MODERN NONLINEAR OPTICS

Part 3

Second Edition

ADVANCES IN CHEMICAL PHYSICS
VOLUME 119

Edited by

Myron W. Evans

Series Editors

I. PRIGOGINE

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INTRODUCTION

Few of us can any longer keep up with the flood of scientific literature, even in specialized subfields. Any attempt to do more and be broadly educated with respect to a large domain of science has the appearance of tilting at windmills. Yet the synthesis of ideas drawn from different subjects into new, powerful, general concepts is as valuable as ever, and the desire to remain educated persists in all scientists. This series, *Advances in Chemical Physics*, is devoted to helping the reader obtain general information about a wide variety of topics in chemical physics, a field that we interpret very broadly. Our intent is to have experts present comprehensive analyses of subjects of interest and to encourage the expression of individual points of view. We hope that this approach to the presentation of an overview of a subject will both stimulate new research and serve as a personalized learning text for beginners in a field.

I. PRIGOGINE
STUART A. RICE

PREFACE

This volume, produced in three parts, is the Second Edition of Volume 85 of the series, *Modern Nonlinear Optics*, edited by M. W. Evans and S. Kielich. Volume 119 is largely a dialogue between two schools of thought, one school concerned with quantum optics and Abelian electrodynamics, the other with the emerging subject of non-Abelian electrodynamics and unified field theory. In one of the review articles in the third part of this volume, the Royal Swedish Academy endorses the complete works of Jean-Pierre Vigier, works that represent a view of quantum mechanics opposite that proposed by the Copenhagen School. The formal structure of quantum mechanics is derived as a linear approximation for a generally covariant field theory of inertia by Sachs, as reviewed in his article. This also opposes the Copenhagen interpretation. Another review provides reproducible and repeatable empirical evidence to show that the Heisenberg uncertainty principle can be violated. Several of the reviews in Part 1 contain developments in conventional, or Abelian, quantum optics, with applications.

In Part 2, the articles are concerned largely with electrodynamical theories distinct from the Maxwell–Heaviside theory, the predominant paradigm at this stage in the development of science. Other review articles develop electrodynamics from a topological basis, and other articles develop conventional or U(1) electrodynamics in the fields of antenna theory and holography. There are also articles on the possibility of extracting electromagnetic energy from Riemannian spacetime, on superluminal effects in electrodynamics, and on unified field theory based on an SU(2) sector for electrodynamics rather than a U(1) sector, which is based on the Maxwell–Heaviside theory. Several effects that cannot be explained by the Maxwell–Heaviside theory are developed using various proposals for a higher-symmetry electrodynamical theory. The volume is therefore typical of the second stage of a paradigm shift, where the prevailing paradigm has been challenged and various new theories are being proposed. In this case the prevailing paradigm is the great Maxwell–Heaviside theory and its quantization. Both schools of thought are represented approximately to the same extent in the three parts of Volume 119.

As usual in the *Advances in Chemical Physics* series, a wide spectrum of opinion is represented so that a consensus will eventually emerge. The prevailing paradigm (Maxwell–Heaviside theory) is ably developed by several groups in the field of quantum optics, antenna theory, holography, and so on, but the paradigm is also challenged in several ways: for example, using general relativity, using O(3) electrodynamics, using superluminal effects, using an extended electrodynamics based on a vacuum current, using the fact that longitudinal waves may appear in vacuo on the U(1) level, using a reproducible and repeatable device, known as the *motionless electromagnetic generator*, which extracts electromagnetic energy from Riemannian spacetime, and in several other ways. There is also a review on new energy sources. Unlike Volume 85, Volume 119 is almost exclusively dedicated to electrodynamics, and many thousands of papers are reviewed by both schools of thought. Much of the evidence for challenging the prevailing paradigm is based on empirical data, data that are reproducible and repeatable and cannot be explained by the Maxwell–Heaviside theory. Perhaps the simplest, and therefore the most powerful, challenge to the prevailing paradigm is that it cannot explain interferometric and simple optical effects. A non-Abelian theory with a Yang–Mills structure is proposed in Part 2 to explain these effects. This theory is known as O(3) *electrodynamics* and stems from proposals made in the first edition, Volume 85.

As Editor I am particularly indebted to Alain Beaulieu for meticulous logistical support and to the Fellows and Emeriti of the Alpha Foundation's Institute for Advanced Studies for extensive discussion. Dr. David Hamilton at the U.S. Department of Energy is thanked for a Website reserved for some of this material in preprint form.

Finally, I would like to dedicate the volume to my wife, Dr. Laura J. Evans.

Myron W. Evans

Ithaca, New York

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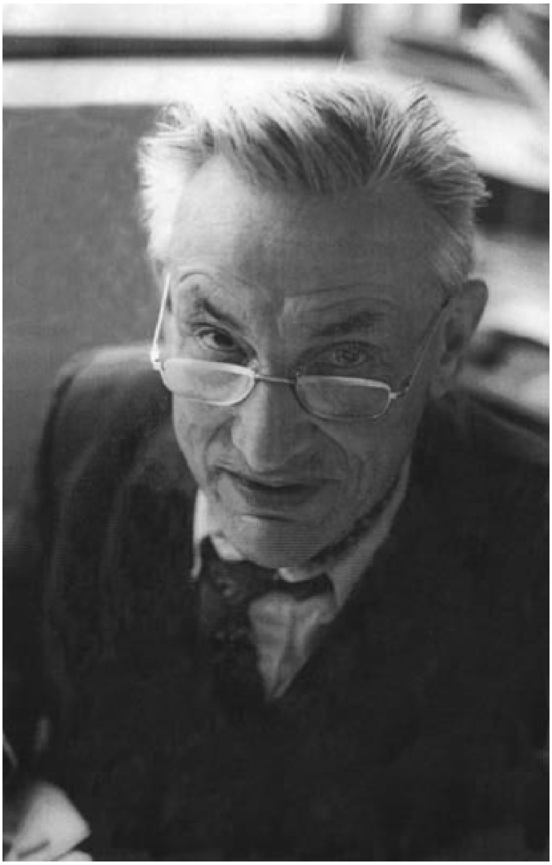

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ADVANCES IN CHEMICAL PHYSICS

VOLUME 119

THE PRESENT STATUS OF THE QUANTUM THEORY OF LIGHT

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CONTENTS

I. INTRODUCTION

If one takes as the birth of the quantum theory of light, the publication of Planck's famous paper solving the difficulties inherent in the blackbody spectrum [1], then we are currently marking its centenary. Many developments have occurred since 1900 or so and are briefly reviewed below. (See Selleri [27] or Milloni [6] for a more comprehensive historical review). The debates concerning wave–particle duality are historically rooted in the seventeenth century with the publication of Newton's *Optiks* [2] and the *Treatise on Light* by Christian Huygens [3]. For Huygens, light was a form of wave motion propagating through an ether that was conceived as a substance that was "as nearly approaching to perfect hardness and possessing a springiness as prompt as we choose." For Newton, however, light comprised material particles and he argues, contra Huygens, "Are not all hypotheses erroneous, in which Light is supposed to consist of Pression, or Motion propagated through a Fluid medium?" (see Newton [2], Query 28). Newton attempts to refute Huygens' approach by pointing to the difficulties in explaining double refraction if light is simply a form of wave motion and asks, "Are not the Rays of Light very small bodies emitted from shining substances? For such bodies will pass through uniform Mediums in right Lines without bending into Shadow, which is the Nature of the Rays of Light?" (Ref. 2, Query 29). The corpuscular theory received a major blow in the nineteenth century with the publication of Fresnel's essay [4] on the diffraction of light. Poisson argued on the basis of Fresnel's analysis that a perfectly round object should diffract so as to produce a bright spot on the axis behind it. This was offered as a reductio ad absurdum argument against wave theory. However, Fresnel and Arago carried out the actual experiment and found that there is indeed a diffracted bright spot. The nineteenth century also saw the advent of accurate methods for the determination of the speed of light by Fizeau and Foucault that were used to verify the prediction from Maxwell's theory relating the velocity of light to known electric and magnetic constants. Maxwell's magnificent theory of electromagnetic waves arose from the work of Oersted, Ampère, and Faraday, which proved the intimate interconnection between electric and magnetic phenomena.

This volume discusses the consequences of modifying the traditional, classical view of light as a transverse electromagnetic wave whose electric and magnetic field components exist only in a plane perpendicular to the axis of propagation, and posits the existence of a longitudinal magnetic field component. These considerations are of relatively recent vintage, however [5].

The corpuscular view was revived in a different form early in twentieth century with Planck's solution of the blackbody problem and Einstein's adoption of the photon model in 1905. Milloni [6] has emphasized the fact that Einstein's famous 1905 paper [7] "Concerning a heuristic point of view toward the emission and transformation of light'' argues strongly for a model of light that *simultaneously* displays the properties of waves and particles. He quotes Einstein:

> The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena and will probably never be replaced by another theory. It should be kept in mind, however, that the optical observations refer to time averages rather than instantaneous values. In spite of the complete experimental confirmation of the theory as applied to diffraction, reflection, refraction, dispersion, etc., it is still conceivable that the theory of light which operates with continuous spatial functions may lead to contradictions with experience when it is applied to the phenomena of emission and transformation of light.
>
> According to the hypothesis that I want here to propose, when a ray of light expands starting from a point, the energy does not distribute on ever increasing volumes, but remains constituted of a finite number of energy quanta localized in space and moving without subdividing themselves, and unable to be absorbed or emitted partially.

This is the famous paper where Einstein, adopting Planck's idea of light quanta, gives a complete account of the photoelectric effect. He predicts the linear relationship between radiation frequency and stopping potential: ''As far as I can see, there is no contradiction between these conceptions and the properties of the photoelectric effect observed by Herr Lenard. If each energy quantum of the incident light, independently of everything else, delivers its energy to electrons, then the velocity distribution of the ejected electrons will be independent of the intensity of the incident light. On the other hand the number of electrons leaving the body will, if other conditions are kept constant, be proportional to the intensity of the incident light.''

Textbooks frequently cite this work as strong empirical evidence for the existence of photons as quanta of electromagnetic energy localized in space and time. However, it has been shown that [8] a complete account of the photoelectric effect can be obtained by treating the electromagnetic field as a classical Maxwellian field and the detector is treated according to the laws of quantum mechanics.

In view of his subsequent discomfort with dualism in physics, it is ironic that Einstein [9] gave a treatment of the fluctuations in the energy of electromagnetic waves that is fundamentally dualistic insofar that, if the Rayleigh–Jeans formula is adopted, the fluctuations are characteristic of electromagnetic waves. However, if the Wien law is used, the fluctuations are characteristic of particles. Einstein made several attempts to derive the Planck radiation law without invoking quantization of the radiation but without success. There was no alternative but to accept the quantum. This raised immediately the difficult question as to how such quanta gave rise to interference phenomena. Einstein suggested that perhaps light quanta need not interfere with themselves, but might interfere with other quanta as they propagated. This suggestion was soon ruled out by interference experiments conduced at extremely low light levels. Dirac, in his well-known textbook [10] on quantum mechanics, stated "Each photon interferes only with itself. Interference between two different photons never occurs." The latter part of this statement is now known to be wrong [11]. The advent of highly coherent sources has enabled two-beam interference with two separate sources. In these experiments, the classic interference pattern is not observed but rather intensity correlations between the two beams are measured [12]. The recording of these intensity correlations is proof that the electromagnetic fields from the two lasers have superposed. As Paul [11] argues, any experiment that indicates that such a superposition has occurred should be called an interference experiment.

Taylor [13] was the first to report on two-beam interference experiments undertaken at extremely low light levels such that one can assert that, on average, there is never more than one photon in the apparatus at any given time. Such experiments have been repeated many times. However, given that the sources used in these experiments generated light beams that exhibited photon bunching [14], the basic assumption that there is only ever one photon in the apparatus at any given time is not sound. More recent experiments using sources that emit single-photon states have been performed [15–17].

In 1917 Einstein [18] wrote a paper on the dualistic nature of light in which he discusses emission "without excitation from external causes," in other words stimulated emission and also spontaneous absorption and emission. He derives Planck's formula but also discusses the recoil of molecules when they emit photons. It is the latter discussion that Einstein regarded as the most significant aspect of the paper: "If a radiation bundle has the effect that a molecule struck by it absorbs or emits a quantity of energy $h\nu$ in the form of radiation (ingoing radiation), then a momentum $h\nu/c$ is always transferred to the molecule. For an absorption of energy, this takes place in the direction of propagation of the radiation bundle; for an emission, in the opposite direction."

In 1923, Compton [19] gave convincing experimental evidence for this process: "The experimental support of the theory indicates very convincingly that a radiation quantum carries with itself, directed momentum as well as energy." Einstein's dualism raises the following difficult question: If the particle carries all the energy and momentum then, in what sense can the wave be regarded as real? Einstein's response was to refer to such waves as "ghost fields" (Gespensterfelder). Such waves are also referred to as "empty" - a wave propagating in space and time but (virtually) devoid of energy and momentum. If described literally, then such waves could not induce any physical changes in matter. Nevertheless, there have been serious proposals for experiments that might lead to the detection of "empty" waves associated with either photons [20] or neutrons [21]. However, by making additional assumptions about the nature of such ‘‘empty’’ waves [22], experiments have been proposed that might reveal their actual existence. One such experiment [23] has not yielded any such definitive evidence. Other experiments designed to determine whether empty waves can induce coherence in a two-beam interference experiment have not revealed any evidence for their existence [24], although Croca [25] now argues that this experiment should be regarded as inconclusive as the count rates were very low.

Controversies still persist in the interpretation of the quantum theory of light and indeed more generally in quantum mechanics itself. This happens notwithstanding the widely held view that all the difficult problems concerning the correct interpretation of quantum mechanics were resolved a long time ago in the famous encounters between Einstein and Bohr. Recent books have been devoted to foundational issues [26] in quantum mechanics, and some seriously question Bohrian orthodoxy [27,28]. There is at least one experiment described in the literature [29] that purports to do what Bohr prohibits: demonstrate the simultaneous existence of wave and particle-like properties of light.

Einstein’s dualistic approach to electromagnetic radiation was generalized by de Broglie [30] to electrons when he combined results from the special theory of relativity (STR) and Planck’s formula for the energy of a quantum to produce his famous formula relating wavelength to particle momentum. His model of a particle was one that contained an internal periodic motion plus an external wave of different frequency that acts to guide the particle. In this model, we have a wave–particle unity—both objectively exist. To quote de Broglie [31]: ‘‘The electron . . . must be associated with a wave, and this wave is no myth; its wavelength can be measured and its interferences predicted.’’ De Broglie’s approach to physics has been described by Lochak [32] as quoted in Selleri [27]:

> Louis de Broglie is an intuitive spirit, concrete and realist, in love with simple images in three-dimensional space. He does not grant ontological value to mathematical models, in particular to geometrical representations in abstract spaces; he does not consider and does not use them other than as convenient mathematical instruments, among others, and it is not in their handling that his physical intuition is directly applied; faced with these abstract representations, he always keeps in mind the idea of all phenomena actually taking place in physical space, so that these mathematical modes of reasoning have a true meaning in his eyes only insofar as he perceives at all times what physical laws they correspond to in usual space.

De Broglie’s views are not widely subscribed to today since as with ‘‘empty’’ waves, there is no compelling experimental evidence for the existence of physical waves accompanying the particle’s motion (see, however, the discussion in Selleri [27]). Models of particles based on de Broglian ideas are still advanced by Vigier, for example [33].

As is well known, de Broglie abandoned his attempts at a realistic account of quantum phenomena for many years until David Bohm's discovery of a solution of Schrödinger's equation that lends itself to an interpretation involving a physical particle traveling under the influence of a so-called quantum potential.

As de Broglie stated:

> For nearly twenty-five years, I remained loyal to the Bohr-Heisenberg view, which has been adopted almost unanimously by theorists, and I have adhered to it in my teaching, my lectures and my books. In the summer of 1951, I was sent the preprint of a paper by a young American physicist David Bohm, which was subsequently published in the January 15, 1952 issue of the Physical Review. In this paper, Mr. Bohm takes up the ideas I had put forward in 1927, at least in one of the forms I had proposed, and extends them in an interesting way on some points. Later, J.P. Vigier called my attention to the resemblance between a demonstration given by Einstein regarding the motion of particles in General Relativity and a completely independent demonstration I had given in 1927 in an exercise I called the "theory of the double solution."

A comprehensive account of the views of de Broglie, Bohm, and Vigier is given in Jeffers et al. [34]. In these models, contra Bohr particles actually do have trajectories. Trajectories computed for the double-slit experiment show patterns that reproduce the interference pattern observed experimentally [35]. Furthermore, the trajectories so computed never cross the plane of symmetry so that one can assert *with certainty* through which the particles traveled. This conclusion was also reached by Prosser [36,37] in his study of the double-slit experiment from a strictly Maxwellian point of view. Poynting vectors were computed whose distribution mirrors the interference pattern, and these never cross the symmetry plane as in the case of the de Broglie–Bohm–Vigier models. Prosser actually suggested an experimental test of this feature of his calculations. The idea was to illuminate a double-slit apparatus with very short microwave pulses and examine the received radiation at a suitable point off-axis behind the double slits. Calculations showed that for achievable experimental parameters, one could detect either two pulses if the orthodox view were correct, or only one pulse if the Prosser interpretation were correct. However, further investigation [38] showed that the latter conclusion was not correct. Two pulses would be observed, and their degree of separation (i.e., distinguishability) would be inversely related to the degree of contrast in the interference fringes.

Contemporary developments include John Bell's [39] discovery of his famous inequality that is predicated on the assumptions of both locality and realism. Bell's inequality is violated by quantum mechanics, and consequently, it is frequently argued, one cannot accept quantum mechanics, realism, and locality. Experiments on correlated particles appear to demonstrate that the Bell inequalities are indeed violated. Of the three choices, the most acceptable one is to abandon locality. However, Afriat and Selleri [40] have extensively reviewed both the current theoretical and experimental situation regarding the status of Bell's inequalities. They conclude, contrary to accepted wisdom, that one can construct local and realistic accounts of quantum mechanics that violate Bell's inequalities, and furthermore, there remain several loopholes in the experiments that have not yet been closed that allow for local and realist interpretations. No actual experiment that has been performed to date has *conclusively* demonstrated that locality has to be abandoned. However, experiments that approximate to a high degree the original gedanken experiment discussed by David Bohm, and that potentially close all known loopholes, will soon be undertaken. See the review article by Fry and Walther [41]. To quote these authors: "Quantum mechanics, even 50 years after its formulation, is still full of surprises." This underscores Einstein's famous remark: "All these years of conscious brooding have brought me no nearer to the answer to the question "What are light quanta?" Nowadays, every Tom, Dick, and Harry thinks he knows it, but he is mistaken."

II. THE PROCA EQUATION

The first inference of photon mass was made by Einstein and de Broglie on the assumption that the photon is a particle, and behaves as a particle in, for example, the Compton and photoelectric effects. The wave–particle duality of de Broglie is essentially an extension of the photon, as the quantum of energy, to the photon, as a particle with quantized momentum. The Beth experiment in 1936 showed that the photon has angular momentum, whose quantum is $\hbar$. Other fundamental quanta of the photon are inferred in Ref. 42. In 1930, Proca [43] extended the Maxwell–Heaviside theory using the de Broglie guidance theorem:

$$\hbar\omega_0 = m_0 c^2 \tag{1}$$

where m_0 is the rest mass of the photon and m_0c^2 is its rest energy, equated to the quantum of rest energy $\hbar\omega_0$. The original derivation of the Proca equation therefore starts from the Einstein equation of special relativity:

$$p^{\mu}p_{\mu} = m_0^2c^2 \tag{2a}$$

The usual quantum ansatz is applied to this equation to obtain a wave equation:

$$En = i\hbar\frac{\partial}{\partial t}; \qquad \boldsymbol{p} = -i\hbar\nabla \tag{2b}$$

This is an example of the de Broglie wave–particle duality. The resulting wave equation is

$$\left(\Box + \frac{m_0^2 c^4}{\hbar^2}\right)\psi = 0 \tag{3}$$

where ψ is a wave function, whose meaning was first inferred by Born in 1926. If the wave function is a scalar, Eq. (3) becomes the Klein–Gordon equation. If ψ is a 2-spinor, Eq. (3) becomes the van der Waerden equation, which can be related analytically to the Dirac equation, and if ψ is the electromagnetic 4-potential A^μ, Eq. (3) becomes the Proca equation:

$$\Box A^\mu = -\left(\frac{m_0 c^2}{\hbar}\right)^2 A^\mu \tag{4}$$

So A^μ can act as a wave function and the Proca equation can be regarded as a quantum equation if A^μ is a wave function in configuration space, and as a classical equation in momentum space.

It is customary to develop the Proca equation in terms of the vacuum charge current density

$$\Box A^\mu = -\left(\frac{m_0 c^2}{\hbar}\right)^2 A^\mu = -\kappa^2 A^\mu = \frac{1}{\varepsilon_0} J^\mu(\text{vac}) \tag{5}$$

The potential A^μ therefore has a physical meaning in the Proca equation because it is directly proportional to J^μ(vac). The Proca equations in the vacuum are therefore

$$\partial_\mu F^{\mu\nu} + \left(\frac{m_0 c^2}{\hbar}\right)^2 A^\nu = 0 \tag{6}$$

$$\partial_\mu A^\mu = 0 \tag{7}$$

and, as described in the review by Evans in Part 2 of this compilation [44], these have the structure of the Panofsky, Phillips, Lehnert, Barrett, and O(3) equations, a structure that can also be inferred from the symmetry of the Poincaré group [44]. Lehnert and Roy [45] self-consistently infer the structure of the Proca equations from their own equations, which use a vacuum charge and current.

The problem with the Proca equation, as derived originally, is that it is not gauge-invariant because, under the U(1) gauge transform [46]

$$A^\mu \rightarrow A^\mu + \frac{1}{g}\partial^\mu \Lambda \tag{8}$$

the left-hand side of Eq. (4) is invariant but an arbitrary quantity $\frac{1}{g}\partial^{\mu}\Lambda$ is added to the right-hand side. This is paradoxical because the Proca equation is well founded in the quantum ansatz and the Einstein equation, yet violates the fundamental principle of gauge invariance. The usual resolution of this paradox is to assume that the mass of the photon is identically zero, but this assumption leads to another paradox, because a particle must have mass by definition, and the wave-particle dualism of de Broglie becomes paradoxical, and with it, the basis of quantum mechanics.

In this section, we suggest a resolution of this >70-year-old paradox using O(3) electrodynamics [44]. The new method is based on the use of covariant derivatives combined with the first Casimir invariant of the Poincaré group. The latter is usually written in operator notation [42,46] as the invariant $P_{\mu}P^{\mu}$, where P^{μ} is the generator of spacetime translation:

$$P^{\mu} = i\partial^{\mu} = \frac{p^{\mu}}{\hbar} \tag{9}$$

The ordinary derivative in gauge theory becomes the covariant derivative

$$\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} - igA_{\mu} \tag{10}$$

for all gauge groups. The generator D_{μ} is a generator of the Poincaré group because it obeys the Jacobi identity

$$\sum_{\sigma,\nu,\mu} \left[D_{\sigma}, \left[D_{\nu}, D_{\mu}\right]\right] \equiv 0 \tag{11}$$

and the covariant derivative (10) can be regarded as a sum of spacetime translation generators.

The basic assumption is that the photon acquires mass through the invariant

$$D_{\mu}D^{\mu *}\psi = 0 \tag{12}$$

for any gauge group. This equation can be developed for any gauge group as

$$\left(\partial_{\mu} - igA_{\mu}\right)(\partial^{\mu} + igA^{\mu *})\psi = 0 \tag{13}$$

and can be expressed as

$$\begin{aligned}
&\Box\psi - igA_{\mu}\partial^{\mu}\psi + ig\partial_{\mu}(A^{\mu}\psi) + g^2 A_{\mu}A^{\mu}\psi \\
&\quad = 0 \\
&\quad = \Box\psi - igA_{\mu}\partial^{\mu}\psi + ig\psi\partial_{\mu}A^{\mu} + igA^{\mu}\partial_{\mu}\psi + g^2 A_{\mu}A^{\mu}\psi \\
&\quad = \left(\Box + ig\partial_{\mu}A^{\mu} + g^2 A_{\mu}A^{\mu}\right)\psi \\
&\quad = 0
\end{aligned} \tag{14}$$

This equation reduces to

$$\left(\Box + \kappa^2\right)\psi = -ig\partial_\mu A^\mu \psi \tag{15}$$

for any gauge group because

$$g = \frac{\kappa}{A^{(0)}}; \qquad A_\mu A^\mu = A^{(0)2} \tag{16}$$

In the plane-wave approximation:

$$\partial_\mu A^\mu = 0 \tag{17a}$$

and the Proca equation for any gauge group becomes

$$\left(\Box + \kappa^2\right)\psi = 0 \tag{17b}$$

for any gauge group.

Therefore Eq. (18) has been shown to be an invariant of the Poincaré group, Eq. (12), and a product of two Poincaré covariant derivatives. In momentum space, this operator is equivalent to the Einstein equation under any condition. The conclusion is reached that the factor g is nonzero in the vacuum.

In gauge theory, for any gauge group, however, a rotation

$$\psi' = e^{i\Lambda}\psi \equiv S\psi \tag{18}$$

in the internal gauge space results in the gauge transformation of A_μ as follows

$$A'_\mu = SA_\mu S^{-1} - \frac{i}{g}\left(\partial_\mu S\right)S^{-1} \tag{19}$$

and to construct a gauge-invariant Proca equation from the operator (16), a search must be made for a potential A_μ that is invariant under gauge transformation. It is not possible to find such a potential on the U(1) level because the inhomogeneous term is always arbitrary. On the O(3) level, however, the potential can be expressed as

$$\boldsymbol{A}_\mu = A_\mu^{(2)}\boldsymbol{e}^{(1)} + A_\mu^{(1)}\boldsymbol{e}^{(2)} + A_\mu^{(3)}\boldsymbol{e}^{(3)} \tag{20}$$

if the internal gauge space is a physical space with O(3) symmetry described in the complex circular basis ((1),(2),(3)) [3]. A rotation in this physical gauge space can be expressed in general as

$$\psi' = \exp\left(i\mathrm{M}^a\Lambda^a(x^\mu)\right)\psi \tag{21}$$

where M^a are the rotation generators of O(3) and where $\Lambda^{(1)}, \Lambda^{(2)}$, and $\Lambda^{(3)}$ are angles.

Developing Eq. (13), we obtain

$$\begin{aligned}(\partial_\mu - igA_\mu^{(1)})(\partial^\mu + igA^{\mu(2)})\psi &= 0\\ (\partial_\mu - igA_\mu^{(2)})(\partial^\mu + igA^{\mu(1)})\psi &= 0\\ (\partial_\mu - igA_\mu^{(3)})(\partial^\mu + igA^{\mu(3)})\psi &= 0\end{aligned} \tag{22}$$

The eigenfunction ψ may be written in general as the O(3) vector

$$\psi \equiv \boldsymbol{A}^\nu \tag{23}$$

and under gauge transformation

$$\boldsymbol{A}^{\nu'} = \exp(i\mathrm{M}^a \Lambda^a(x^\mu))\boldsymbol{A}^\nu \tag{24}$$

from Eq. (21). Here, $\Lambda^{(1)}, \Lambda^{(2)}$, and $\Lambda^{(3)}$ are angles in the physical internal gauge space of O(3) symmetry.

Therefore Eqs. (22) become

$$\Box^2 \boldsymbol{A}^\nu = -\kappa^2 \boldsymbol{A}^\nu = \frac{1}{\varepsilon_0}\boldsymbol{J}^\nu(\mathrm{vac}) \tag{25}$$

where

$$\boldsymbol{J}^\nu = \left(\rho^{(i)}, \frac{\boldsymbol{J}^{(i)}}{c}\right) \qquad i = 1, 2, 3 \tag{26}$$

and Eqs. (25) become

$$\Box A^{\nu(1)} = -\kappa^2 A^{\nu(1)} = \frac{J^{\nu(1)}}{\varepsilon_0} \tag{27}$$

$$\Box A^{\nu(2)} = -\kappa^2 A^{\nu(2)} = \frac{J^{\nu(2)}}{\varepsilon_0} \tag{28}$$

$$\Box A^{\nu(3)} = 0 \tag{29}$$

It can be seen that the photon mass is carried by $A^{\nu(1)}$ and $A^{\nu(2)}$, but not by $A^{\nu(3)}$. This result is also obtained by a different route using the Higgs mechanism in Ref. 42, and is also consistent with the fact that the mass associated with $A^{\nu(3)}$ corresponds with the superheavy boson inferred by Crowell [42], reviewed in

Ref. 42 and observed in a LEP collaboration [42]. The effect of a gauge transformation on Eqs. (27)–(29) is as follows:

$$\square\left(A_{\mu}^{(1)}+\frac{1}{g}\partial_{\mu}\Lambda^{(1)}\right)=-\kappa^{2}\left(A_{\mu}^{(1)}+\frac{1}{g}\partial_{\mu}\Lambda^{(1)}\right) \tag{30}$$

$$\square\left(A_{\mu}^{(2)}+\frac{1}{g}\partial_{\mu}\Lambda^{(2)}\right)=-\kappa^{2}\left(A_{\mu}^{(2)}+\frac{1}{g}\partial_{\mu}\Lambda^{(2)}\right) \tag{31}$$

$$\square\left(A_{\mu}^{(3)}+\frac{1}{g}\partial_{\mu}\Lambda^{(3)}\right)=0 \tag{32}$$

Equations (30) and (31) are eigenequations with the same eigenvalue, $-\kappa^2$, as Eqs. (27) and (28). On the O(3) level, the eigenfunctions $A_{\mu}^{(1)}+\frac{1}{g}\partial_{\mu}\Lambda^{(1)}$ are not arbitrary because $\Lambda^{(1)}$ and $\Lambda^{(2)}$ are angles in a physical internal gauge space. The original Eq. (12) is gauge-invariant, however, because on gauge transformation

$$g^{2}A_{\mu}A^{\mu*}\rightarrow g^{2}A'_{\mu}A^{\mu*'};\qquad g'=\frac{\kappa}{A^{(0)'}} \tag{33}$$

and

$$D_{\mu}D^{\mu*}\psi\rightarrow D_{\mu}D^{\mu*}(S\psi)=\psi D_{\mu}D^{\mu*}S+SD_{\mu}D^{\mu*}\psi=0 \tag{34}$$

because S must operate on ψ.

In order for Eq. (34) to be compatible with Eqs. (30) and (31), we obtain

$$\square(\partial_{\mu}\Lambda^{(1)})=-\kappa^{2}(\partial_{\mu}\Lambda^{(1)}) \tag{35}$$

$$\square(\partial_{\mu}\Lambda^{(2)})=-\kappa^{2}(\partial_{\mu}\Lambda^{(2)}) \tag{36}$$

which are also Proca equations. So the >70-year-old problem of the lack of gauge invariance of the Proca equation is solved by going to the O(3) level.

The field equations of electrodynamics for any gauge group are obtained from the Jacobi identity of Poincaré group generators [42,46]:

$$\sum_{\sigma,\mu,\nu}\left[D_{\sigma},\left[D_{\mu},D_{\nu}\right]\right]\equiv 0 \tag{37}$$

If the potential is classical, the Jacobi identity (37) can be written out as

$$D_{\sigma}G_{\mu\nu}+D_{\mu}G_{\nu\sigma}+D_{\nu}G_{\sigma\mu}-G_{\mu\nu}D_{\sigma}-G_{\nu\sigma}D_{\mu}-G_{\sigma\mu}D_{\nu}\equiv 0 \tag{38}$$

This equation implies the Jacobi identity:

$$[A_{\sigma},G_{\mu\nu}]+[A_{\mu},G_{\nu\sigma}]+[A_{\nu},G_{\sigma\mu}]\equiv 0 \tag{39}$$

which in vector form can be written as

$$\begin{aligned} \boldsymbol{A}_\mu \times \tilde{\boldsymbol{G}}^{\mu\nu} &= \boldsymbol{A}^\sigma \times \boldsymbol{G}^{\mu\nu} + \boldsymbol{A}^\mu \times \boldsymbol{G}^{\nu\sigma} + \boldsymbol{A}^\nu \times \boldsymbol{G}^{\sigma\mu} \\ &\equiv \boldsymbol{0} \end{aligned} \tag{40}$$

As a result of this Jacobi identity, the homogeneous field equation

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{41}$$

reduces to

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{42}$$

for all gauge group symmetries. The implication is that instantons or pseudoparticles do not exist in Minkowski spacetime in a pure gauge theory, because magnetic monopoles and currents vanish for all internal gauge group symmetries. Therefore, the homogeneous field equation of electrodynamics, considered as a gauge theory of any internal symmetry, can be obtained from the Jacobi identity (42) of the Poincaré group of Minkowski spacetime. The homogeneous field equation is gauge-covariant for any internal symmetry. Analogously, the Proca equation is the mass Casimir invariant (12) of the Poincaré group of Minkowski spacetime.

There are several major implications of the Jacobi identity (40), so it is helpful to give some background for its derivation. On the U(1) level, consider the following field tensors in $c = 1$ units and contravariant covariant notation in Minkowski spacetime:

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -B^1 & -B^2 & -B^3 \\ B^1 & 0 & E^3 & -E^2 \\ B^2 & -E^3 & 0 & E^1 \\ B^3 & E^2 & -E^1 & 0 \end{bmatrix}; \qquad \tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{bmatrix}$$

$$F_{\rho\sigma} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}; \qquad F^{\rho\sigma} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{bmatrix} \tag{43}$$

These tensors are generated from the duality relations [47]

$$\begin{aligned} \tilde{G}^{\mu\nu} &= \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}; \qquad & G^{\mu\nu} &= -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\tilde{G}_{\rho\sigma} \\ \tilde{G}_{\mu\nu} &= \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}; \qquad & G_{\mu\nu} &= -\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\tilde{G}^{\rho\sigma} \end{aligned} \tag{44}$$

where the totally antisymmetric unit tensor is defined as

$$\varepsilon^{0123} = 1 = -\varepsilon_{0123} \tag{45}$$

and result in the following Jacobi identity:

$$\partial_\mu \tilde{F}^{\mu\nu} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} \equiv 0 \tag{46}$$

It also follows that

$$\partial_\mu F^{\mu\nu} = \partial_\sigma \tilde{F}^{\mu\nu} + \partial_\mu \tilde{F}^{\nu\sigma} + \partial_\nu \tilde{F}^{\sigma\mu} \tag{47}$$

The proof of the Jacobi identity (46) can be seen by considering a development such as

$$\begin{aligned}\partial_\mu \tilde{F}^{\mu\nu} &= \frac{1}{2}\partial_\mu(\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma})\\ &= \frac{1}{2}\partial_\mu\big(\varepsilon^{\mu\nu 01}F_{01} + \varepsilon^{\mu\nu 02}F_{02} + \varepsilon^{\mu\nu 03}F_{03} + \varepsilon^{\mu\nu 10}F_{10} + \varepsilon^{\mu\nu 20}F_{20} + \varepsilon^{\mu\nu 30}F_{30}\\ &\quad + \varepsilon^{\mu\nu 12}F_{12} + \varepsilon^{\mu\nu 13}F_{13} + \varepsilon^{\mu\nu 21}F_{21} + \varepsilon^{\mu\nu 31}F_{31} + \varepsilon^{\mu\nu 23}F_{23} + \varepsilon^{\mu\nu 32}F_{32}\big)\end{aligned} \tag{48}$$

If $\nu = 0$, then

$$\partial_1 \tilde{F}^{10} + \partial_2 \tilde{F}^{20} + \partial_3 \tilde{F}^{30} = -\partial_1 F^{23} - \partial_2 F^{13} - \partial_3 F^{12} \equiv 0 \tag{49}$$

Equation (47) may be proved similarly. On the O(3) level there exist the analogous equations (40) and

$$\boldsymbol{A}_\mu \times \boldsymbol{G}^{\mu\nu} = \boldsymbol{A}_\sigma \times \tilde{\boldsymbol{G}}^{\mu\nu} + \boldsymbol{A}_\mu \times \tilde{\boldsymbol{G}}^{\nu\sigma} + \boldsymbol{A}_\nu \times \tilde{\boldsymbol{G}}^{\sigma\mu} \tag{50}$$

which is not zero in general.

It follows from the Jacobi identity (40) that there also exist other Jacobi identities such as [42]

$$\boldsymbol{A}_\lambda^{(2)} \times (\boldsymbol{A}_\mu^{(1)} \times \boldsymbol{A}_\nu^{(2)}) + \boldsymbol{A}_\mu^{(2)} \times (\boldsymbol{A}_\nu^{(1)} \times \boldsymbol{A}_\lambda^{(2)}) + \boldsymbol{A}_\nu^{(2)} \times (\boldsymbol{A}_\lambda^{(1)} \times \boldsymbol{A}_\mu^{(2)}) \equiv \boldsymbol{0} \tag{51}$$

The Jacobi identity (40) means that the homogeneous field equation of electrodynamics for any gauge group is

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv 0 \tag{52}$$