# **Global Positioning** Systems, **Inertial Navigation,** and Integration

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### *Preface*

This book is intended for people who will use Global Positioning Systems (GPS), Inertial Navigation Systems (INS), and Kalman filters. Our objective is to give our readers a working familiarity with both the *theoretical and practical* aspects of these subjects. For that purpose we have included "real-world" problems from practice as illustrative examples. We also cover the more practical aspects of implementation: how to represent problems in a mathematical model, analyze performance as a function of model parameters, implement the mechanization equations in numerically stable algorithms, assess its computational requirements, test the validity of results, and monitor performance in operation with sensor data from GPS and INS. These important attributes, often overlooked in theoretical treatments, are essential for effective application of theory to real-world problems.

The accompanying diskette contains MATLAB® m-files to demonstrate the workings of the Kalman filter algorithms with GPS and INS data sets, so that the reader can better discover how the Kalman filter works by observing it in action with GPS and INS. The implementation of GPS, INS, and Kalman filtering on computers also illuminates some of the practical considerations of finite-wordlength arithmetic and the need for alternative algorithms to preserve the accuracy of the results. If the student wishes to apply what she or he learns, then it is essential that she or he experience its workings and failings—and learn to recognize the difference.

The book is organized for use as a text for an introductory course in GPS technology at the senior level or as a first-year graduate level course in GPS, INS, and Kalman filtering theory and application. It could also be used for self-instruction or review by practicing engineers and scientists in these fields.

Chapter 1 informally introduces the general subject matter through its history of development and application. Chapters 2-5 and 9 cover the basic theory of GPS and present material for a senior-level class in geomatics, electrical engineering, systems engineering, and computer science. Chapters 6-8 cover the application of GPS and INS integration with Kalman filtering. These chapters could be covered in a graduate level course in Electrical, computer, and systems engineering.

Chapter 6 gives the basics of INS. Chapter 7 covers linear optimal filters, predictors, and nonlinear estimation by "extended" Kalman filters. Applications of these techniques to the identification of unknown parameters of systems are given as examples. Chapter 8 deals with Kalman filter engineering, with algorithms provided for computer implementation. Chapter 9 covers current developments in the Wide Area Augmentation System (WAAS) and Local-Area Augmentation System (LAAS), including Local Area Differential GPS (LADGPS) and Wide-Area Differential GPS (WADGPS).

The following chapter-level dependency graph shows the book's organization and how the subject of each chapter depends upon material in other chapters. The arrows in the figure indicate the recommended order of study. Boxes above another box and



connected by arrows indicate that the material represented by the upper boxes is background material for the subject in the lower box.

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### *Acronyms*











#### *Global Positioning Systems, Inertial Navigation, and Integration*

*1*

## *Introduction*

The five basic forms of navigation are as follows:

- 1. Pilotage, which essentially relies on recognizing landmarks to know where you are. It is older than human kind.
- 2. Dead reckoning, which relies on knowing where you started from, plus some form of heading information and some estimate of speed.
- 3. Celestial navigation, using time and the angles between local vertical and known celestial objects (e.g., sun, moon, or stars) [115].
- 4. Radio navigation, which relies on radio-frequency sources with known locations (including Global Positioning System satellites).
- 5. Inertial navigation, which relies on knowing your initial position, velocity, and attitude and thereafter measuring your attitude rates and accelerations. It is the only form of navigation that does not rely on external references.

These forms of navigation can be used in combination as well [16, 135]. The subject of this book is a combination of the fourth and fifth forms of navigation using Kalman filtering.

Kalman filtering exploits a powerful synergism between the *Global Positioning System* (GPS) and an *inertial navigation system* (INS). This synergism is possible, in part, because the INS and GPS have very complementary error characteristics. Short-term position errors from the INS are relatively small, but they degrade without bound over time. GPS position errors, on the other hand, are not as good over the short term, but they do not degrade with time. The Kalman filter is able to take advantage of these characteristics to provide a common, integrated navigation implementation with performance superior to that of either subsystem (GPS or INS). By using statistical information about the errors in both systems, it is able to combine a system with tens of meters position uncertainty (GPS) with another system whose position uncertainty degrades at kilometers per hour (INS) and achieve bounded position uncertainties in the order of centimeters [with differential GPS (DGPS)] to meters.

A key function performed by the Kalman filter is the statistical combination of GPS and INS information to track drifting parameters of the sensors in the INS. As a result, the INS can provide enhanced inertial navigation accuracy during periods when GPS signals may be lost, and the improved position and velocity estimates from the INS can then be used to make GPS signal reacquisition happen much faster when the GPS signal becomes available again.

This level of integration necessarily penetrates deeply into each of these subsystems, in that it makes use of partial results that are not ordinarily accessible to users. To take full advantage of the offered integration potential, we must delve into technical details of the designs of both types of systems.

#### **1.1 GPS AND GLONASS OVERVIEW**

#### **1.1.1 GPS**

The GPS is part of a satellite-based navigation system developed by the U.S. Department of Defense under its NAVSTAR satellite program [54, 56, 58-63, 96- 98].

*1.1.1.1 GPS Orbits* The fully operational GPS includes 24 or more (28 in March 2000) active satellites approximately uniformly dispersed around six circular orbits with four or more satellites each. The orbits are inclined at an angle of 55° relative to the equator and are separated from each other by multiples of  $60^{\circ}$  right ascension. The orbits are nongeostationary and approximately circular, with radii of 26,560 km and orbital periods of one-half sidereal day ( $\approx$ 11.967h). Theoretically, three or more GPS satellites will always be visible from most points on the earth's surface, and four or more GPS satellites can be used to determine an observer's position anywhere on the earth's surface 24 h per day.

*1.1.1.2 GPS Signals* Each GPS satellite carries a cesium and/or rubidium atomic clock to provide timing information for the signals transmitted by the satellites. Internal clock correction is provided for each satellite clock. Each GPS satellite transmits two spread spectrum, L-band carrier signals—an  $L_1$  signal with carrier frequency  $f_i = 1575.42 \text{ MHz}$  and an  $L_2$  signal with carrier frequency  $f_2$  = 1227.6 MHz. These two frequencies are integral multiples  $f_1$  = 1540 $f_0$  and  $f_2 = 1200f_0$  of a base frequency  $f_0 = 1.023$  MHz. The  $L_1$  signal from each satellite uses *binary phase-shift keying* (BPSK), modulated by two *pseudorandom noise* (PRN) codes in phase quadrature, designated as the C/A-code and P-code. The *L2* signal from each satellite is BPSK modulated by only the P-code. A brief description of the nature of these PRN codes follows, with greater detail given in Chapter 3.

*Compensating for Propagation Delays* This is one motivation for use of two different carrier signals  $L_1$  and  $L_2$ . Because delay varies approximately as the inverse square of signal frequency f (delay  $\propto f^{-2}$ ), the measurable differential delay between the two carrier frequencies can be used to compensate for the delay in each carrier. (See [86] for details.)

*Code Division Multiplexing* Knowledge of the PRN codes allows users independent access to multiple GPS satellite signals on the same carrier frequency. The signal transmitted by a particular GPS signal can be selected by generating and matching, or correlating, the PRN code for that particular satellite. All PRN codes are known and are generated or stored in GPS satellite signal receivers carried by ground observers. A first PRN code for each GPS satellite, sometimes referred to as a precision code or P-code, is a relatively long, fine-grained code having an associated clock or chip rate of  $10f<sub>0</sub> = 10.23 \text{ MHz}$ . A second PRN code for each GPS satellite, sometimes referred to as a clear or coarse acquisition code or C/Acode, is intended to facilitate rapid satellite signal acquisition and hand-over to the Pcode. It is a relatively short, coarser grained code having an associated clock or chip rate  $f_0 = 1.023$  MHz. The C/A-code for any GPS satellite has a length of 1023 chips or time increments before it repeats. The full P-code has a length of 259 days, during which each satellite transmits a unique portion of the full P-code. The portion of Pcode used for a given GPS satellite has a length of precisely one week (7.000 days) before this code portion repeats. Accepted methods for generating the C/A-code and P-code were established by the satellite developer<sup>1</sup> in 1991 [42, 66].

*Navigation Signal* The GPS satellite bit stream includes navigational information on the ephemeris of the transmitting GPS satellite and an almanac for all GPS satellites, with parameters providing approximate corrections for ionospheric signal propagation delays suitable for single-frequency receivers and for an offset time between satellite clock time and true GPS time. The navigational information is transmitted at a rate of 50 baud. Further discussion of the GPS and techniques for obtaining position information from satellite signals can be found in Chapter 3 and in [84, pp. 1-90].

*1.1.1.3 Selective Availability* Selective Availability (SA) is a combination of methods used by the U.S. Department of Defense for deliberately derating the accuracy of GPS for "nonauthorized" (i.e., non-U.S. military) users. The current satellite configurations use only pseudorandom dithering of the onboard time reference [134], but the full configuration can also include truncation of the

<sup>1</sup> Satellite Systems Division of Rockwell International Corporation, now part of the Boeing Company.

transmitted ephemerides. This results in three grades of service provided to GPS users. SA has been removed as of May 1, 2000.

*Precise Positioning Service* Precise Positioning Service (PPS) is the fullaccuracy, single-receiver GPS positioning service provided to the United States and its allied military organizations and other selected agencies. This service includes access to the unencrypted P-code and the removal of any SA effects.

*Standard Positioning Service without SA* Standard Positioning Service (SPS) provides GPS single-receiver (stand-alone) positioning service to any user on a continuous, worldwide basis. SPS is intended to provide access only to the C/Acode and the  $L<sub>i</sub>$  carrier.

*Standard Positioning Service with SA* The horizontal-position accuracy, as degraded by SA, currently is advertised as 100 m, the vertical-position accuracy as 156m, and time accuracy as 334ns—all at the 95% probability level. SPS also guarantees the user-specified levels of coverage, availability, and reliability.

#### **1.1.2 GLONASS**

A second configuration for global positioning is the Global Orbiting Navigation Satellite System (GLONASS), placed in orbit by the former Soviet Union, and now maintained by the Russian Republic [75, 80].

*1.1.2.1 GLONASS Orbits* GLONASS also uses 24 satellites, but these are distributed approximately uniformly in three orbital plans (as opposed to four for GPS) of eight satellites each (six for GPS). Each orbital plane has a nominal inclination of 64.8° relative to the equator, and the three orbital planes are separated from each other by multiples of 120° right ascension. GLONASS orbits have smaller radii than GPS orbits, about 25,510km, and a satellite period of revolution of approximately  $\frac{8}{17}$  of a sidereal day. A GLONASS satellite and a GPS satellite will complete 17 and 16 revolutions, respectively, around the earth every 8 days.

**/.** *1.2.2 GLONASS Signals* The GLONASS system uses frequency division multiplexing of independent satellite signals. Its two carrier signals corresponding to  $L_1$ and  $L_2$  have frequencies  $f_1 = (1.602 + 9k/16)$  GHz and  $f_2 =$  $(1.246 + 7k/16)$  GHz, where  $k = 0, 1, 2, ..., 23$  is the satellite number. These frequencies lie in two bands at  $1.597-1.617$  GHz ( $L_1$ ) and  $1240-1260$  GHz ( $L_2$ ). The  $L_1$  code is modulated by a C/A-code (chip rate  $= 0.511 \text{ MHz}$ ) and by a P-code (chip rate  $= 5.11$  MHz). The  $L_2$  code is presently modulated only by the P-code. The GLONASS satellites also transmit navigational data at a rate of 50 baud. Because the satellite frequencies are distinguishable from each other, the P-code and the  $C/A$ code are the same for each satellite. The methods for receiving and analyzing GLONASS signals are similar to the methods used for GPS signals. Further details can be found in the patent by Janky [66].

GLONASS does not use any form of SA.

#### **1.2 DIFFERENTIAL AND AUGMENTED GPS**

#### **1.2.1 Differential GPS**

Differential GPS (DGPS) is a technique for reducing the error in GPS-derived positions by using additional data from a reference GPS receiver at a known position. The most common form of DGPS involves determining the combined effects of navigation message ephemeris and satellite clock errors (including propagation delays and the effects of SA) at a reference station and transmitting pseudorange corrections, in real time, to a user's receiver, which applies the corrections in the process of determining its position [63, 96, 98].

#### **1.2.2 Local-Area Differential GPS**

Local-area differential GPS (LAGPS) is a form of DGPS in which the user's GPS receiver also receives real-time pseudorange and, possibly, carrier phase corrections from a local reference receiver generally located within the line of sight. The corrections account for the combined effects of navigation message ephemeris and satellite clock errors (including the effects of SA) and, usually, atmospheric propagation delay errors at the reference station. With the assumption that these errors are also common to the measurements made by the user's receiver, the application of the corrections will result in more accurate coordinates.

#### **1.2.3 Wide-Area Differential GPS**

Wide-area DGPS (WADGPS) is a form of DGPS in which the user's GPS receiver receives corrections determined from a network of reference stations distributed over a wide geographical area. Separate corrections are usually determined for specific error sources—such as satellite clock, ionospheric propagation delay, and ephemeris. The corrections are applied in the user's receiver or attached computer in computing the receiver's coordinates. The corrections are typically supplied in real time by way of a geostationary communications satellite or through a network of ground-based transmitters. Corrections may also be provided at a later date for postprocessing collected data [63].

#### **1.2.4 Wide-Area Augmentation System**

Three space-based augmentation systems (SBASs) were under development at the beginning of the third millenium. These are the Wide Area Augmentation System (WAAS), European Geostationary Navigation Overlay System (EGNOS),

and Multifunctional Transport Satellite (MTSAT) Based Augmentation System (MSAS).

The WAAS enhances the GPS SPS over a wide geographical area. The U.S. Federal Aviation Administration (FAA), in cooperation with other agencies, is developing WAAS to provide WADGPS corrections, additional ranging signals from geostationary earth orbit (GEO) satellites, and integrity data on the GPS and GEO satellites.

#### **1.2.5 Inmarsat Civil Navigation**

The Inmarsat overlay is an implementation of a wide-area differential service. Inmarsat is the International Mobile Satellite Organization, an 80-nation international consortium, originally created in 1979 to provide maritime<sup>2</sup> mobile services on a global basis but now offering a much wider range of mobile satellite services. Inmarsat launched four geostationary satellites that provide complete coverage of the globe from  $\pm 70^{\circ}$  latitude. The data broadcast by the satellites are applicable to users in regions having a corresponding ground station network. The U.S. region is the continental U.S. (CONUS) and uses Atlantic Ocean Region West (AOR-W) and Pacific Ocean Region (POR) geostationary satellites. This is called the WAAS and is being developed by the FAA. The ground station network is operated by the service provider, that is, the FAA, whereas Inmarsat is responsible for operation of the space segment. Inmarsat affiliates operate the uplink earth stations (e.g., COMSAT in the United States). WAAS is discussed further in Chapter 9.

#### **1.2.6 Satellite Overlay**

The Inmarsat Civil Navigation Geostationary Satellite Overlay extends and complements the GPS and GLONASS satellite systems. The overlay navigation signals are generated at ground based facilities. For example, for WAAS, two signals are generated from Santa Paula, California—one for AOR-W and one for POR. The back-up signal for POR is generated from Brewster, Washington. The backup signal for AOR-W is generated from Clarksburg, Maryland. Signals are uplinked to Inmarsat-3 satellites such as AOR-W and POR. These satellites contain special satellite repeater channels for rebroadcasting the navigation signals to users. The use of satellite repeater channels differs from the navigation signal broadcast techniques employed by GLONASS and GPS. GLONASS and GPS satellites carry their own navigation payloads that generate their respective navigation signals.

#### **1.2.7 Future Satellite Systems**

In Europe, activities supported by the European TRIPARTITE Group [European Space Agency (ESA), European Commission (EC), EUROCONTROL] are under-

 $2$  The "mar" in the name originally stood for "maritime."

way to specify, install, and operate a future civil Global Navigation Satellite System (GNSS) (GNSS-2 or GALILEO).

Based on the expectation that GNSS-2 will be developed through an evolutionary process as well as long-term augmentations [e.g., GNSS-1 or European GNSS Navigation Overlay Service (EGNOS)], short- to midterm augmentation systems (e.g., differential systems) are being targeted.

The first steps toward GNSS-2 will be made by the TRIPARTITE Group. The augmentations will be designed such that the individual elements will be suitable for inclusion in GNSS-2 at a later date. This design process will provide the user with maximum continuity in the upcoming transitions.

In Japan, the Japanese Commercial Aviation Board (JCAB) is developing the MSAS.

#### **1.3 APPLICATIONS**

Both GPS and GLONASS have evolved from dedicated military systems into true dual-use systems. Satellite navigation technology is utilized in numerous civil and military applications, ranging from golf and leisure hiking to spacecraft navigation. Further discussion on applications can be found in Chapters 8 and 9.

#### **1.3.1 Aviation**

The aviation community has propelled the use of GNSS and various augmentations (e.g., WAAS, EGNOS, and MSAS). These systems provide guidance for en route through precision approach phases of flight. Incorporation of a data link with a GNSS receiver enables the transmission of aircraft location to other aircraft and/or to air traffic control (ATC). This function is called automatic dependent surveillance (ADS) and is in use in the POR. Key benefits are ATC monitoring for collision avoidance and optimized routing to reduce travel time and fuel consumption [98].

#### **1.3.2 Spacecraft Guidance**

The space shuttle utilizes GPS for guidance in all phases of its operation (e.g., ground launch, on-orbit and reentry, and landing). NASA's small satellite programs use and plan to use GPS, as does the military on SBIRLEO (space-based infrared low earth orbit) and GBI (ground-based interceptor) kill vehicles.

#### **1.3.3 Maritime**

GNSS has been used by both commercial and recreational maritime communities. Navigation is enhanced on all bodies of waters, from oceanic travel to river ways, especially in bad weather.