

Statistical Design and Analysis of Experiments

With Applications to Engineering
and Science

Second Edition

Robert L. Mason

Southwest Research Institute
San Antonio, Texas

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To Carmen, Ann, Janis Sue

Preface

Statistical Design and Analysis of Experiments is intended to be a practitioner's guide to statistical methods for designing and analyzing experiments. The topics selected for inclusion in this book represent statistical techniques that we feel are most useful to experimenters and data analysts who must either collect, analyze, or interpret data. The material included in this book also was selected to be of value to managers, supervisors, and other administrators who must make decisions based in part on the analyses of data that may have been performed by others.

The intended audience for this book consists of two groups. The first group covers a broad spectrum of practicing engineers and scientists, including those in supervisory positions, who utilize or wish to utilize statistical approaches to solving problems in an experimental setting. This audience includes those who have little formal training in statistics but who are motivated by industrial or academic experiences in laboratory or process experimentation. These practicing engineers and scientists should find the contents of this book to be self-contained, with little need for reference to other sources for background information.

The second group for whom this book is intended is students in introductory statistics courses in colleges and universities. This book is appropriate for courses in which statistical experimental design and the analysis of data are the main topics. It is appropriate for upper-level undergraduate or introductory graduate-level courses, especially in disciplines for which the students have had or will have laboratory or similar data-collection experiences. The focus is on the use of statistical techniques, not on the theoretical underpinnings of those techniques. College algebra is the only prerequisite. A limited amount of supplemental material makes use of vector and matrix operations, notably the coverage of multiple linear regression. This material has been placed in appendices and is not essential for an understanding of the methods and applications contained in this book.

The emphasis in this book is on the strategy of experimentation, data analysis, and the interpretation of experimental results. The text features numerous examples using actual engineering and scientific studies. It presents statistics as an integral component of experimentation from the planning stage to the presentation of the conclusions.

This second edition constitutes a significant revision. A number of users of the first edition were surveyed and their feedback was incorporated in the revision. This resulted in deleting some material that wasn't intimately connected to the main thrust of the book, adding some new topics that supplemented existing topical coverage, and rearranging the presentation. For example, some introductory material was eliminated in order to introduce experimental design topics more quickly. A number of new examples were included in several of the chapters. New exercises were added to each of the chapters. In decisions regarding topics, we were guided by our collective experiences as statistical consultants and by our desire to produce a book that would be informative and readable. The topics selected for inclusion in both editions of this book can be implemented by practitioners and do not require a high level of training in statistics.

A key feature of the book, one that was cited as pedagogically beneficial by reviewers, is the depth and concentration of experimental design coverage, with equivalent but separate emphasis on the analysis of data from the various designs. In contrast to the previous edition, however, in the second edition chapters on the analysis of designed experiments have been placed immediately following the corresponding chapters on the respective designs. This was viewed as especially beneficial for classroom use. Instructors and readers can still emphasize design issues in a cohesive manner and can now have the analysis of the data resulting from the use of the respective designs reinforce the important features of the designs by having both the design and the analysis covered in close proximity to one another.

This second edition of *Statistical Design and Analysis of Experiments* is divided into four sections. Part I consists of Chapters 1 to 3 and presents a quick overview of many conceptual foundations of modern statistical practice. These three chapters introduce the reader to the basic issues surrounding the statistical analysis of data. The distinctions between populations or processes and samples, parameters and statistics, and mathematical and statistical modeling are discussed. In addition, elementary descriptive statistics and graphical displays are presented. Throughout the presentation, the informational content of simple graphical and numerical methods of viewing data is stressed.

Chapters 4 to 8 constitute Part II and Chapters 9–13 constitute Part III. These are the heart of the experimental design and analysis portions of the book. Unlike many other statistics books, this book intentionally separates discussions of the design of an experiment from those of the analysis of the resulting data from these experiments. Readers benefit from the reinforcement

of concepts by considering the topics on experimental design in close proximity to one another. In addition, alternatives to the various designs are easily cross-referenced, making the distinctions between the designs clearer. Following the concentrated attention on experimental-design issues, separate chapters immediately provide for the analysis of data from these designs. All too often, texts devote a paragraph to the design of an experiment and several pages to the analysis of the resulting data. Our experiences with this approach are that the material on experimental design is slighted when designs and analyses are presented in the same chapter. A much clearer understanding of proper methods for designing experiments is achieved by separating the topics.

The chapters in Part II concentrate on the design and analysis of experiments with factorial structures. New in the second edition is expanded coverage of statistical graphics (e.g., trellis plots in Chapter 6), three-level and combined two- and three-level fractional factorial experiments (Chapter 7), and expanded coverage on the analysis of data from unbalanced experiments (Chapter 8).

The chapters in Part III stress the design and analysis of data from designed experiments with random factor effects. Added to the second edition is additional material on the analysis of data from incomplete block designs (Chapter 9) and split-plot designs (Chapter 11), new analyses for data from process improvement designs (Chapter 12), and analyses of data from gage R&R studies and data from some designs popularized by Genichi Taguchi (Chapter 13).

Throughout the analysis chapters in Parts II and III, confidence-interval and hypothesis-testing procedures are detailed for single-factor and multifactor experiments. Statistical models are used to describe responses from experiments, with careful attention to the specification of the terms of the various models and their relationship to the possible individual and joint effects of the experimental factors.

Part IV consists of Chapters 14 to 19 and is devoted to the analysis of experiments containing quantitative predictors and factors. Linear regression modeling using least-squares estimators of the model parameters is detailed, along with various diagnostic techniques for assessing the assumptions typically made with both regression and analysis-of-variance models. Analysis-of-covariance procedures are introduced, and the design and analysis needed for use in fitting response surfaces are presented. Identification of influential observations and the concepts of model assessment and variable selection are also discussed.

We are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F. R. S., to Dr. Frank Yates, F. R. S., and to the Longman Group, Ltd., London, for permission to reprint part of Table XXIII from their book *Statistical Tables for Biological, Agricultural, and Medical Research* (6th edition, 1974).

In the first edition, Bea Schube was the John Wiley editor who helped initiate this project, and later Kate Roach was the editor who completed it. We are thankful to both of them as well as to the current Wiley editor, Steve Quigley, for their contributions. For this second edition, we also express our appreciation to Andrew Prince of John Wiley and Joan Wolk of Joan Wolk Editorial Services for their excellent work during the editorial and production process.

We are indebted to many individuals for contributing to this work. Several colleagues read earlier versions of the first edition and made many valuable suggestions on content and readability. We also are thankful to many users of the first edition of this book. Their comments and suggestions, as well as those received from several anonymous reviewers, have been very useful as we developed the second edition.

Contents

Preface	vii
PART I FUNDAMENTAL STATISTICAL CONCEPTS	1
1. Statistics in Engineering and Science	3
1.1. The Role of Statistics in Experimentation,	5
1.2. Populations and Samples,	9
1.3. Parameters and Statistics,	19
1.4. Mathematical and Statistical Modeling,	24
Exercises,	28
2. Fundamentals of Statistical Inference	33
2.1. Traditional Summary Statistics,	33
2.2. Statistical Inference,	39
2.3. Probability Concepts,	42
2.4. Interval Estimation,	48
2.5. Statistical Tolerance Intervals,	50
2.6. Tests of Statistical Hypotheses,	52
2.7. Sample Size and Power,	56
Appendix: Probability Calculations,	59
Exercises,	64

3. Inferences on Means and Standard Deviations	69
3.1. Inferences on a Population or Process Mean, 72	
3.1.1. Confidence Intervals, 73	
3.1.2. Hypothesis Tests, 76	
3.1.3. Choice of a Confidence Interval or a Test, 78	
3.1.4. Sample Size, 79	
3.2. Inferences on a Population or Process Standard Deviation, 81	
3.2.1. Confidence Intervals, 82	
3.2.2. Hypothesis Tests, 84	
3.3. Inferences on Two Populations or Processes Using Independent Pairs of Correlated Data Values, 86	
3.4. Inferences on Two Populations or Processes Using Data from Independent Samples, 89	
3.5. Comparing Standard Deviations from Several Populations, 96	
Exercises, 99	
 PART II DESIGN AND ANALYSIS WITH FACTORIAL STRUCTURE	 107
4. Statistical Principles in Experimental Design	109
4.1. Experimental-Design Terminology, 110	
4.2. Common Design Problems, 115	
4.2.1. Masking Factor Effects, 115	
4.2.2. Uncontrolled Factors, 117	
4.2.3. Erroneous Principles of Efficiency, 119	
4.2.4. One-Factor-at-a-Time Testing, 121	
4.3. Selecting a Statistical Design, 124	
4.3.1. Consideration of Objectives, 125	
4.3.2. Factor Effects, 126	
4.3.3. Precision and Efficiency, 127	
4.3.4. Randomization, 128	
4.4. Designing for Quality Improvement, 128	
Exercises, 132	

5. Factorial Experiments in Completely Randomized Designs	140
5.1. Factorial Experiments,	141
5.2. Interactions,	146
5.3. Calculation of Factor Effects,	152
5.4. Graphical Assessment of Factor Effects,	158
Appendix: Calculation of Effects for Factors with More than Two Levels,	160
Exercises,	163
6. Analysis of Completely Randomized Designs	170
6.1. Balanced Multifactor Experiments,	171
6.1.1. Fixed Factor Effects,	171
6.1.2. Analysis-of-Variance Models,	173
6.1.3. Analysis-of-Variance Tables,	176
6.2. Parameter Estimation,	184
6.2.1. Estimation of the Error Standard Deviation,	184
6.2.2. Estimation of Effects Parameters,	186
6.2.3. Quantitative Factor Levels,	189
6.3. Statistical Tests,	194
6.3.1. Tests on Individual Parameters,	194
6.3.2. F -Tests for Factor Effects,	195
6.4. Multiple Comparisons,	196
6.4.1. Philosophy of Mean-Comparison Procedures,	196
6.4.2. General Comparisons of Means,	203
6.4.3. Comparisons Based on t -Statistics,	209
6.4.4. Tukey's Significant Difference Procedure,	212
6.5. Graphical Comparisons,	213
Exercises,	221
7. Fractional Factorial Experiments	228
7.1. Confounding of Factor Effects,	229
7.2. Design Resolution,	237
7.3. Two-Level Fractional Factorial Experiments,	239

7.3.1. Half Fractions,	239
7.3.2. Quarter and Smaller Fractions,	243
7.4. Three-Level Fractional Factorial Experiments,	247
7.4.1. One-Third Fractions,	248
7.4.2. Orthogonal Array Tables,	252
7.5. Combined Two- and Three-Level Fractional Factorials,	254
7.6. Sequential Experimentation,	255
7.6.1. Screening Experiments,	256
7.6.2. Designing a Sequence of Experiments,	258
Appendix: Fractional Factorial Design Generators,	260
Exercises,	266
 8. Analysis of Fractional Factorial Experiments	 271
8.1. A General Approach for the Analysis of Data from Unbalanced Experiments,	272
8.2. Analysis of Marginal Means for Data from Unbalanced Designs,	276
8.3. Analysis of Data from Two-Level, Fractional Factorial Experiments,	278
8.4. Analysis of Data from Three-Level, Fractional Factorial Experiments,	287
8.5. Analysis of Fractional Factorial Experiments with Combinations of Factors Having Two and Three Levels,	290
8.6. Analysis of Screening Experiments,	293
Exercises,	299
 PART III Design and Analysis with Random Effects	 309
 9. Experiments in Randomized Block Designs	 311
9.1. Controlling Experimental Variability,	312
9.2. Complete Block Designs,	317
9.3. Incomplete Block Designs,	318
9.3.1. Two-Level Factorial Experiments,	318
9.3.2. Three-Level Factorial Experiments,	323
9.3.3. Balanced Incomplete Block Designs,	325

9.4. Latin-Square and Crossover Designs,	328
9.4.1. Latin Square Designs,	328
9.4.2. Crossover Designs,	331
Appendix: Incomplete Block Design Generators,	332
Exercises,	342
10. Analysis of Designs with Random Factor Levels	347
10.1. Random Factor Effects,	348
10.2. Variance-Component Estimation,	350
10.3. Analysis of Data from Block Designs,	356
10.3.1. Complete Blocks,	356
10.3.2. Incomplete Blocks,	357
10.4. Latin-Square and Crossover Designs,	364
Appendix: Determining Expected Mean Squares,	366
Exercises,	369
11. Nested Designs	378
11.1. Crossed and Nested Factors,	379
11.2. Hierarchically Nested Designs,	381
11.3. Split-Plot Designs,	384
11.3.1. An Illustrative Example,	384
11.3.2. Classical Split-Plot Design Construction,	386
11.4. Restricted Randomization,	391
Exercises,	395
12. Special Designs for Process Improvement	400
12.1. Assessing Quality Performance,	401
12.1.1. Gage Repeatability and Reproducibility,	401
12.1.2. Process Capability,	404
12.2. Statistical Designs for Process Improvement,	406
12.2.1. Taguchi's Robust Product Design Approach,	406
12.2.2. An Integrated Approach,	410
Appendix: Selected Orthogonal Arrays,	414
Exercises,	418

13. Analysis of Nested Designs and Designs for Process Improvement	423
13.1. Hierarchically Nested Designs,	423
13.2. Split-Plot Designs,	428
13.3. Gage Repeatability and Reproducibility Designs,	433
13.4. Signal-to-Noise Ratios,	436
Exercises,	440
 PART IV Design and Analysis with Quantitative Predictors and Factors	 459
14. Linear Regression with One Predictor Variable	461
14.1. Uses and Misuses of Regression,	462
14.2. A Strategy for a Comprehensive Regression Analysis,	470
14.3. Scatterplot Smoothing,	473
14.4. Least-Squares Estimation,	475
14.4.1. Intercept and Slope Estimates,	476
14.4.2. Interpreting Least-Squares Estimates,	478
14.4.3. No-Intercept Models,	480
14.4.4. Model Assumptions,	481
14.5. Inference,	481
14.5.1. Analysis-of-Variance Table,	481
14.5.2. Tests and Confidence Intervals,	484
14.5.3. No-Intercept Models,	485
14.5.4. Intervals for Responses,	485
Exercises,	486
 15. Linear Regression with Several Predictor Variables	 496
15.1. Least Squares Estimation,	497
15.1.1. Coefficient Estimates,	497
15.1.2. Interpreting Least-Squares Estimates,	499
15.2. Inference,	503
15.2.1. Analysis of Variance,	503
15.2.2. Lack of Fit,	505
15.2.3. Tests on Parameters,	508
15.2.4. Confidence Intervals,	510

15.3. Interactions Among Quantitative Predictor Variables, 511	
15.4. Polynomial Model Fits, 514	
Appendix: Matrix Form of Least-Squares Estimators, Exercises, 525	522
16. Linear Regression with Factors and Covariates as Predictors	535
16.1. Recoding Categorical Predictors and Factors, 536	
16.1.1. Categorical Variables: Variables with Two Values, 536	
16.1.2. Categorical Variables: Variables with More Than Two Values, 539	
16.1.3. Interactions, 541	
16.2. Analysis of Covariance for Completely Randomized Designs, 542	
16.3. Analysis of Covariance for Randomized Complete Block Designs, 552	
Appendix: Calculation of Adjusted Factor Averages, Exercises, 558	556
17. Designs and Analyses for Fitting Response Surfaces	568
17.1. Uses of Response-Surface Methodology, 569	
17.2. Locating an Appropriate Experimental Region, 575	
17.3. Designs for Fitting Response Surfaces, 580	
17.3.1. Central Composite Design, 582	
17.3.2. Box–Behnken Design, 585	
17.3.3. Some Additional Designs, 586	
17.4. Fitting Response-Surface Models, 588	
17.4.1. Optimization, 591	
17.4.2. Optimization for Robust Parameter Product-Array Designs, 594	
17.4.3. Dual Response Analysis for Quality Improvement Designs, 597	
Appendix: Box–Behnken Design Plans; Locating Optimum Responses, Exercises, 606	600

18. Model Assessment	614
18.1. Outlier Detection,	614
18.1.1. Univariate Techniques,	615
18.1.2. Response-Variable Outliers,	619
18.1.3. Predictor-Variable Outliers,	626
18.2. Evaluating Model Assumptions,	630
18.2.1. Normally Distributed Errors,	630
18.2.2. Correct Variable Specification,	634
18.2.3. Nonstochastic Predictor Variables,	637
18.3. Model Respecification,	639
18.3.1. Nonlinear-Response Functions,	640
18.3.2. Power Reexpressions,	642
Appendix: Calculation of Leverage Values and Outlier Diagnostics,	647
Exercises,	651
19. Variable Selection Techniques	659
19.1. Comparing Fitted Models,	660
19.2. All-Possible-Subset Comparisons,	662
19.3. Stepwise Selection Methods,	665
19.3.1. Forward Selection,	666
19.3.2. Backward Elimination,	668
19.3.3. Stepwise Iteration,	670
19.4. Collinear Effects,	672
Appendix: Cryogenic-Flowmeter Data,	674
Exercises,	678
APPENDIX: Statistical Tables	689
1. Table of Random Numbers,	690
2. Standard Normal Cumulative Probabilities,	692
3. Student t Cumulative Probabilities,	693
4. Chi-Square Cumulative Probabilities,	694
5. F Cumulative Probabilities,	695
6. Factors for Determining One-sided Tolerance Limits,	701
7. Factors for Determining Two-sided Tolerance Limits,	702

8. Upper-Tail Critical Values for the F -Max Test, 703
9. Orthogonal Polynomial Coefficients, 705
10. Critical Values for Outlier Test Using L_k and S_k , 709
11. Critical Values for Outlier Test Using E_k , 711
12. Coefficients Used in the Shapiro–Wilk Test for Normality, 713
13. Critical Values for the Shapiro–Wilk Test for Normality, 716
14. Percentage Points of the Studentized Range, 718

INDEX**723****DATA INDEX****729**

PART I

Fundamental Statistical Concepts

CHAPTER 1

Statistics in Engineering and Science

In this chapter we introduce basic statistical concepts and terminology that are fundamental to the use of statistics in experimental work. These concepts include:

- *the role of statistics in engineering and scientific experimentation,*
- *the distinction between samples and populations,*
- *relating sample statistics to populations parameters, and*
- *characterizing deterministic and empirical models.*

The term *scientific* suggests a process of objective investigation that ensures that valid conclusions can be drawn from an experimental study. Scientific investigations are important not only in the academic laboratories of research universities but also in the engineering laboratories of industrial manufacturers. *Quality* and *productivity* are characteristic goals of industrial processes, which are expected to result in goods and services that are highly sought by consumers and that yield profits for the firms that supply them. Recognition is now being given to the necessary link between the scientific study of industrial processes and the quality of the goods produced. The stimulus for this recognition is the intense international competition among firms selling similar products to a limited consumer group.

The setting just described provides one motivation for examining the role of statistics in scientific and engineering investigations. It is no longer satisfactory just to monitor on-line industrial processes and to ensure that products are within desired specification limits. Competition demands that a better product be produced within the limits of economic realities. Better products are initiated in academic and industrial research laboratories, made feasible in

pilot studies and new-product research studies, and checked for adherence to design specifications throughout production. All of these activities require experimentation and the collection of data. The definition of the discipline of statistics in Exhibit 1.1 is used to distinguish the field of statistics from other academic disciplines and is oriented toward the experimental focus of this text. It clearly identifies statistics as a scientific discipline, which demands the same type of rigor and adherence to basic principles as physics or chemistry. The definition also implies that when problem solving involves the collection of data, the science of statistics should be an integral component of the process.

EXHIBIT 1.1

Statistics. Statistics is the science of problem-solving in the presence of variability.

Perhaps the key term in this definition is the last one. The problem-solving process involves a degree of uncertainty through the natural variation of results that occurs in virtually all experimental work.

When the term *statistics* is mentioned, many people think of games of chance as the primary application. In a similar vein, many consider statisticians to be “number librarians,” merely counters of pertinent facts. Both of these views are far too narrow, given the diverse and extensive applications of statistical theory and methodology.

Outcomes of games of chance involve uncertainty, and one relies on probabilities, the primary criteria for statistical decisions, to make choices. Likewise, the determination of environmental standards for automobile emissions, the forces that act on pipes used in drilling oil wells, and the testing of commercial drugs all involve some degree of uncertainty. Uncertainty arises because the level of emissions for an individual automobile, the forces exerted on a pipe in one well, and individual patient reactions to a drug vary with each observation, even if the observations are taken under “controlled” conditions. These types of applications are only a few of many that could be mentioned. Many others are discussed in subsequent chapters of this book.

Figure 1.1 symbolizes the fact that statistics should play a role in every facet of data collection and analysis, from initial problem formulation to the drawing of final conclusions. This figure distinguishes two types of studies: experimental and observational. In experimental studies the variables of interest often can be controlled and fixed at predetermined values for each test run in the experiment. In observational studies many of the variables of interest cannot be controlled, but they can be recorded and analyzed. In this book we emphasize experimental studies, although many of the analytic procedures discussed can be applied to observational studies.

- Model (*continued*)
 random-effect, 349
 regression, 462, 497, 536
 respecification, 639
 response surface, 588
 saturated, 185
 second-order, 513, 515
 specification, 462, 472, 497, 634
 statistical, 25
 sum of squares, *see* Sum of squares
- Multi-panel conditioning, 214
- Multiple comparison procedures, 196
- Nested design, 378, 423
- Nested factors, *see* Factor
- Noncentral composite design, 588
- Nonlinear
 relationship, 635, 640
 response function, 640
- Nonorthogonal designs, 252
- Normal density function, 43
- Normal equations, 499
- Normal probability distribution, 20, 43, 59
- Null hypothesis, 52, 55
- Observation, 12
- Observational
 data, 587
 studies, 4
- Observed value, 11
- One-Factor-at-a-Time (OFAT) Testing, 121
- Operating characteristic curve, 59, 80
- Optimum response, 573, 576
- Orthogonal arrays, 252, 407, 414
- Orthogonal contrast, *see* Contrast
- Orthogonal polynomials, 207
- Outer array, 408
- Outliers, 70
 accommodation, 615
 detection, 614
 in predictor variables, 626
 in response variables, 619
- Parameter, 19
 analysis of variance model, 174
 constraints, 174
 estimation, 186
 interaction, 174
 main-effect, 174
- Parsimony, 516
- Partial regression coefficient estimate, 499
- Pearson's r , 468
- Pick the winner, 437, 457
- PISEAS, 470
- Plackett–Burman design, 256
 analysis, 293
- Plots
 boxplot, 70
 contour, 122, 571
 cube, 213
 factor effects, 158
 interaction, 150, 216
 labeled scatterplot, 148
 least significant interval, 218
 normal quantile-quantile, 159, 630
 overlaying, 575
 partial-regression, 637
 partial-residual, 635
 point, 39
 residual, 634
 scatter, 6, 148
 studentized deleted residual, 644
 trellis, 214
- Pooled standard deviation estimate, 90
- Population, 10
- Power, 56
- Precision, 127
- Prediction
 equation, 475, 498
 interval, 485
- Probability concepts, 42
- Product array design, 594
- Process, 10
 capability, 404
- Pure error, 506
- p -value, 55, 77
- Quadratic model, 515, 588
- Quality control procedures, 128
- Quality loss function, 407
- Quantile, 159
- Quartile, 37
- Random sample, 14
- Randomization, 128, 142, 391
 restricted, 391
- Randomized complete block (RCB)
 design, 317, 552
- Range, 35

- Reduction in error sum of squares, 273
- Reexpression, 642, 647
- Regression analysis
 - assumptions, 463, 481
 - common uses and misuses, 462
 - analysis linear, 470
 - local fit, 473
 - strategy, 470
 - sum of squares, 481, 503
- Regression coefficient, 462, 497
 - beta-weight, 502, 519
 - standardized, 502, 519
- Regression fallacy, 480
- Repeat test, 110, 144, 312
- Repeatability, 315, 402, 434
- Replication, 110, 312
- Reproducibility, 315, 402, 434
- Residuals, 475, 498
 - partial, 635
 - studentized deleted, 623
- Response
 - dispersion, 130
 - location, 130
 - predicted, 475, 498
 - variable, 120
- Response surface designs, 129, 410, 568, 580
 - Box–Behnken, 413, 585
 - central composite, 413, 582
- Rising ridge, 572
- Robust design, 401
 - parameter design, 587, 594
- Robustness, 401
- Rotatable design, 581
- Ruggedness tests, 267, 293
- Saddle, 572
- Sample correlation coefficient, 468
- Sample, 13
 - mean 34
 - sampling distribution of, 45
 - median, 34
 - size, 56, 79
 - standard deviation, 36, 73
 - types of, 13
 - variance, 46
- Sampling distribution, 21, 45, 73
- Saturated designs, 253
- Scatterplot 6; smoothing, 473
- Screening design, 144, 256
 - analysis, 293
- Screening experiments, 129, 158, 256
- Semi-interquartile range, 37, 70
- Sequential experimentation, 255, 331
- Shapiro–Wilk test for normality, 633
- Signal-to-noise ratio, 436
- Significance level, 55
- Significance probability, 55
- Simplex design, 588
- Small composite design, 588
- Smoothing, 473
- Span, 473
- Split plot, 384, 388
- Split-plot design, 384, 388, 428
- Stable process, 404
- Staggered nested design, 382, 427
- Standard deviation, 36, 81
- Standard error, 45
- Standard normal variate, 44
- Standard process, 404
- Standardized predictor variable, 519, 591
- Stationary point, 593
- Stationary ridge, 571
- Statistic, 4, 19
- Steepest ascent method, 602
- Stepwise variable selection techniques, 665
 - collinear effects, 672
- Student *t* distribution, 46, 60
 - approximate, 91
- Sum of squares
 - contrast, 200
 - error, 180, 481, 498
 - interaction effect, 178
 - main effect, 178
 - model, 177
 - partitioned, 177
 - regression, 481, 503
 - total, 177, 481
- Supersaturated design analysis, 297
- Taguchi approach, 129, 406
- Taguchi design, 406, 436
- t*-distribution, 46
- Test run, 110
- Tolerance intervals, 50
- Transformation
 - Box–Cox, 642
 - logarithmic, 640
 - power-family, 642
- Transmitted variation, 411
- t*-statistic, 73, 209

Tukey's significant difference, 212
Type I error, 55, 202
Type II error, 55, 202

Unbalanced design
analysis, 272

Variable 11
categorical, 536
collinear, 518
continuous, 42
indicator, 461, 536
predictor, 461, 462, 497

response, 12, 110
selection techniques, 659
standardized, 579
Variance component, 350, 434
Variance inflation factors, 673
Variation
assignable cause, 173
measurement, 402
random, 173
sources of, 173
transmitted, 411
Whole plots, 388

Data Index

Chapter	Title/Description	Page	Other Chapter	Data Location*
1	Chemical analysis variation study	7	no	T1.1
1	Employee identification numbers	16	no	T1.3
1	Skin elasticity measurements	18	no	T1.4
2	Diesel fuel economy, 4 vehicles, 8 fuels	38	no	T2.1
2	Wire tensile strength	40	2,3	T2.2
3	Percent solids in two pipeline locations	86	no	T3.5
4	Cylinder pressure measurements	116	no	T4.4
4	Skin color measurements	116	12	T4.5
5	Cutting tool life	147	no	T5.4
6	Chemical yield from pilot plant	172	5,7,8,9	T6.1
6	Filtration flow rates	184	no	T6.5
6	Copper plate warping	191	no	T6.7
8	Chemical yield from pilot plant (unbalanced)	274	5,6,7	T8.1
8	Product color in a chemical process	280	no	T8.4
8	Density of bricks	288	no	T8.9
8	Diesel fuel NO _x experiment	292	no	T8.14
8	Solder-coverage ruggedness testing data	294	no	T8.17
8	Ruggedness test viscosity data	296	no	T8.20
8	Fatigue life of weld-repaired castings	298	no	T8.23
10	Asphalt pavement rating study	358	no	T10.4
10	Tire brands road test	365	no	T10.9
11	NO _x emissions for 2 vehicles from 3 models	385	no	T11.2
11	NO _x emissions for 6 vehicles and 3 fuels	386	no	T11.3
12	Gage R&R study on innerspring length	405	9	F12.2