

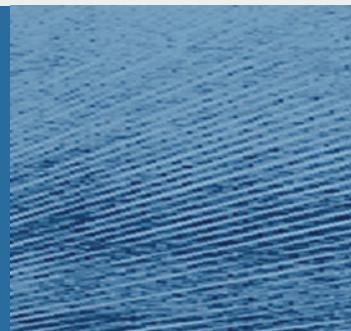
# NOISE and VIBRATION ANALYSIS

Signal Analysis  
and Experimental Procedures



ANDERS BRANDT

 WILEY





# **NOISE AND VIBRATION ANALYSIS**



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## SIGNAL ANALYSIS AND EXPERIMENTAL PROCEDURES

**Anders Brandt**

*Department of Industrial and Civil Engineering  
University of Southern Denmark*



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# About the Author

Anders Brandt has more than twenty years experience as a consultant and short-course instructor in experimental vibration analysis. During his entire career, he has worked on providing increased understanding of the measurement and analysis procedures used in experimental vibration analysis. Currently, Anders Brandt is an Associate Professor of Experimental Dynamics and Signal Processing at the University of Southern Denmark, where his main research interests are in applied signal processing and operational modal analysis. Anders is a popular short-course instructor and lecturer on the topics covered by this book.



# Preface

The material in this book has been developing in my mind for more than twenty years of teaching. During these years I have been teaching over 200 shortcourses for engineers in the industry on techniques for experimental noise and vibration analysis and also on how to use commercial measurement and analysis systems. In addition, in the late 1990s I developed and taught three master's level courses in experimental analysis of vibrations at Blekinge Institute of Technology in Sweden. Noise and vibration analysis is an interdisciplinary field, incorporating diverse subjects such as mechanical dynamics, sensor technology, statistics, and signal processing. Whereas there are many excellent and comprehensive books in each of these disciplines, there has been a lack of introductory material for the engineering student who first starts to make noise and/or vibration measurements, or the engineer who needs a reference in his or her daily life. In addition, there are few textbooks in this field presenting the techniques as they are actually used in practice. This book is an attempt to fill this void.

My aim for this book is for it to serve both as a course book and as supplementary reading in university courses, as well as providing a handbook for engineers or researchers who measure and analyze acoustic or vibration signals. The level of the book makes it appropriate both for undergraduate and graduate levels, with a proper selection of the content. In addition the book should be a good reference for analysts who use experimental results and need to interpret them. To satisfy these rather different purposes, for some of the topics in the book I have included more detail than would be necessary for an introductory text. To facilitate its use as a handbook, I have also included a short summary at the end of each chapter where some of the key points of the chapter are repeated.

This book contains background theory explaining the majority of analysis methods used in modern commercial software for noise and vibration measurement and analysis, with one exception: experimental modal analysis is only briefly introduced, as this is a specialized field with some excellent textbooks already available. This book also includes a number of tools which are usually not found in commercial systems, but which are still useful for the practitioner. With modern computer-based software, it is easy to export data to, e.g., MATLAB/Octave (see below), and apply the techniques there.

Since it is an introductory text, most of the content of this book is of course available in more specialized textbooks and scientific papers. A few parts, however, include some improvements of existing techniques. I will mention these points in the descriptions of the appropriate chapters below.

Signal analysis is traditionally a field within electrical engineering, whereas most engineers and students pursuing noise and vibration measurements are mechanical or civil engineers. The aim has therefore been to make the material accessible, particularly to students and engineers of these latter disciplines. For this reason I have included introductions to the Laplace and Fourier transforms – both essential tools for understanding, analyzing and solving problems in dynamics. Electrical engineering students and practitioners should still find many of the topics in the book interesting.

Signal analysis is a subject which is best learned by practicing the theories (as, perhaps, all subjects are). I have therefore incorporated numerous examples using MATLAB or GNU Octave throughout the book. Further examples and an accompanying toolbox which can be used with either MATLAB or GNU

Octave can be downloaded from Internet. More information about this is located in Section 1.6. I strongly recommend the use of these tools as a complement to reading this book, regardless of whether you are a student, a researcher or an industry practitioner.

Chapter 2 introduces dynamic signals and systems with the aim of being an introduction particularly for mechanical and civil engineering students. In this chapter the classification of signals into periodic, random and transient signals is introduced. The chapter also includes linear system theory and a comprehensive introduction to the Laplace and Fourier transforms, both important tools for understanding and analyzing dynamic systems.

In Chapter 3 some fundamental concepts of sampled signals are presented. Starting with the sampling theorem and continuing with digital filter theory, this chapter presents some important applications of digital filters for fractional octave analysis and for integrating and differentiating measured signals.

Chapter 4 introduces some applied statistics and random process theory from a practical perspective. It includes an introduction to hypothesis testing as this tool is sometimes used for testing normality and stationarity of data. This chapter also gives an introduction to the application of statistics for data quality assessment, which is becoming more important with the large amounts of data collected in many applications of noise and vibration analysis.

Chapters 5 and 6 provide an introduction to the theory of mechanical vibrations. I anticipate that the contents of these two chapters will already be known to many readers, but I have found it important to include them because my presentation focuses on the experimental implications of the theory, unlike the presentation in most mechanical vibration textbooks, and because some later chapters in the book need a foundation with a common nomenclature. Chapter 6 also includes an accurate and fast method for computing forced response of linear systems in the time domain which is very attractive, e.g., to produce known experimental signals for testing out signal analysis procedures. This method, developed by Professor Kjell Ahlin, has been presented at conferences, but deserves better dissemination.

In Chapter 7 the most important transducers used for measurements of noise and vibration signals are presented; specifically the accelerometer, the force sensor and the microphone. Because piezoelectric sensors with built-in signal conditioning (so-called IEPE sensors) are widely used today, this technology is presented in some depth. In this chapter I also present some personal ideas on how to become a good experimentalist.

The analysis techniques mostly used in this field are based on the Discrete Fourier Transform (DFT), computed by the FFT. Spectrum analysis is therefore an important part of this book and Chapters 8 through 10 are spent on this topic. Chapter 8 introduces basic frequency analysis theory by presenting the different signal classes, and the different spectra used to describe the frequency content of these signals.

In Chapter 9 the DFT and some other techniques used to experimentally determine the frequency content of signals are presented. The properties of the DFT, which are very important to understand when interpreting experimental frequency spectra, are presented relatively comprehensively.

Chapter 10 includes a comprehensive presentation of how spectra from periodic, random and transient signals, and mixes of these signal classes, should be estimated in practice. Also, I mention a convenient technique for removing harmonics in spectral density estimates using the smoothed periodogram method; which, to my knowledge, has never been presented before. Chapter 10 also includes a comprehensive explanation of Welch's method for PSD estimation, including overlap processing, as this is the method used in virtually all commercial software. The treatment of practical spectral analysis in this chapter should also be of use to engineers outside the field of acoustics and vibrations who want to calculate and/or interpret spectra by using the FFT.

In Chapter 11 the design of modern data acquisition and measurement systems is described from a user perspective. In this chapter both hardware and software issues are penetrated.

Chapter 12 addresses order tracking, which is a common technique for analysis of rotating machinery equipment. The chapter describes the most common techniques used to measure such signals both with fixed sampling frequency and with synchronous sampling.

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Frequency response functions are important measurement functions in experimental noise and vibration analysis and are used, for example, in experimental modal analysis. Chapter 13 therefore covers techniques for measuring frequency responses for single-input/single-output (SISO) systems. Both impact excitation and shaker excitation techniques are presented in detail. In Chapter 14 the techniques are extended to multiple-input/multiple-output (MIMO) systems. In Chapters 13 and 14 I also present a technique which has not, to the best of my knowledge, been presented before. Using well-known periodic excitation signals, I show that the bias error in frequency response estimates with extraneous noise present in both input and output signals can be eliminated by time domain averaging, for single-input as well as multiple-input systems.

Chapter 15 presents some relatively advanced techniques used for multichannel analysis, namely principal components and virtual signals. These techniques are commonly used for noise path analysis and noise source identification in many of the sophisticated software packages available commercially. I present these concepts in some depth, since they are not readily available in other textbooks.

In Chapter 16 I have collected a number of more advanced techniques that engineers in this field should be acquainted with. This chapter presents, in order, the shock response spectrum, the Hilbert transform with applications, the cepstrum and envelope spectrum, how to produce Gaussian time signals with known spectral density, and finally two very important tools: operational deflection shapes, and experimental modal analysis. The latter is a comprehensive technique and only briefly introduced.

In the Appendix section I have included some fundamentals on complex numbers, logarithmic diagrams and the decibel unit, matrix theory, and eigenvalues and the singular value decomposition. The reader who does not feel confident with some of these concepts will hopefully find enough theory in these appendices to follow the text in this book. The last appendix contains some references to good sources for more information within the noise and vibration community. I hope the newcomer to this field can benefit from this list.



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# List of Abbreviations

2DOF	Two degrees-of-freedom system
AC	Alternating current
ADC	Analog-to-digital converter
BT	Bandwidth-time (product)
CSD	Cross-spectral density function
DAC	Digital-to-analog converter
DC	Direct current
DFT	Discrete Fourier transform
DOF	Degree-of-freedom (point and direction)
ESD	Energy spectral density
FE	Finite element
FEM	Finite element method
FFT	Fast Fourier transform
FIR	Finite impulse response (filter)
FRF	Frequency response function
HF	High frequency
HP	Highpass
IDFT	Inverse discrete Fourier transform
IEPE	Integrated electronics piezoelectric (sensor)
IFFT	Inverse fast Fourier transform
IIR	Infinite impulse response (filter)
IRF	Impulse response function
LF	Low frequency
ISO	International standardization organization
MDOF	Multiple degrees-of-freedom
MIF	Mode indicator function
MIMO	Multiple-input/multiple-output
MISO	Multiple-input/single-output
MPSS	Multi-phase stepped sine
MrMIF	Modified real mode indicator function
MvMIF	Multivariate mode indicator function
NSI	Noise source identification
NSR	Noise-to-signal ratio
ODS	Operating deflection shape
PDF	Probability density function
PSD	Power spectral density
RMS	Root mean square

RPM	Revolutions per minute
SDOF	Single degree-of-freedom
SIMO	Single-input/multiple-output
SISO	Single-input/single-output
SNR	Signal-to-noise ratio
SRS	Shock response spectrum
SVD	Singular value decomposition
TEDS	Transducer electronic data sheet

# Notation

$\langle x \rangle$	Average of $x$
$\mathcal{F}[\cdot]$	Fourier transform of [ ]
$\mathcal{H}[\cdot]$	Hilbert transform of [ ]
$\mathcal{L}[\cdot]$	Laplace transform of [ ]
$E[\cdot]$	Expected value
$a, a(t)$	Vibration acceleration
$A_{pqr}$	Residue of mode $r$ , between points $p$ and $q$
$A_{xx}$	Autopower spectrum of $x$
$B$	Bandwidth in [Hz]
$B_e$	Equivalent (statistical) bandwidth in Hz
$B_{en}$	Normalized equivalent bandwidth (dimensionless)
$B_r$	Resonance bandwidth in Hz
$c_p$	Power cepstrum
$c_r$	Modal (viscous) damping of mode $r$
$\delta(t)$	Dirac's unit impulse
$\Delta f$	Frequency increment of discrete Fourier transform
$\Delta t$	Time increment in [s]
$\varepsilon$	Normalized error
$f$	Frequency in [Hz]
$f_n, f_r$	Undamped natural frequency
$g^2(f)$	Virtual coherence function
$\gamma_{yx}^2$	Coherence function between $x$ (input) and $y$ (output)
$\gamma_{yxx}^2$	Multiple coherence of $y$ (output) with all $x_q$ (inputs)
$G_{xx}(f)$	Single-sided autospectral density of $x$
$G'_{xx}$	Principal component
$[G_{xx}]$	Single-sided input cross-spectral matrix
$G_{yx}(f)$	Single-sided cross-spectral density between $x$ (input) and $y$ (output)
$[G_{yx}]$	Single-sided input/output cross-spectral matrix
$h(n)$	Discrete impulse response
$h(t)$	Analog impulse response
$H(f)$	Analog frequency response function
$H(k)$	Discrete frequency response function
$H(s)$	Transfer function
$\text{Im}[\cdot]$	Imaginary part of [ ]
$j$	Imaginary number, $\sqrt{-1}$
$k$	Discrete (dimensionless) frequency variable
$k_r$	Modal stiffness of mode $r$

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$K_x$	Kurtosis of $x$
$\lambda$	Eigenvalue
$\mu_x$	(Theoretical) mean value of $x$
$m_r$	Modal mass of mode $r$
$M_n$	$N$ th statistical (central) moment
$n$	Discrete (dimensionless) time variable
$\phi$	Phase, general random variable
$p_x(x)$	Probability density of $x$
$P(x)$	Probability distribution of $x$
$\{\psi\}_r$	Mode shape vector of mode $r$
$[\Psi]_r$	Mode shape matrix of mode $r$
$Q$	Quality factor ( $Q$ -factor)
$Q_r$	Modal scale constant of mode $r$
$R_{xx}(\tau)$	Autocorrelation of $x$
$R_{yx}(\tau)$	Cross-correlation between $x$ (input) and $y$ (output)
$\text{Re}[\cdot]$	Real part of $[\cdot]$
$s$	Laplace operator (in [rad/s])
$s_r$	Pole, root to characteristic polynomial
$\sigma_x$	Standard deviation of $x$
$S_x$	Skewness of $x$
$S_{xx}(f)$	Double-sided autospectral density of $x$
$S_{yx}(f)$	Double-sided cross-spectral density between $x$ (input) and $y$ (output)
$[G_{yx}]$	Single-sided input/output cross-spectral matrix
$t$	Analog time
$T$	Measurement time
$\tau$	Time delay, time lag variable for correlation functions
$T_x(k)$	Discrete transient spectrum of $x$
$u, u(t)$	Vibration displacement
$v, v(t)$	Vibration velocity
$w(n)$	Discrete time window
$x(n)$	Discrete/sampled (input) signal
$x(t)$	Analog (input) signal
$\tilde{x}(t)$	Hilbert transform of $x(t)$
$X(f)$	(Continuous) Fourier transform of $x(t)$
$X'$	Spectrum of virtual signal
$X(k)$	Discrete Fourier transform of $x(n)$
$X_L(k)$	Linear (RMS) spectrum of $x(n)$
$y(n)$	Discrete/sampled (output) signal
$y(t)$	Analog (output) signal
$\omega$	Angular frequency in [radians/s]
$\zeta_r$	Relative (viscous) damping

# 1

## Introduction

This chapter provides a short introduction to the field of noise and vibration analysis. Its main objective is to show new students in this field the wide range of applications and engineering fields where noise and vibration issues are of interest. If you are a researcher or an engineer who wants to use this book as a reference source, you may want to skim this chapter. If you decide to do so, I would recommend you to read Section 1.6, in which I present some personal ideas on how to use this book, as well as on how to go about becoming a good experimentalist – the ultimate goal after reading this book.

I want to show you not only the width of disciplines where noise and vibrations are found. I also want to show you that noise and vibration *analysis*, the particular topic of this book, is truly a fascinating and challenging discipline. One of the reasons I personally find noise and vibration analysis so fascinating is the interdisciplinary character of this field. Because of this interdisciplinary character, becoming an expert in this area is indeed a real challenge, regardless of which engineering field you come from. If you are a student just entering this field, I can only congratulate you for selecting (which I hope you do!) this field as yours for a lifetime. You will find that you will never cease learning, and that every day offers new challenges.

### 1.1 Noise and Vibration

Noise and vibration are constantly present in our high-tech society. Noise causes serious problems both at home and in the workplace, and the task of reducing community noise is a subject currently focused on by authorities in many countries. Similarly, manufacturers of mechanical products with vibrations causing acoustic noise, more and more find themselves forced to compete on the noise levels of their products. Such competition has so far occurred predominantly in the automotive industry, where the issues with sound and noise have long attracted attention, but, at least in Europe, e.g., domestic appliances are increasingly marketed stressing low noise levels.

Let us list some examples of reasons why vibration is of interest.

- Vibration can cause *injuries and disease* in humans, with ‘white fingers’ due to long-term exposure to vibration, and back injuries due to severe shocks, as examples.
- Vibration can cause *discomfort*, such as sickness feelings in high-rise buildings during storms, or in trains or other vehicles, if vibration control is not successful.
- Vibration can cause *fatigue*, i.e., products break after being submitted to vibrations for a long (or sometimes not so long) time.

- Vibration can cause *dysfunction* in both humans and things we manufacture, such as bad vision if the eye is subjected to vibration, or a radar on a ship performing poorly due to vibration of the radar antenna.
- Vibration can be used for cleaning, etc.
- Vibration can cause noise, i.e., unpleasant sound, which causes annoyance as well as disease and discomfort.

To follow up on the last point in the list above, once noise is created by vibrations, noise is of interest, e.g., for the following reasons.

- Excessive noise can cause hearing impairment.
- Noise can cause discomfort.
- Noise can (probably) cause disease, such as increased risk of cardiac disease, and stress.
- Noise can be used for burglar alarms and in weapons (by disabling human ability to concentrate or to cope with the situation).

The lists above are examples, meant to show that vibrations and noise are indeed interesting for a wide variety of reasons, not only to protect ourselves and our products, but also because vibration can cause good things.

Besides simply reducing sound levels, much work is currently being carried out within many application areas concerning the concept of *sound quality*. This concept involves making a psychoacoustic judgment of how a particular sound is experienced by a human being. Harley Davidson is an often-cited example of a company that considers the sound from its product so important that it tried to protect that sound by trademark, although the application was eventually withdrawn.

Besides generating noise, vibrations can cause mechanical fatigue. Now and then we read in the newspaper that a car manufacturer is forced to recall thousands of cars in order to exchange a component. In those cases it is sometimes mechanical fatigue that has occurred, resulting in cracks initiating after the car has been driven a long distance. When these cracks grow they can cause component breakdown and, as a consequence, accidents.

## 1.2 Noise and Vibration Analysis

This book is about the *analysis methods* for analyzing noise and vibrations, rather than the mechanisms causing them. In order to identify the sources of vibrations and noise, extensive analysis of measured signals from different tests is often necessary. The measurement techniques used to carry out such analyses are well developed, and in universities as well as in industry, advanced equipment is often used to investigate noise and vibration signals in laboratory and in field environments.

The area of experimental noise and vibration analysis is an intriguing field, as I hope this book will reveal. It is so partly because this field is multidisciplinary, and partly because dynamics (including vibrations) is a complicated field where the most surprising things can happen. Using measurement and analysis equipment often requires a good understanding of mechanics, sensor technology, electronic measurement techniques, and signal analysis.

Vibrations and noise are found in many disciplines in the academic arena. Perhaps we first think of mechanics, with engines, vehicles, and pumps, etc. However, vibrations are also found also in civil engineering, in bridges, buildings, etc. Many of the measurement instruments and sensors we use in the field of analyzing vibrations and noise are, of course, electrical, and so the field of electrical engineering is heavily involved. This makes the initial study of noise and vibration analysis difficult, perhaps, because you are forced to get into some of the other fields of academia. Hopefully, this book can help bridge some of the gaps between disciplines.