MODERN NONLINEAR OPTICS

Part 2

Second Edition

ADVANCES IN CHEMICAL PHYSICS
VOLUME 119

Edited by

Myron W. Evans

Series Editors

I. PRIGOGINE

Center for Studies in Statistical Mechanics and Complex Systems
The University of Texas
Austin, Texas
and
International Solvay Institutes
Université Libre de Bruxelles
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Department of Chemistry
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INTRODUCTION

Few of us can any longer keep up with the flood of scientific literature, even in specialized subfields. Any attempt to do more and be broadly educated with respect to a large domain of science has the appearance of tilting at windmills. Yet the synthesis of ideas drawn from different subjects into new, powerful, general concepts is as valuable as ever, and the desire to remain educated persists in all scientists. This series, *Advances in Chemical Physics*, is devoted to helping the reader obtain general information about a wide variety of topics in chemical physics, a field that we interpret very broadly. Our intent is to have experts present comprehensive analyses of subjects of interest and to encourage the expression of individual points of view. We hope that this approach to the presentation of an overview of a subject will both stimulate new research and serve as a personalized learning text for beginners in a field.

I. Prigogine
Stuart A. Rice

PREFACE

This volume, produced in three parts, is the Second Edition of Volume 85 of the series, *Modern Nonlinear Optics*, edited by M. W. Evans and S. Kielich. Volume 119 is largely a dialogue between two schools of thought, one school concerned with quantum optics and Abelian electrodynamics, the other with the emerging subject of non-Abelian electrodynamics and unified field theory. In one of the review articles in the third part of this volume, the Royal Swedish Academy endorses the complete works of Jean-Pierre Vigier, works that represent a view of quantum mechanics opposite that proposed by the Copenhagen School. The formal structure of quantum mechanics is derived as a linear approximation for a generally covariant field theory of inertia by Sachs, as reviewed in his article. This also opposes the Copenhagen interpretation. Another review provides reproducible and repeatable empirical evidence to show that the Heisenberg uncertainty principle can be violated. Several of the reviews in Part 1 contain developments in conventional, or Abelian, quantum optics, with applications.

In Part 2, the articles are concerned largely with electrodynamical theories distinct from the Maxwell–Heaviside theory, the predominant paradigm at this stage in the development of science. Other review articles develop electrodynamics from a topological basis, and other articles develop conventional or U(1) electrodynamics in the fields of antenna theory and holography. There are also articles on the possibility of extracting electromagnetic energy from Riemannian spacetime, on superluminal effects in electrodynamics, and on unified field theory based on an SU(2) sector for electrodynamics rather than a U(1) sector, which is based on the Maxwell–Heaviside theory. Several effects that cannot be explained by the Maxwell–Heaviside theory are developed using various proposals for a higher-symmetry electrodynamical theory. The volume is therefore typical of the second stage of a paradigm shift, where the prevailing paradigm has been challenged and various new theories are being proposed. In this case the prevailing paradigm is the great Maxwell–Heaviside theory and its quantization. Both schools of thought are represented approximately to the same extent in the three parts of Volume 119.

As usual in the *Advances in Chemical Physics* series, a wide spectrum of opinion is represented so that a consensus will eventually emerge. The prevailing paradigm (Maxwell–Heaviside theory) is ably developed by several groups in the field of quantum optics, antenna theory, holography, and so on, but the paradigm is also challenged in several ways: for example, using general relativity, using O(3) electrodynamics, using superluminal effects, using an extended electrodynamics based on a vacuum current, using the fact that longitudinal waves may appear in vacuo on the U(1) level, using a reproducible and repeatable device, known as the *motionless electromagnetic generator*, which extracts electromagnetic energy from Riemannian spacetime, and in several other ways. There is also a review on new energy sources. Unlike Volume 85, Volume 119 is almost exclusively dedicated to electrodynamics, and many thousands of papers are reviewed by both schools of thought. Much of the evidence for challenging the prevailing paradigm is based on empirical data, data that are reproducible and repeatable and cannot be explained by the Maxwell–Heaviside theory. Perhaps the simplest, and therefore the most powerful, challenge to the prevailing paradigm is that it cannot explain interferometric and simple optical effects. A non-Abelian theory with a Yang–Mills structure is proposed in Part 2 to explain these effects. This theory is known as O(3) *electrodynamics* and stems from proposals made in the first edition, Volume 85.

As Editor I am particularly indebted to Alain Beaulieu for meticulous logistical support and to the Fellows and Emeriti of the Alpha Foundation's Institute for Advanced Studies for extensive discussion. Dr. David Hamilton at the U.S. Department of Energy is thanked for a Website reserved for some of this material in preprint form.

Finally, I would like to dedicate the volume to my wife, Dr. Laura J. Evans.

Myron W. Evans

Ithaca, New York

CONTENTS

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Second Edition

ADVANCES IN CHEMICAL PHYSICS

VOLUME 119

OPTICAL EFFECTS OF AN EXTENDED ELECTROMAGNETIC THEORY

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CONTENTS

I. INTRODUCTION

Conventional electromagnetic field theory based on Maxwell's equations and quantum mechanics has been very successful in its application to numerous problems in physics, and has sometimes manifested itself in an extremely good agreement with experimental results. Nevertheless, in certain areas these joint theories do not seem to provide fully adequate descriptions of physical reality. Thus there are unsolved problems leading to difficulties with Maxwell's equations that are not removed by and not directly associated with quantum mechanics [1,2].

Because of these circumstances, a number of modified and new approaches have been elaborated since the late twentieth century. Among the reviews and conference proceedings describing this development, those by Lakhtakia [3], Barrett and Grimes [4], Evans and Vigier [5], Evans et al. [6,7], Hunter et al. [8], and Dvoeglazov [9] can be mentioned here. The purpose of these approaches can be considered as twofold:

- To contribute to the understanding of so far unsolved problems
- To predict new features of the electromagnetic field

The present chapter is devoted mainly to one of these new theories, in particular to its possible applications to photon physics and optics. This theory is based on the hypothesis of a nonzero divergence of the electric field in vacuo, in combination with the condition of Lorentz invariance. The nonzero electric field divergence, with an associated "space-charge current density," introduces an extra degree of freedom that leads to new possible states of the electromagnetic field. This concept originated from some ideas by the author in the late 1960s, the first of which was published in a series of separate papers [10,12], and later in more complete forms and in reviews [13–20].

As a first step, the treatment in this chapter is limited to electromagnetic field theory in orthogonal coordinate systems. Subsequent steps would include more advanced tensor representations and a complete quantization of the extended field equations.

II. UNSOLVED PROBLEMS IN CONVENTIONAL ELECTROMAGNETIC THEORY

The failure of standard electromagnetic theory based on Maxwell's equations is illustrated in numerous cases. Here the following examples can be given.

1. Light appears to be made of waves and simultaneously of particles. In conventional theory the individual photon is on one hand conceived to be a massless particle, still having an angular momentum, and is on the other hand regarded as a wave having the frequency ν and the energy $h\nu$, whereas the angular momentum is independent of the frequency. This dualism of the wave and particle concepts is so far not fully understandable in terms of conventional theory [5].
2. The photon can sometimes be considered as a plane wave, but some experiments also indicate that it can behave like a bullet. In investigations on interference patterns created by individual photons on a screen [21], the impinging photons produce dot-like marks on the latter, such as those made by needle-shaped objects.
3. In attempts to develop conventional electrodynamic models of the individual photon, it is difficult to finding axisymmetric solutions that both converge at the photon center and vanish at infinity. This was already realized by Thomson [22] and later by other investigators [23].
4. During the process of total reflection at a vacuum boundary, the reflected beam has been observed to be subject to a parallel displacement with respect to the incident beam. For this so-called Goos–Hänchen effect, the displacement was further found to have a maximum for parallel polarization of the incident electric field, and a minimum for perpendicular polarization [24,25]. At an arbitrary polarization angle, however, the displacement does not acquire an intermediate value, but splits into the two values for parallel and perpendicular polarization. This behaviour cannot be explained by conventional electromagnetic theory.
5. The Fresnel laws of reflection and refraction of light in nondissipative media have been known for over 180 years. However, these laws do not apply to the total reflection of an incident wave at the boundary between a dissipative medium and a vacuum region [26].
6. In a rotating interferometer, fringe shifts have been observed between light beams that propagate parallel and antiparallel with the direction of rotation [4]. This Sagnac effect requires an unconventional explanation.
7. Electromagnetic wave phenomena and the related photon concept remain somewhat of an enigma in more than one respect. Thus, the latter concept should in principle apply to wavelengths ranging from about 10^{-15} m of gamma radiation to about 10^{5} m of long radiowaves. This leads to an as yet not fully conceivable transition from a beam of individual photons to a nearly plane electromagnetic wave.
8. As the only explicit time-dependent solution of Cauchy's problem, the Lienard–Wiechert potentials are claimed be inadequate for describing

the entire electromagnetic field [2]. With these potentials only, the implicitly time-independent part of the field is then missing, namely, the part that is responsible for the interparticle long-range Coulomb interaction. This question may need further analysis.

9. There are a number of observations which seem to indicate that superluminal phenomena are likely to exist [27]. Examples are given by the concept of negative square-mass neutrinos, fast galactic miniquasar expansion, photons tunneling through a barrier at speeds greater than c, and the propagation of so called X-shaped waves. These phenomena cannot be explained in terms of the purely transverse waves resulting from Maxwell's equations, and they require a longitudinal wave component to be present in the vacuum [28].
10. A photon gas cannot have changes of state that are adiabatic and isothermal at the same time, according to certain studies on the distribution laws for this gas. To eliminate such a discrepancy, longitudinal modes, which do not exist in conventional theory, must be present [29,30].
11. It is not possible for conventional electromagnetic models of the electron to explain the observed property of a "point charge" with an excessively small radial dimension [20]. Nor does the divergence in self-energy of a point charge vanish in quantum field theory where the process of renormalization has been applied to solve the problem.

III. BASIS OF PRESENT APPROACH

The present modified form of Maxwell's equations in vacuo is based on two mutually independent hypotheses:

- The divergence of the electric field may differ from zero, and a corresponding "space-charge current" may exist in vacuo. This concept should not become less conceivable than the earlier one regarding introduction of the displacement current, which implies that a nonvanishing curl of the magnetic field and a corresponding current density can exist in vacuo. Both these concepts can be regarded as intrinsic properties of the electromagnetic field. The nonzero electric field divergence can thereby be interpreted as a polarization of the vacuum ground state [13] which has a nonzero energy as predicted by quantum physics [5], as confirmed by the existence of the Casimir effect. That electric polarization can occur out of a neutral state is also illustrated by electron–positron pair formation from a photon [18].
- This extended form of the field equations should remain Lorentz-invariant. Physical experience supports such a statement, as long as there are no results that conflict with it.

A. Formulation in Terms of Electromagnetic Field Theory

1. Basic Equations

On the basis of these two hypotheses the extended field equations in vacuo become

$$\operatorname{curl}\frac{\mathbf{B}}{\mu_0} = \mathbf{j} + \frac{\varepsilon_0 \partial \mathbf{E}}{\partial t} \tag{1}$$

$$\operatorname{curl}\mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\mathbf{j} = \bar{\rho}\,\mathbf{C} \tag{3}$$

in SI units. Here $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric fields, $\mathbf{j}$ is the current density, and $\bar{\rho}$ the charge density arising from a nonzero electric field divergence in vacuo. As a consequence of the divergence of equations (1) and (2),

$$\operatorname{div}\mathbf{E} = \frac{\bar{\rho}}{\varepsilon_0} \tag{4}$$

$$\operatorname{div}\mathbf{B} = 0 \qquad \mathbf{B} = \operatorname{curl}\mathbf{A} \tag{5}$$

and

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \tag{6}$$

The space-charge current density in vacuo expressed by Eqs. (3) and (4) constitutes the essential part of the present extended theory. To specify the thus far undetermined velocity $\mathbf{C}$, we follow the classical method of recasting Maxwell's equations into a four-dimensional representation. The divergence of Eq. (1) can, in combination with Eq. (4), be expressed in terms of a four-dimensional operator, where $(\mathbf{j}, ic\bar{\rho})$ thus becomes a 4-vector. The potentials $\mathbf{A}$ and ϕ are derived from the sources $\mathbf{j}$ and $\bar{\rho}$, which yield

$$-\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\mathbf{A}, \frac{i\phi}{c}\right) \equiv \square\left(\mathbf{A}, \frac{i\phi}{c}\right) = \mu_0(\mathbf{j}, ic\bar{\rho}) = \mu_0\bar{\rho}(\mathbf{C}, ic) \equiv \mu_0\mathbf{J} \tag{7}$$

when being combined with the condition of the Lorentz gauge. The Lorentz condition is further discussed in Appendix A.

It should be observed that Eq. (7) is of a “Proca type,” here being due to generation of a space-charge density $\bar{\rho}$ in vacuo (free space). Such an equation can describe a particle with the spin value unity [31].

Returning to the form (3) of the space-charge current density, and observing that $(\mathbf{j}, ic\bar{\rho})$ is a 4-vector, the Lorentz invariance thus leads to

$$j^2 - c^2\bar{\rho}^2 = \bar{\rho}^2\left(C^2 - c^2\right) = \text{const} = 0 \qquad C^2 = c^2 \tag{8}$$

where $j^2 = \mathbf{j}^2$ and $C^2 = \mathbf{C}^2$. The constant in this relation has to vanish because it should be universal to any inertial frame, and because the charge density varies from frame to frame. This result is further reconcilable with the relevant condition that the current density $\mathbf{j}$ of Eq. (3) should vanish in absence of the space-charge density $\bar{\rho}$. In this way Eqs. (1)–(6) and (8) provide an extended Lorentz invariant form of Maxwell’s equations that includes all earlier treated electromagnetic phenomena but also contains new classes of time-dependent and steady solutions, as illustrated later.

Concerning the velocity field $\mathbf{C}$, the following general features can now be specified:

- The vector $\mathbf{C}$ is time-independent.
- The direction of the unit vector of $\mathbf{C}$ depends on the geometry of the particular configuration to be analyzed, as is also the case for the unit vector of the current density $\mathbf{j}$ in any configuration treated in terms of conventional electromagnetic theory. As will be shown later, the direction of $\mathbf{C}$ thus depends on the necessary boundary conditions.
- Both $\operatorname{curl}\mathbf{C}$ and $\operatorname{div}\mathbf{C}$ can differ form zero, but here we restrict ourselves to

$$\operatorname{div}\mathbf{C} = 0 \tag{9}$$

We finally observe that a combination of Eqs. (1) and (4) leads to the classical relation

$$\operatorname{div}\mathbf{j} = -\frac{\partial\bar{\rho}}{\partial t} \tag{10}$$

of the 4-vector $(\mathbf{j}, ic\bar{\rho})$.

The introduction of the current density (3) in 3-space is, in fact, less intuitive than what could appear at first glance. As soon as the charge density (4) is permitted to exist as the result of a nonzero electric field divergence, the Lorentz invariance of a 4-current (7) with the time part $ic\bar{\rho}$ namely requires the associated space part to adopt the form (3), that is, by necessity.

The degree of freedom introduced by a nonzero electric field divergence leads both to new features of the electromagnetic field and to the possibility of satisfying boundary conditions in cases where this would not become possible in conventional theory.

In connection with the basic ideas of the present approach, the question may be raised as to why only div **E**, and not also div **B**, is permitted to be nonzero. This issue can be considered to be both physical and somewhat philosophical. Here we should remember that the electric field is associated with an equivalent "charge density" $\bar{\rho}$ considered as a source, whereas the magnetic field has its source in the current density **j**. The electric field lines can thereby be "cut off" by ending at a corresponding "charge," whereas the magnetic field lines generated by a line element of the current density are circulating around the same element. From the conceptual point of view it thus appears more difficult to imagine how these circulating magnetic field lines could be cut off to form magnetic poles by assuming div **B** to be nonzero, than to have electric field lines ending on charges with a nonzero div **E**.

Some investigators have included magnetic monopoles in extended theories [32,33], also from the quantum-theoretic point of view [20]. According to Dirac [34], the magnetic monopole concept is an open question. In this connection it should finally be mentioned that attempts have been made to construct theories based on general relativity where gravitation and electromagnetism are derived from geometry, as well as theories including both a massive photon and a Dirac monopole [20].

2. *The Momentum and Energy Balance*

We now turn to the momentum and energy balance of the electromagnetic field. In analogy with conventional deductions, Eq. (1) is multiplied vectorially by **B** and Eq. (2), by $\varepsilon_0\mathbf{E}$. The sum of the resulting equations is then rearranged into the local momentum balance equation

$$\operatorname{div} {}^2\mathbf{S} = \bar{\rho}(\mathbf{E} + \mathbf{C} \times \mathbf{B}) + \varepsilon_0 \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B}) \tag{11}$$

where ${}^2\mathbf{S}$ is the electromagnetic stress tensor [35] and Eq. (3) has been employed. The integral form of Eq. (11) becomes

$$\int {}^2\mathbf{S} \cdot \mathbf{n}\, dS = \mathbf{F}_e + \mathbf{F}_m + \frac{\partial}{\partial t} \int \mathbf{g}\, dV \tag{12}$$

where dS and dV are surface and volume elements, respectively,

$$\mathbf{F}_e = \int \bar{\rho}\mathbf{E}\, dV \quad \mathbf{F}_m = \int \bar{\rho}\mathbf{C} \times \mathbf{B}\, dV \tag{13}$$

are the electric and magnetic volume forces, and

$$\mathbf{g} = \varepsilon_0 \mathbf{E} \times \mathbf{B} = \frac{1}{c^2} \mathbf{S} \tag{14}$$

can be interpreted as an electromagnetic momentum with $\mathbf{S}$ denoting the Poynting vector. Here the component S_{jk} of the tensor ${}^2\mathbf{S}$ is the momentum that in unit time crosses in the j- direction for a unit element of surface whose normal is oriented along the k axis [35]. The difference in the present results (11) and (12) as compared to conventional theory is in the appearance of the terms, which include the nonzero charge density $\bar{\rho}$ in vacuo.

In a similar way scalar multiplications of Eq. (1) by $\mathbf{E}$ and Eq. (2) by $\mathbf{B}/\mu_0$ yields, after subtraction of the resulting equations, the local energy balance equation

$$-\mathrm{div}\,\mathbf{S} = -\left(\frac{1}{\mu_0}\right)\mathrm{div}(\mathbf{E} \times \mathbf{B}) = \bar{\rho}\mathbf{E} \cdot \mathbf{C} + \frac{1}{2}\,\varepsilon_0\,\frac{\partial}{\partial t}\,(\mathbf{E}^2 + c^2\mathbf{B}^2) \tag{15}$$

This equation differs from that of the conventional Poynting theorem, due to the existence of the term $\bar{\rho}\mathbf{E} \cdot \mathbf{C}$ in vacuo. That there should arise a difference has also been emphasized by Evans et al. [6] as well as by Chubykalo and Smirnov-Rueda [2]. These investigators note that the Poynting vector in vacuo is only defined in terms of transverse plane waves, that the case of a longitudinal magnetic field $\mathbf{B}^{(3)}$ leads to a new form of the Poynting theorem, and that the Poynting vector can be associated only with the free magnetic field. We shall return to this question later, when considering axisymmetric wavepackets and the photon interpreted as a particle with an associated pilot wave. It will also be seen later in this context that $\mathbf{F}_e$, $\mathbf{F}_m$, and the integral of $\bar{\rho}\mathbf{E} \cdot \mathbf{C}$ can disappear in the special case of axisymmetric wavepackets, and that $\bar{\rho}\mathbf{E} \cdot \mathbf{C}$ disappears for plane waves.

3. *The Energy Density*

The last term in Eq. (15) includes the local "field energy density"

$$w_f = \frac{1}{2}\left(\varepsilon_0\mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0}\right) \tag{16}$$

interpreted in terms of the electromagnetic field strengths $\mathbf{E}$ and $\mathbf{B}$. An alternative form [35], which at least holds for steady states and for waves where the field quantities vary as $\exp(-i\omega t)$ and have the same phases, is given by the local "source energy density"

$$w_s = \frac{1}{2}\,(\bar{\rho}\phi + \mathbf{j} \cdot \mathbf{A}) = \frac{1}{2}\,\bar{\rho}(\phi + \mathbf{C} \cdot \mathbf{A}) \tag{17}$$

interpreted in terms of the sources $\bar{\rho}$ and $\mathbf{j}$, which generate the electromagnetic field, and where the form (17) is a direct measure of the local work performed on the electric charges and currents. The total field energy becomes

$$W = \int w_f \, dV = \int w_s \, dV \tag{18}$$

provided it leads to surface integrals that vanish at infinity, and at the origin. Thus, Eq. (18) does not hold when the field quantities become divergent at the origin or at infinity.

In the present approach a physically relevant expression for the local energy density is sometimes needed. In such a case we shall prefer the form (17) to that of Eq. (16). Thus there are situations where the moment has to be taken of the local energy density, with some space-dependent function f. Since w_f and w_s represent entirely different spatial distributions of energy, it is then observed that

$$\int f \cdot w_f \, dV \neq \int f \cdot w_s \, dV \tag{19}$$

A further feature of physical interest is that the *local* energy density (17) can become positive as well as *negative* in some regions of space, even if the total energy W becomes positive as long as relation (18) holds. It is, however, not clear at this stage whether the form (17) could open up a possibility of finding negative energy states.

When considering the energy density of the form (17), it is sometimes convenient to divide the electromagnetic field into two parts when dealing with charge and current distributions that are limited to a region in space near the origin. This implies that the potentials are written as

$$\mathbf{A} = \mathbf{A}_s + \mathbf{A}_v \quad \phi = \phi_s = \phi_v \tag{20}$$

Here $\mathrm{curl}^2\, \mathbf{A}_s \neq 0$ and $\nabla^2 \phi_s \neq 0$ refer to the "source part" of the field that is nonzero within such a limited region, whereas $\mathrm{curl}^2\, \mathbf{A}_v = 0$ and $\nabla^2 \phi_v = 0$ refer to the "vacuum part" outside the same region [13,20], and the notation $\mathrm{curl}^2 \equiv \mathrm{curl\,curl}$ is used henceforth. For a model of a charged particle such as the electron, the potentials $\mathbf{A}_v$ and ϕ_v would thus be connected with its long-distance magnetic dipole field and electrostatic Coulomb field, respectively [20]. The total energy becomes

$$\begin{aligned} W = \frac{1}{2}\,\varepsilon_0 \int \left(c^2 \mathbf{A}_s \cdot \mathrm{curl}^2\, \mathbf{A}_s - \phi_s \nabla^2 \phi_s\right) dV + \frac{1}{2}\,\varepsilon_0 \int \mathbf{n} \cdot \big[c^2(\mathbf{A}_s \times \mathrm{curl}\, \mathbf{A}_v \\ - \mathbf{A}_v \times \mathrm{curl}\, \mathbf{A}_s) + \phi_s \nabla \phi_v - \phi_v \nabla \phi_s\big]\, dS \end{aligned} \tag{21}$$

where S now stands for the bounding surfaces to be taken into account. There are, in principle, two possibilities:

- When there is a single bounding surface S that can be extended to infinity where the electromagnetic field vanishes, only the space-charge parts $\mathbf{A}_s$ and ϕ_s will contribute to the energy (21). This possibility is of special interest in this context, which concentrates mainly on photon physics.
- When there is also an inner surface S_i enclosing the origin and at which the field diverges, special conditions have to be imposed for $\mathbf{A}_s$ and ϕ_s to represent a total energy, and for convergent integrated expressions still to result from the analysis [13,20]. These conditions will apply to a model of charged particle equilibrium states, such as those representing charged leptons discussed in Section V.A and Appendix B.

B. Formulation in Terms of Quantum Mechanics

An adaptation of quantum mechanics implies that a number of constraints are imposed on the system as follows.

- The energy is given in terms of the quantum $h\nu$, where ν is the frequency.
- The angular momentum (spin) of a particle-like state becomes $h/2\pi$ for a boson and $h/4\pi$ for a fermion.
- The magnetic moment of a charged particle, such as the electron, is quantized according to the Dirac theory of the electron [36], including a small modification according to Feynman [37], which results in an excellent agreement with experiments. As based on a tentative model of "self-confined" (bound) circulating radiation [11,13,20], the quantization of energy and its alternative form mc^2 can also be shown to result in an angular momentum equal to about $h/4\pi$, and a magnetic moment of the magnitude obtained in the theory by Dirac. One way to obtain exact agreement with the results by Dirac and Feynman is provided by different spatial distributions of electric charge and energy density. This is possible within the frame of the present theory [13,20]. However, it has also to be observed that these results apply to an electron in an electromagnetic field, and they could therefore differ from the result obtained for a free electron.
- With e as a given elementary electric charge, there is also a condition on the quantization of magnetic flux. This could be reinterpreted as a subsidiary condition in an effort to quantize the electron charge and deduce its absolute value by means of the present theory [13,18,20], but the details of such an analysis are not yet available. Magnetic flux quantization is discussed in further detail in Appendix B.

In a first step, these conditions can be imposed on the general solutions of the present electromagnetic field equations. At a later stage the same equations should be quantized by the same procedure as that applied earlier in quantum electrodynamics to Maxwell's equations [39].

C. Derivation from Gauge Theory

It should finally be mentioned that the basic equations (1)–(8) have been derived from gauge theory in the vacuum, using the concept of covariant derivative and Feynman's universal influence [38]. These equations and the Proca field equations are shown to be interrelated to the well-known de Broglie theorem, in which the photon rest mass m_0 can be interpreted as nonzero and be related to a frequency $\nu' = m_0 c^2/h$. A gauge-invariant Proca equation is suggested by this analysis and relations (1)–(8). It is also consistent with the earlier conclusion that gauge invariance does not require the photon rest mass to be zero [20,38].

IV. MAIN CHARACTERISTICS OF MODIFIED FIELD THEORIES

Before turning to the details of the present analysis, we describe and compare the main features of some of the modified and extended theories that have been proposed and elaborated on with the purpose of replacing Maxwell's equations. This description includes a Proca-type equation as a starting point. Introducing the 4-potential $A_\mu = (\mathbf{A}, i\phi/c)$ and the 4-current J_μ, the latter equation can be written as

$$\Box A_\mu = \mu_0 J_\mu \tag{22}$$

A. Electron Theory by Dirac

According to the Dirac [36] electron theory, the relativistic wavefunction Ψ has four components in spin-space. With the Hermitian adjoint wave function $\bar{\Psi}$, the quantum mechanical forms of the charge and current densities become [31,40]

$$\bar{\rho} = e\bar{\Psi}\Psi \tag{23}$$

and

$$\mathbf{j} = ce(\bar{\Psi}\alpha_i\Psi) \qquad i = 1, 2, 3 \tag{24}$$

where α_i are the Dirac matrices of the three spatial directions (x, y, z). There is more than one set of choices of these matrices [41].

Expressions (23) and (24) could be interpreted as the result of the electronic charge being "smeared out" over the volume of an electron with a very small but nonzero radius. The 4-current of the right-hand side of equation (22) thus becomes

$$J_\mu = ce(\bar{\Psi}\alpha_i\Psi, i\bar{\Psi}\Psi) \tag{25}$$

in this case.

B. Photon Theory by de Broglie, Vigier, and Evans

At an early stage Einstein [42] as well as Bass and Schrödinger [43] considered the possibility for the photon to have a very small but nonzero rest mass m_0. Later de Broglie and Vigier [44] and Evans and Vigier [5] derived a corresponding form of the 4-current in the Proca-type equation (22) as given by

$$J_\mu = \left(\frac{1}{\mu_0}\right)\left(\frac{2\pi m_0 c}{h}\right)^2\left(\mathbf{A}, \frac{i\phi}{c}\right) \tag{26}$$

As a consequence, the solutions of the field equations were also found to include longitudinal fields. Thereby Evans [45] was the first to give attention to a longitudinal magnetic field part, $\mathbf{B}^{(3)}$, of the photon in the direction of propagation.

C. Present Nonzero Electric Field Divergence Theory

The present approach of Eqs. (1)–(8) includes the four-current

$$J_\mu = \bar{\rho}(\mathbf{C}, ic) = \varepsilon_0(\operatorname{div}\mathbf{E})(\mathbf{C}, ic) \tag{27}$$

The solutions of the corresponding field equations have a wide area of application. They can be integrated to yield such quantities as the electric charge of a steady particle-like state, as well as a nonzero rest mass in a dynamic state representing an individual photon that also includes longitudinal field components in the direction of propagation. Thereby application of de Broglie's theorem for the photon rest mass links the concepts of expressions (26) and (27) together, as well as those of the longitudinal magnetic fields. This point is illuminated further in the following sections.

The present theory should be interpreted as microscopic in nature, in the sense that it is based only on the electromagnetic field itself. This applies to both free states of propagating wavefronts and the possible existence of bound steady axisymmetric states in the form of self-confined circulating radiation. Consequently, the extended theory does not need to include the concept of an initial particle rest mass. The latter concept does not enter into the differential equations of the electromagnetic field, simply because a rest mass should first originate from a spatial integration of the electromagnetic energy density, such as in a bound state [11–13].

When further relating the present approach to Eqs. (23) and (24) of the Dirac theory, we therefore have to consider wavefunctions that only represent states without a rest mass. One functions of this special class is given by [40]

$$\Psi = u(x, y, z)\begin{bmatrix} U \\ 0 \\ \pm U \\ 0 \end{bmatrix} \tag{28}$$

where u is an arbitrary function and U a constant. This form yields a charge density

$$\bar{\rho} = 2e\bar{U}U\bar{u}u \tag{29}$$

and the corresponding current density components

$$j_z = c\bar{\rho};\ j_x = 0 \qquad \text{and} \qquad j_y = 0 \tag{30}$$

where a bar over U and u indicates the complex conjugate value. Other forms analogous to the wavefunction (28) can be chosen to correspond to the cases

$$j_y = \pm c\bar{\rho}: \qquad j_z = j_x = 0 \tag{31}$$

$$j_x = \pm c\bar{\rho}; \qquad j_y = j_z = 0 \tag{32}$$

This result, as well as the form of expressions (23) and (24), shows that the charge and current density relations (3), (4), and (8) of the present extended theory become consistent with and related to the Dirac theory. It also implies that this extended theory can be developed in harmony with the basis of quantum electrodynamics.

The introduced current density $\mathbf{j} = \varepsilon_0(\operatorname{div}\mathbf{E})\mathbf{C}$ is thus consistent with the corresponding formulation in the Dirac theory of the electron, but this introduction also applies to electromagnetic field phenomena in a wider sense.

D. Nonzero Conductivity Theory by Bartlett, Harmuth, Vigier, and Roy

Bartlett and Corle [46] proposed modification of Maxwell's equations in the vacuum by assigning a small nonzero electric condictivity to the formalism. As pointed out by Harmuth [47], there was never a satisfactory concept of propagation velocity of signals within the framework of Maxwell's theory. Thus, the equations of the latter fail for waves with nonnegligible relative frequency bandwidth when propagating in a dissipative medium. To resolve this problem, a nonzero electric conductivity σ and a corresponding current density

$$\mathbf{j}_\sigma = \sigma\mathbf{E} \tag{33}$$