme, together with the developed hierarchy of physics-based models, that contributes to its uniqueness. This text has evolved from two previous texts, Candy ([1], [2]) and has been broadened by a wealth of practical applications to real-world model-based problems.

The place of such a text in the signal processing textbook community can best be explained by tracing the technical ingredients that form its contents. It can be argued that it evolves from the digital signal processing area, primarily from those texts that deal with random or statistical signal processing or possibly more succinctly “signals contaminated with noise.” The texts by Kay ([3], [4], [5]), Therrien [6], and Brown [7] provide the basic background information in much more detail than this text, so there is little overlap with them.

This text additionally prepares the advanced senior or graduate student with enough theory to develop a fundamental basis and go onto more rigorous texts like Jazwinski [8], Sage [9], Gelb [10], Anderson [11], Maybeck [12], Bozic [13],

Kailath [14], and more recently, Mendel [15], Grewel [16], and Bar-Shalom [17]. These texts are rigorous and tend to focus on Kalman filtering techniques, ranging from continuous to discrete with a wealth of detail in all of their variations. The model-based approach discussed in this text certainly includes the state-space models as one of its model classes (probably the most versatile), but the emphasis is on various classes of models and how they may be used to solve a wide variety of signal processing problems. Some more recent texts of about the same technical level, but again, with a different focus, are Widrow [18], Orfanidis [19], Sharf [20], Haykin [21], Hayes [22], Brown [7], and Stoica [23]. Again, the focus of these texts is not the model-based approach but rather a narrow set of specific models and the development of a variety of algorithms to estimate them. The system identification literature and texts therein also provide some overlap with this text, but the approach is again focused on estimating a model from noisy data sets and is not really aimed at developing a model-based solution to a particular signal processing problem. The texts in this area are Ljung ([24], [25]), Goodwin [26], Norton [27] and Soderstrom [28].

The approach we take is to introduce the basic idea of model-based signal processing (MBSP) and show where it fits in terms of signal processing. It is argued that MBSP is a natural way to solve basic processing problems. The more a priori information we know about data and its evolution, the more information we can incorporate into the processor in the form of mathematical models to improve its overall performance. This is the theme and structure that echoes throughout the text. Current applications (e.g., structures, tracking, equalization, and biomedical) and simple examples to motivate the organization of the text are discussed. Next, in Chapter 2, the “basics” of stochastic signals and systems are discussed, and a suite of models to be investigated in the text, going from simple time series models to state-space and wave-type models, is introduced. The state-space models are discussed in more detail because of their general connection to “physical models” and their availability limited to control and estimation texts rather than the usual signal processing texts. Examples are discussed to motivate all the models and prepare the reader for further developments in subsequent chapters. In Chapter 3, the basic estimation theory required to comprehend the model-based schemes that follow are developed establishing the groundwork for performance analysis (bias, error variance, Cramer-Rao bound, etc.). The remaining chapters then develop the model-based processors for various model sets with real-world-type problems discussed in the individual case studies and examples. Chapter 4 develops the model-based scheme for the popular model sets (*AR*, *MA*, *ARMA*, etc.) abundant in the signal processing literature and texts today, following the model-based approach outlined in the first chapter and presenting the unified framework for the algorithms and solutions. Highlights of this chapter include the real-world case studies as well as the “minimum variance” approach to processor design along with accompanying performance analysis. Next we begin to lay the foundation of the “physics-based” processing approach by developing the linear state-space, Gauss-Markov model-set, leading to the well-known Kalman filter solution in Chapter 5. The Kalman filter is developed from the innovations viewpoint with its optimality properties

analyzed within. The solution to the minimum variance design is discussed (tuned filter) along with a “practical” cookbook approach (validated by theory). Next critical special extensions of the linear filter are discussed along with a suite of solutions to various popular signal processing problems (identification, deconvolution/bias estimation, etc.). Here the true power of the model-based approach using state-space models is revealed and developed for difficult problems that are easily handled within this framework. A highlight of the chapter is a detailed processor design for a storage tank, unveiling all of the steps required to achieve a minimum variance design. Chapter 6 extends these results even further to the case of nonlinear state-space models. Theoretically each processor is developed in a logical fashion leading to some of the more popular structures with example problems throughout. This chapter ends with a case study of tracking the motion of a super tanker during docking. Next the adaptive version of the previous algorithms is developed, again, within the model-based framework. Here many interesting and exciting examples and applications are presented along with some detailed case studies demonstrating their capability when applied to real-world problems. Here, in Chapter 7, we continue with the basic signal processing models and apply them to a suite of applications. Next, in Chapter 8, we extend the state-space model sets (linear and nonlinear) to the adaptive regime. We develop the adaptive Kalman-type filters and apply them to a real-world ocean acoustic application (case study). Finally, in Chapter 9, we develop a suite of physics-based models ranging from reentry vehicle dynamics (*ARMAX*), to nondestructive evaluation using laser ultrasound (*FIR*), to a suite of state-space models for vibrating structures, ocean acoustics, dispersive waves, and distributed groundwater flow. In each case the processor along with accompanying simulations is discussed and applied to various data sets, demonstrating the applicability and power of the model-based approach.

In closing, we must mention some of the new and exciting work currently being performing in nonlinear estimation. Specifically, these are the unscented Kalman filter [29] (Chapter 6), which essentially transforms the nonlinear problem into an alternate space without linearization (and its detrimental effects) to enhance performance, and the particle filter, which uses probabilistic sampling-resampling theory (Markov chain/Monte Carlo methods) (MCMC) to handle the non-gaussian type problems. Both approaches are opening new avenues of thought in estimation that has been stagnant since the 1970s. These approaches have evolved because of the computer power (especially the MCMC techniques) now becoming available ([29], [30]).

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JVC



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# INTRODUCTION

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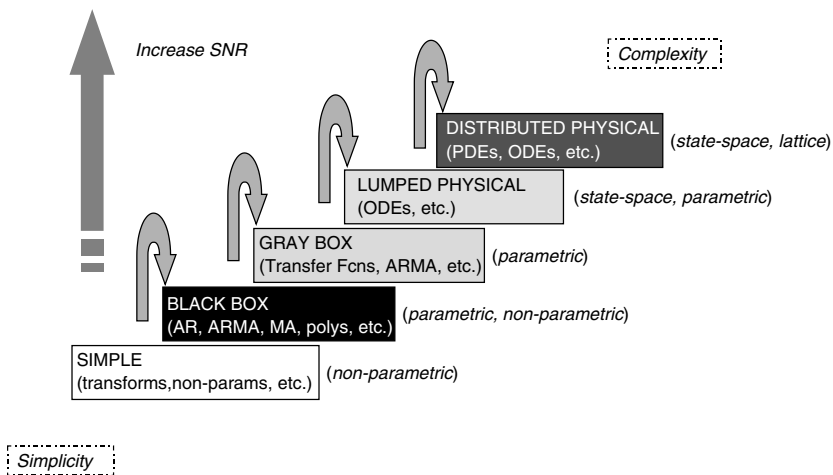
## 1.1 BACKGROUND

Perhaps the best way to start a text such as this is through an example that will provide the basis for this discussion and motivate the subsequent presentation. The processing of noisy measurements is performed with one goal in mind—to extract the desired information and reject the extraneous [1]. In many cases this is easier said than done. The first step, of course, is to determine what, in fact, *is* the desired information, and typically this is not the task of the signal processor but that of the phenomenologist performing the study. In our case we assume that the investigation is to extract information stemming from measured data. Many applications can be very complex, for instance, in the case of waves propagating through various media such as below the surface of the earth [2] or through tissue in biomedical [3] or through heterogeneous materials of critical parts in nondestructive evaluation (NDE) investigations [4]. In any case, the processing typically involves manipulating the measured data to extract the desired information, such as location of an epicenter, or the detection of a tumor or flaw in both biomedical and NDE applications.

Another view of the underlying processing problem is to decompose it into a set of steps that capture the strategic essence of the processing scheme. Inherently, we believe that the more “a priori” knowledge about the measurement and its underlying phenomenology we can incorporate into the processor, the better we can expect the processor to perform—as long as the information that is included is correct! One strategy called the *model-based approach* provides the essence of model-based

signal processing [1]. Some believe that all signal processing schemes can be cast into this generic framework. Simply, the model-based approach is “incorporating mathematical models of both physical phenomenology and the measurement process (including noise) into the processor to extract the desired information.” This approach provides a mechanism to incorporate knowledge of the underlying physics or dynamics in the form of mathematical process models along with measurement system models and accompanying noise as well as model uncertainties directly into the resulting processor. In this way the model-based processor enables the interpretation of results directly in terms of the problem physics. The model-based processor is actually a modeler’s tool enabling the incorporation of any a priori information about the problem to extract the desired information. As depicted in Figure 1.1, the fidelity of the model incorporated into the processor determines the complexity of the model-based processor with the ultimate goal of increasing the inherent signal-to-noise ratio ( $SNR$ ). These models can range from simple, implicit, nonphysical representations of the measurement data such as the Fourier or wavelet transforms to parametric black-box models used for data prediction, to lumped mathematical representations characterized by ordinary differential equations, to distributed representations characterized by partial differential equation models to capture the underlying physics of the process under investigation. The dominating factor of which model is the most appropriate is usually determined by how severe the measurements are contaminated with noise and the underlying uncertainties. If the  $SNR$  of the measurements is high, then simple nonphysical techniques can be used to extract the desired information; however, for low  $SNR$  measurements more and more of the physics and instrumentation must be incorporated for the extraction.

This approach of selecting the appropriate processing technique is pictorially shown in Figure 1.1. Here we note that as we progress up the *modeling steps*



**Figure 1.1.** Model-based signal processing: model staircase.

- State, 93–95
  - covariance, 106, 320
  - equations, 96, 97
  - estimate, 290, 312, 315, 320, 370, 377
  - estimation error, 288, 290, 296, 303, 315
  - estimation problem, 285, 295
  - information, 399
  - input transfer matrix, 97
  - mean vector, 106
  - orthogonality condition*, 293
  - perturbation, 370, 372
  - predictor gradient weighting matrix, 502, 504
  - propagation, 328
  - transition
    - matrix, 98, 101, 102, 414, 426, 597
    - mechanism, 169
    - variable, 93, 95, 397
    - variance, 106
    - vector, 95, 97, 353
- State-space, 95, 123, 176, 281, 284, 291–293, 304, 308, 405, 411, 491, 495, 592, 594, 595, 597, 599, 627
  - description, 608
  - equation, 426, 428
  - estimators, 651
  - feed-forward lattice form, 118
  - feedback lattice form, 119
  - form, 134, 353, 367, 454, 491, 523, 571, 583, 597
  - formulation, 610
  - framework, 614
  - model, 95, 99, 284, 289, 299, 312, 313, 489, 535, 585, 586, 595, 611, 628, 641
  - model-based processor, 290
  - postprocessor, 648
  - preprocessor, 648
  - propagator, 526, 600
  - rational lattice form, 119
  - representation, 94, 95, 101, 281, 284, 427, 492, 522, 573, 608, 609
  - techniques, 430
  - wave model, 120
- Stationary, 41, 107, 198, 419, 443, 468
  - noise processes, 291
  - process, 50, 59
  - signal, 423
- Statistical
  - consistency, 509
  - hypothesis test, 300
  - performance, 571
  - test, 237
  - tests, 299, 321, 553, 616, 618, 619
- Statistically
  - independent, 430
  - white, 306, 575
- Steady-state, 108, 303, 312, 324, 358, 361, 424, 495, 533
  - gain, 510
  - processor, 361
  - Wiener solution, 440
- Step-size, 237, 419, 421, 422, 428, 429, 431–433, 441, 454, 459, 486, 565
  - adjustment, 456
- Stochastic
  - approximation, 430, 509
  - deconvolution problem, 342
  - gradient, 422, 423, 431, 435, 440, 445, 455, 459, 463, 481, 483, 485, 486
  - algorithm, 425, 428–431, 455
  - technique, 424
  - Newton, 440
  - process, 36, 37, 38, 41, 44, 92, 135, 284, 378, 633, 635
  - realization, 496, 534
- Stopping rule, 386
- Storage coefficient, 622, 626
- Storage tank, 351, 358
- Storativity, 621
- Structural
  - displacements, 575
  - failure detection, 577, 581, 583
  - failures, 627
  - model, 534, 579
  - process model, 573, 574

- response, 577
  - system, 94, 573, 575–577
- Suboptimal, 521
  - method, 343
  - processor, 323, 324, 327
  - state estimates, 326
- Subspace, 152, 253, 263
  - decomposition, 258
- Subsurface hydrology, 621
- Sufficient, 137
  - statistic, 145
- Sum decomposition, 53, 58, 59, 130, 166, 496
- Sum-squared
  - error, 436, 438
    - criterion, 147
  - model error, 240
  - prediction error, 242
- Surface displacement, 562, 563
- Synthetic aperture, 584
  - array processing, 523, 628
- System
  - linear time invariant, 27
  - identification, 188, 465, 531, 571
    - problem, 338
  - model, 100
  - order, 220
- Systems theory, 32, 41, 95, 97, 102, 105
- Target, 415
  - bearings, 583
  - localization, 250
- Taylor series, 100, 215, 368, 369, 387, 392, 397, 414, 420, 492, 527, 531, 536, 586
- Temporal frequency, 8
- Test statistic, 301, 302
- Threshold, 268
- Time average, 42
- Time averages, 209, 452
- Time averaging, 445
- Time constant, 432
- Time delay, 88, 91, 201–203, 416, 590–592
- Time
  - reverse, 128
  - series analysis, 175
- Time-to-failure, 577
- Time-correlated, 327, 329, 330
  - noise, 330
- Time-frequency, 557, 560
  - estimation, 556
  - representation, 473
- Time-invariant 27
  - predictor, 499
  - systems, 102
- Time-uncorrelated, 287
- Time-varying, 377, 419, 536
  - bandwidth, 308
  - matrices, 370
  - polynomial, 448
  - problem, 448
- Toeplitz, 246
  - correlation matrix, 550, 564
  - matrix, 176, 178, 179, 185, 186, 191–193, 198, 549
  - structure, 234
- Tomography, 522
- Towed array, 583
- Tracking, 202, 411, 448, 471
  - filter, 411
  - problem, 381, 382, 404, 410
  - radar, 539
  - telescope, 19
- Training sequence, 416
- Trajectory, 149, 159
  - motion compensation, 547, 556, 560
- Transfer function, 3, 29, 31, 61, 73, 74, 76, 96, 101, 104, 117, 127, 167, 276, 486, 540, 560
  - matrix, 97
- Transform, 25
- Transformation matrix, 173, 286, 459
- Transformed statistics, 398
- Transient, 344
  - data, 468
  - performance, 311, 312
  - plasma pulse, 478
  - problems, 342

- Transient (*Continued*)
  - pulse, 479, 481
  - signal, 477
- Transition matrix, 97, 309, 426, 454
- Transmissivity, 621, 622, 626
- Trend, 149
  - estimation, 560
  - removal, 149, 159
- Trial-and-error process, 327
- Triangular matrix, 185
- True measurement, 332
- Truncated SVD, 260
- Truth model, 320, 322, 323, 574
- Tune, 303, 327
- Tuned, 300
- Tuning, 308, 322, 354
  - example, 324
  - problem, 311, 339
- U-D
  - factorization method, 282
  - factorized form, 439, 516
- Ultrasonic
  - signal, 562, 567
  - waves, 562
- Unbiased
  - covariance estimator, 245
  - estimate, 169, 431
  - estimator, 136, 291
- Unconditionally unbiased, 136, 140
- Uncorrelated, 161
  - gaussian variable, 43
  - innovations, 458
  - noise, 111
  - output, 467
- Uniform
  - distribution, 42
  - random variable, 42
- Unit
  - circle, 31, 53, 247, 428
  - impulse, 22
  - ramp, 22
  - step, 22
- Unitary matrix, 167, 427
- Unity constraint, 258
- Unknown input, 343
- Unobservable, 102
- Unpredictable, 289
- Unscented
  - Kalman filter, 392, 397
  - model-based processor, 392
  - transformation, 393, 394
- Van der Monde matrix, 243
- Variance, 12, 65, 634
- Vector
  - measurements, 322
  - recursion, 445
  - space, 150
- Vibrating structure, 577
- Wave, 72
  - source/target parameters, 262
  - dynamics, 617
  - equation, 84, 87, 595
  - estimation, 609
  - model, 72, 90
  - propagation, 608
- Wave-field enhancement problem, 621
- Waveguide, 595
- Wavelength, 85
- Wavenumber, 8, 262, 524, 584, 590, 596, 610, 614
  - vector, 85, 87, 91
  - spectrum, 523
- Wavenumber-frequency, 83
  - space, 85
- Weighted
  - least-squares estimate, 147, 150
  - quadratic cost function, 493
  - sample variance estimator, 494
  - sum-squared error, 492
  - sum-squared residual, 302, 322, 560, 575
- Weighting matrix, 299
- White, 105, 111, 162, 177, 230, 284, 290, 292, 295, 306, 314, 321, 340, 344, 385, 391, 409, 414, 575
  - gaussian noise, 49, 473, 552
  - gaussian sequence, 50

- noise, 48, 59, 131, 191, 275, 330
- prediction errors, 554
- Whiteness, 50, 292, 322, 391, 507, 510, 554, 555
  - detector, 556, 558, 561
  - test, 220, 233, 301, 302, 306, 575, 577, 618
- Whitening filter, 165, 276
- Wide-sense stationary, 41
- Wiener, 142, 443
  - filter, 165, 166, 191, 275, 358, 460, 547, 549, 560
  - problem, 458
  - solution, 142, 165, 192, 193, 198, 203, 275, 423, 424, 437, 459, 468, 497
- Wiener-Kalman filtering, 112
- Wiener-Khintchine, 47, 52
- Wold decomposition, 59, 61
- Yule-Walker, 178
- Z-transform, 58, 101, 104, 108
  - region of convergence, 25
  - unit circle, 25
- Zero
  - mean, 130, 275, 284, 290, 292, 306, 314, 321, 344, 409, 414, 416, 448, 554, 575, 619
  - test, 301, 391
  - misadjustment, 439
- Zero-mean/whiteness test, 321, 322, 326, 409, 504, 510, 520, 593, 619
- Zero-state, 97
- Zeros, 28, 31, 32, 59