machinery vibration and rotordynamics

John Vance • Fouad Zeidan • Brian Murphy

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The first author gratefully dedicates his part in this book to his loving wife Louise, who made the book possible by her unselfish support of the task and devotion to her husband while it was being written.

John M. Vance

CONTENTS

PREFACE

This book follows the first author's book *Rotordynamics of Turbomachinery* in its practical approach and style. Much of the material in that book has been updated and extended with new information, new examples, and a few corrections that reflect what has been learned since then. Of particular interest and significance are the new chapters (4, 5, and 6) on bearings, seals, and computer modeling contributed by the co-authors Dr. Fouad Zeidan and Dr. Brian Murphy. Dr. Zeidan is the president of two companies that design and manufacture high performance bearings and seals. These products often require the design and modeling of the complete rotor-bearing system to ensure reliable operation and compatibility. Dr. Murphy is the author of XLRotor™, one of the most widely used computer programs for rotordynamic analysis. Chapters 1 and 7 are also completely new. Chapter 1 describes the classical analytical techniques used by engineers for troubleshooting vibration problems. Chapter 7 gives a history of the most important rotordynamics analysis and experiments since 1869.

The authors have noted (with some surprise) for many years that the subject material of this book is not taught in most engineering colleges, even though rotating machines are probably the most common application of mechanical engineering. The book is organized so that the first three or four chapters could be used as a text for a senior or graduate college elective course. These chapters have exercises at the end that can be assigned to the students, which will greatly enhance their understanding of the chapter material. The later chapters will serve the same students well after graduation as reference source material with examples of analysis and test results for real machines, bearings, and seals. But for the majority of engineers assigned to troubleshoot a rotating machine, or to design it for reliability, and having no relevant technical background, this entire book can be the substitute for the course they never had.

It is the author's hope that this book will make a significant contribution to the improvement of rotating machines for the service of mankind in the years to come.

John M. Vance
Fouad Y. Zeidan
Brian T. Murphy

1

FUNDAMENTALS OF MACHINE VIBRATION AND CLASSICAL SOLUTIONS

This chapter is focused on practical applications of mechanical vibrations theory. The reader may want to supplement the chapter with one of the vibration textbooks in the reference list at the end of the chapter if he has no background in the theory.

THE MAIN SOURCES OF VIBRATION IN MACHINERY

The most common sources of vibration in machinery are related to the inertia of moving parts in the machine. Some parts have a reciprocating motion, accelerating back and forth. In such a case Newton's laws require a force to accelerate the mass and also require that the force be reacted to the frame of the machine. The forces are usually periodic and therefore produce periodic displacements observed as vibration. For example, the piston motion in the slider-crank mechanism of Fig. 1-1 has a fundamental frequency equal to the crankshaft speed but also has higher frequencies (harmonics). The dominant harmonic is twice crankshaft speed (2nd harmonic). Figure 1-2a shows the displacement of the piston. It looks almost like a sine wave but it is slightly distorted by higher-order harmonics due to the nonlinear kinematics of the mechanism. Fig. 1-2b shows the acceleration of the piston, where the 2nd harmonic is amplified since the acceleration amplitude is frequency-squared times the displacement amplitude.

Even without reciprocating parts, most machines have rotating shafts and wheels that cannot be perfectly balanced, so according to Newton's laws, there must be a rotating force vector at the bearing supports of each

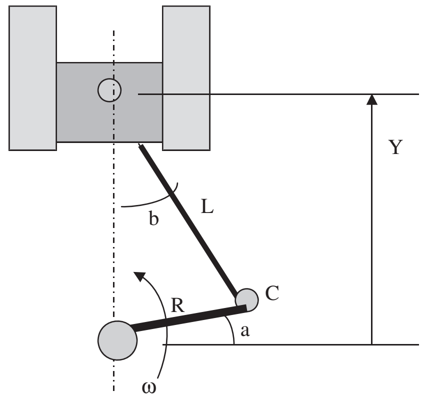

Figure 1-1 Slider-crank mechanism.

rotor to produce the centripetal acceleration of the mass center. Most of these force vectors are rotating and therefore produce a rotating displacement vector (all real machine parts are elastic) that can be observed as an orbit if two orthogonal vibration transducers are employed. Each of the transducers will produce a time trace similar to Fig. 1-2a or 1-2b. Harmonics and resulting distortion similar to Fig. 1-2a and 1-2b can be produced by shaft misalignment or by nonlinearity of the bearing stiffness. The fundamental frequency of the X and Y (orthogonal) vibration vectors is shaft speed ω, so the fundamental vibration is $x(t) = X\cos(\omega t)$ and $y(t) = Y\sin(\omega t)$. This type of vibration is referred to as *forced response* or *synchronous response to unbalance*. The vibration amplitude can become very large if the excitation frequency (rotor speed for example) becomes close to one of the natural frequencies of the machine structure. This is called a *resonance* or a *critical speed*, but it is not an unstable motion since the amplitude does not grow with time (unless there is no damping).

Another type of machine vibration problem, less common but more difficult to deal with, can come from the characteristic natural vibration frequencies (eigenvalues) of the machine structure and its supports, even if no imbalance or excitation is present. Natural frequencies die out in static structures due to the energy dissipated by damping, but in rotating machines they can grow larger with time. This is known as *self-excited instability* or *rotordynamic instability*. It is an innate potential characteristic of some rotating machines, especially when fluid pressures are present (e.g., bearings, impellers, turbine wheels, or seals).

Every real structure has an infinite number of natural frequencies, but many machinery vibration problems involve just one of these frequencies. That is why the simple single degree of freedom (SDOF) model (with just one natural frequency) presented in vibration textbooks [1–3] can be

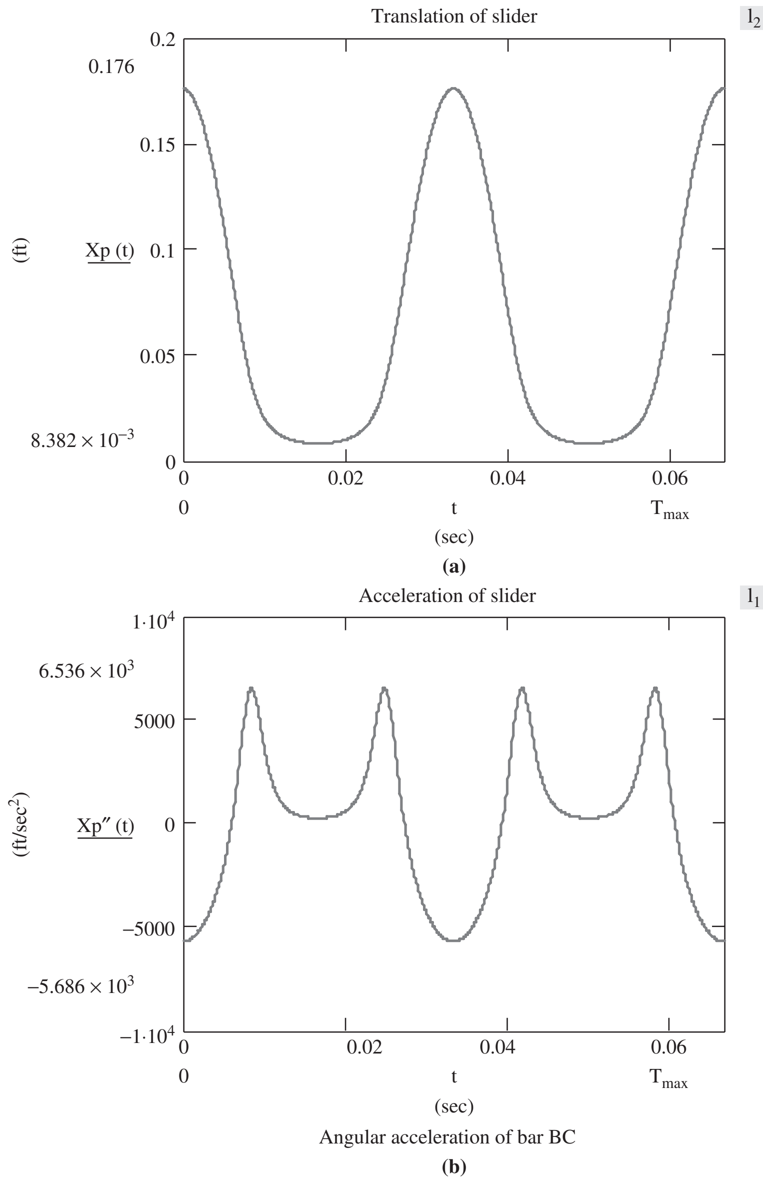

Figure 1-2 (a) Displacement of the piston, and (b) acceleration of the piston.

useful for analyzing vibration in machines. In fact, a SDOF model, consisting of one rigid mass, one spring, and one damper can be constructed to represent the vibration characteristics of any real machine in the neighborhood of a particular natural frequency of interest. This is called a *modal model*. To make physical sense out of complex machinery vibration data, or from realistic computer simulations of machinery vibration, the details of the SDOF mathematical model, its variations, and its solutions must be burned indelibly into the mind of the vibration engineer.

THE SINGLE DEGREE OF FREEDOM (SDOF) MODEL

The SDOF model as seen in most vibration textbooks is shown in Fig. 1-3. Here it will be referred to as system A. The stiffness, damping, and mass are k, c, and m, respectively. The undamped natural frequency is given by

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{rad/sec} \tag{1-1}$$

The circular frequency ω_n can be converted to hertz (Hz) (cycles/sec) as $f_n = \omega_n/2\pi$, or to revolutions per minute (rpm) as $N = 60 f_n$.

With a sinusoidal force applied to the mass, the differential equation of motion

$$m\ddot{x} + c\dot{x} + kx = F\sin(\omega t) \tag{1-2}$$

has a solution made up of two parts: (1) the particular solution for x that gives $F\sin(\omega t)$ on the right-hand side, and (2) the homogeneous solution for x that gives zero on the right-hand side. The sum of the two solutions, of course, gives $F\sin(\omega t)$, which satisfies the equality sign. The two solutions represent the two types of machine vibration described in the previous section, that is, forced response and characteristic (free) vibration. The particular solution for forced response is

$$x_p(t) = F\sin(\omega t + \phi)\Big/\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2} \tag{1-3}$$

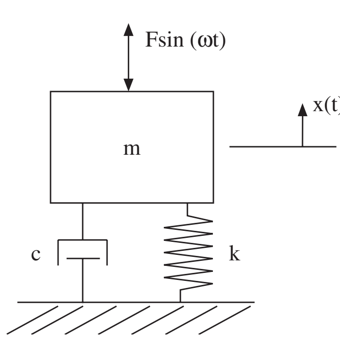

Figure 1-3 Single degree of freedom vibration model (system A).

Notice that the frequency ω of the forced vibration response is the same as the frequency of the excitation. The angle ϕ gives the time ϕ/ω by which the response x lags the excitation force F. For analyzing a vibration problem it is important to understand how k, c, and m influence the response amplitude. They have different effects depending on the frequency ratio ω/ω_n, as we shall see in the section to follow. Looking at Eq. 1-3 we can see that the amplitude X of the forced vibration response is

$$X = F\Big/\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2} \tag{1-4}$$

which depends on k, c, m, ω, and F. Notice that the denominator gets small when the exciting frequency ω is ω_n (Eq. 1-1) unless the damping coefficient c is large. A plot of Eq. 1-4 is shown in Fig. 1-7. It is called the *Bode amplitude plot* or the *frequency response plot* for system A.

The homogeneous part of the solution (for free vibration) with $F = 0$ is given by

$$x_h(t) = Ae^{st} \tag{1-5}$$

where s is a complex number, $s = \lambda + i\omega_d$. s is called the *eigenvalue*. Using the law of exponents, Eq. 1-5 can be rewritten as

$$x_h(t) = Ae^{\lambda t}e^{i\omega_d t} \tag{1-6}$$

where

$$e^{i\omega_d t} = \cos(\omega_d t) + i\,\sin(\omega_d t) \tag{1-7}$$

Equation 1-5 or 1-6 satisfies the differential Eq. 1-2 with $F = 0$ provided that the real part of the eigenvalue is $\lambda = -c/2m$ and the imaginary part is the square root of $\omega_d^2 = k/m - (c/2m)^2$. The amplitude A in Eq. 1-5 is of little interest here since it is determined only by the initial condition that instigates the free vibration. In rotating machinery, the differential equations are more complicated but still are of the same class as (1-2) and have the same form of homogeneous solution as (1-5). The imaginary part of s, ω_d, is the damped natural frequency. Notice that it becomes equal to ω_n, Eq. 1-1, when the damping coefficient $c = 0$.

The real part λ of the eigenvalue s determines how fast the free vibration dies out. It is often converted into a *damping ratio* $\zeta = c/c_{\text{cr}}$, where the critical damping $c_{\text{cr}} = 2m\omega_n$. Critical damping is the amount required to prevent free vibration (and no more). The conversion equation is $\zeta = -\lambda/\omega_n$. Figure 1-4a shows free vibration with $\zeta = 0.05$ (5% of critical damping); Fig. 1-4b shows the same system with $\zeta = 0.25$ (25% of critical damping). If a free vibration is graphed like Fig. 1-4, the damping can be expressed as the natural logarithm of

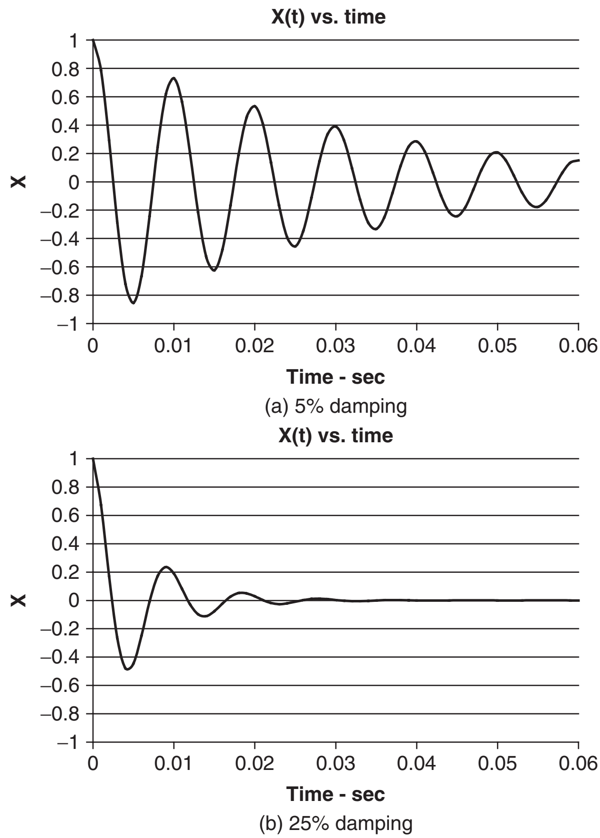

Figure 1-4 (a) Free vibration with 0.05 damping ratio; (b) free vibration with 0.25 damping ratio.

the ratio of successive amplitudes X_n/X_{n+1}. The logarithmic decrement $\delta = \ln(X_n/X_{n+1}) = 2\pi\zeta\big/\left(1+\zeta^2\right)^{1/2}$. The inverse expression is often useful: $\zeta = \delta\big/\left[(2\pi)^2+\delta^2\right]^{1/2}$.

The algebraic sign of the real part of the eigenvalue λ is the mathematical test for vibration stability, i.e., whether the free vibration of frequency ω_d will die out or, in the unstable case, will grow with time. For example, in the simple system of Fig. 1-3, λ becomes positive if the damping c is negative. Negative damping is possible in mechanical systems, especially when fluid pressures are acting.

USING SIMPLE MODELS FOR ANALYSIS AND DIAGNOSTICS

Techniques and methods for solving vibration problems can often be developed by using the simple one degree of freedom model even though the real system is more complicated. The main purpose of the model is to provide an understanding of the type of problem being encountered so that the most effective type of "fix" can be identified. Sometimes a simple model can even yield useful approximations for the optimum parametric values, such as stiffness and damping to be employed. In contrast to the large and detailed finite element models being promoted by some for all diagnostic vibration analysis, this approach suggests that the engineer should first use the simplest possible model that contains the relevant physical characteristics and resort to the more detailed models only when the simple models do not yield sufficient guidance for modifications to the design or when improved accuracy is desired.

In addition to system A of Fig. 1-3, two more single degree of freedom models are shown in Figs. 1-5 and 1-6. All three of these systems have a single natural frequency determined by their modal mass and stiffness, but there are subtle differences between the three models that are related to the type of excitation.

The constant amplitude exciting force F in system A is generally unrealistic. Inertia forces in rotating machinery are proportional to speed squared. Model C in Fig. 1-6 has an unbalanced rotor so that the exciting force $F = m\omega^2 u$, where u is the offset of the center of rotor mass m from the axis of rotation. Note that the mass m is the rotating mass, not the total mass, so m on the left side of differential equation (1-2) must be replaced by the total mass M unless the nonrotating mass is negligible.

In some cases the excitation is a vibration displacement at the base, rather than a force. This is represented by system B in Fig. 1-5.

These small differences in the models produce different frequency response curves. The differences are useful in diagnosing problems and determining solutions. Obviously, to use these differences, the engineer must have a complete and thorough knowledge of the three models and their responses. The three systems illustrated in Figs. 1-3, 1-5, and 1-6 and their mathematical analyses are described in most vibration textbooks [1–3]. In some cases the damping should be included in the most realistic way possible, i.e., as viscous, Coulomb, hysteretic, or aerodynamic damping. However, if the damping is other than viscous, it may usually be

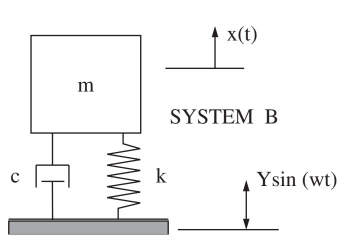

Figure 1-5 SDOF model with base excitation.

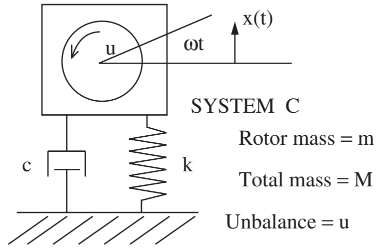

Figure 1-6 SDOF model with rotating unbalance.

represented by an equivalent viscous damping coefficient that varies with frequency [1, page 73]. For purely steel structures, it is usually less than 5% of the critical value. System B may have its predominant damping either (1) between the vibrating base and the modal mass, or (2) from the mass to ground. It is important to recognize the difference and set up the model correctly.

The frequency response curves for systems A, B, and C are plots of the amplitude of forced vibration versus the frequency. The response amplitude for system A is computed from Eq. 1-4 at each frequency, using appropriate values for k, c, m, and F. Figure 1-7 shows the response curve for system A with parameter values from Table 1-1. For plotting the curve, frequency ω (rad/sec) has been converted to rpm (cpm). X_static in the table is F/k, the displacement at zero frequency, which is the deflection of the spring under a static force F. Resonance is the undamped natural

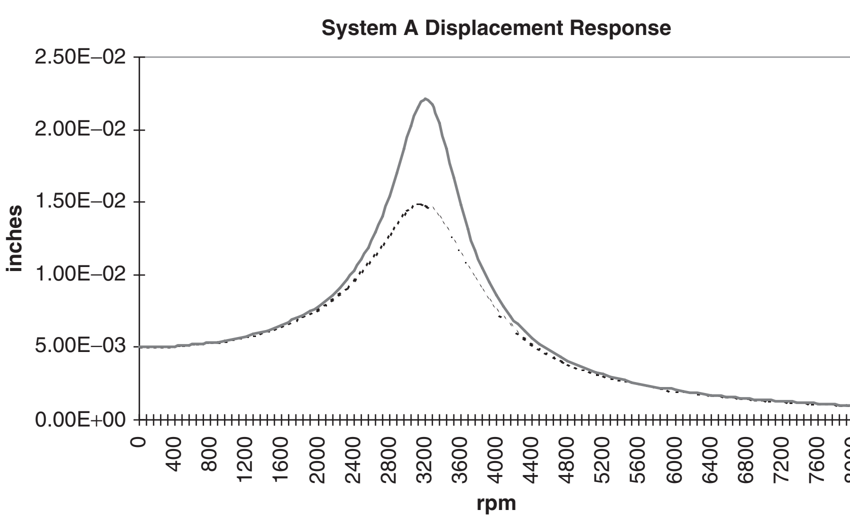

Figure 1-7 Forced response of system A (constant amplitude excitation force *F*).

Table 1-1 System A values for Fig. 1-7

	Data	Units
Input		
Mass	100	lb
Kstiff	30,000	lb/in
Cdamp	20	lb-sec/in
Force	150	lb
Freqstart	0	rpm
Freqstop	8000	rpm
Npoints	101	use 101
Output		
Resonance	3251.252	rpm
Zeta	0.11349	none
X_static	5.00E-03	in

frequency ω_n converted to cpm. Zeta is the critical damping ratio, i.e., the percentage of critical damping divided by 100. The solid curve in Fig. 1-7 has all the parametric values of Table 1-1.

The dashed curve in Fig. 1-7 has all the values of Table 1-1 except that the damping coefficient c has been increased from 20 lb-sec/in. (in the solid curve) to 30 lb-sec/in. The main effect of the increased damping is to reduce the vibration amplitude at the critical speed. It has very little effect at frequencies away from the critical speed. The critical speed (where the peak vibration occurs) is 3200 rpm for the solid curve and about 3150 rpm for the dashed curve. These are both slightly below the undamped natural frequency of 3251 cpm. Thus, damping tends to lower the critical speed. (This effect is reversed in system C (below) when the constant shaking force F is replaced with a rotating unbalance force $m\omega^2 u$). In Fig. 1-7, notice that the response amplitude X (= 5 mils at zero frequency) becomes large near the natural frequency, and approaches zero at very high frequencies. Figure 1-8 shows how the vibration X (the dashed curve) lags the force F with a phase angle ϕ (see Eq. 1-3). Figure 1-9 shows how the phase angle varies with frequency. More damping (the dashed curve) makes the phase angle change more gradually as the excitation frequency passes through ω_n. The phase angle is 90 degrees at the undamped natural frequency ω_n, regardless of the amount of damping. This fact is useful in determining the value of ω_n, since the phase angle can be measured but ω_n cannot be measured.

Graphs like Figs. 1-7 and 1-9 are often referred to as the frequency response curves, or Bode plots. If the parameter values (k, c, m) are changed, then the response curves will look similar but will have different

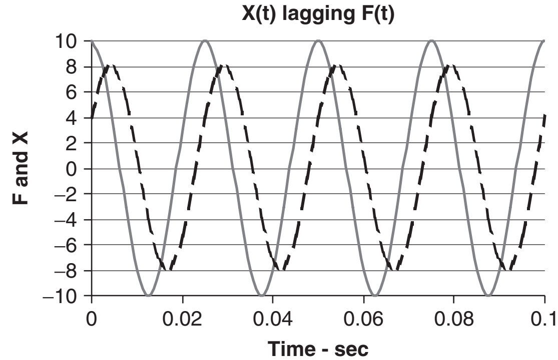

Figure 1-8 *X* (dashed) lagging force (solid).

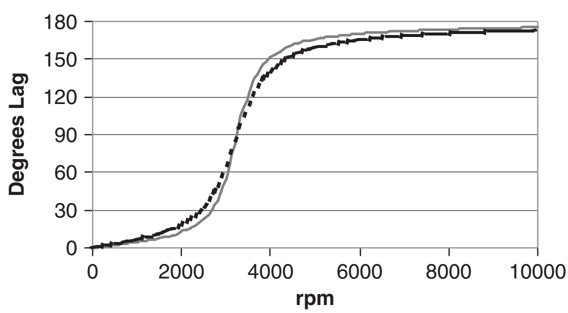

Figure 1-9 Phase lag response of system A.

values of response amplitude and phase. Increasing the damping generally brings the peak amplitude down but has a negligible effect at frequencies away from the natural frequency.

The necessity to plot many different curves for different values of F, k, and m is avoided by plotting the curve with dimensionless ratios as shown in Fig. 1-10. The abscissa in Fig. 1-10 is frequency ratio ω/ω_n; the ordinate Xk/F is X/X_static (the ratio of vibration amplitude to static displacement under the force F).

The frequency response of system B (Fig. 1-5, base vibration excitation) is given by

$$X = Y\sqrt{\frac{k^2 + (\omega c)^2}{\left(k - m\omega^2\right)^2 + (c\omega)^2}} \tag{1-8}$$

Figure 1-11 shows the response amplitude X calculated with the parametric values of Table 1-2. In the table, X_Base is the displacement amplitude Y of the vibrating support. Notice that damping in system B (the dashed

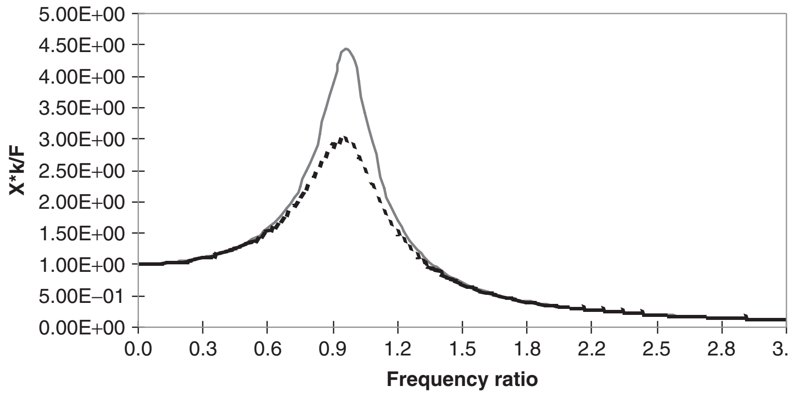

Figure 1-10 Dimensionless response of system A.

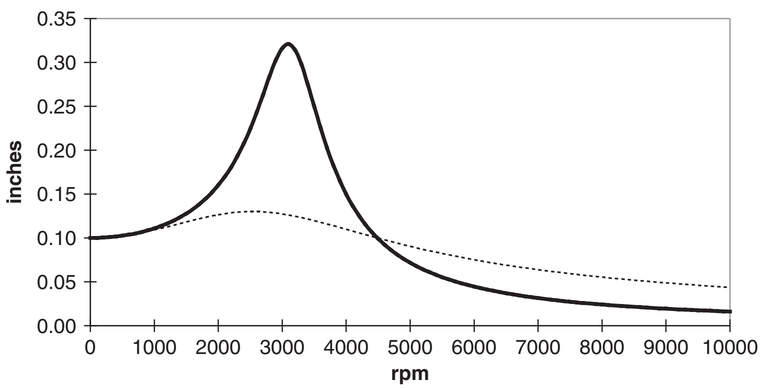

Figure 1-11 Response to base excitation of system B.

curve) actually increases the vibration response at high frequencies. Solving the differential equation for system B [1, page 66] shows that the crossover frequency is 1.4 times the undamped natural frequency. All the curves with different damping values cross at this frequency, and the amplitude there is the same as X_Base. The frequency range above this is called the *isolation range*, since the response there is reduced below what would be obtained with a hard support. A vibrating system with a fixed excitation frequency can be put into the isolation range by softening the spring Kstiff between the vibrating base and the mass.

The frequency response of system C (Fig. 1-6) is given by Eq. 1-9, where u is the unbalance (C.G. offset of the rotor), m is the rotor mass, and M is the total mass:

$$X = m\omega^2 u \Big/ \sqrt{\left(k - M\omega^2\right)^2 + (c\omega)^2} \qquad (1\text{-}9)$$

Table 1-2 System B parameters for Fig. 1-11

	Data	Units
Input		
Mass	0.35	lb
Kstiff	100	lb/in
Cdamp	0.1	lb-sec/in
Cdamp2	0.4	lb-sec/in
X_Base	0.1	in
Freqstart	0	rpm
Freqstop	10,000	rpm
Npoints	101	use 101
Output		
Resonance	3172.897	rpm
Zeta	0.166132	none
Zeta2	0.66453	none

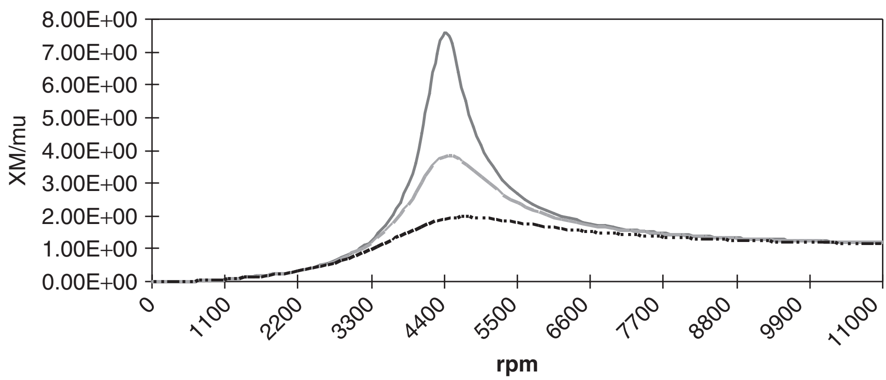

Figure 1-12 Response with an unbalanced rotor for three damping values.

The ratio X/u is often used and is sometimes called the *magnification factor*. The response calculated with the parametric values of Table 1-3 is shown in Fig. 1-12 with dimensionless amplitude XM/mu. In the table, m = Rotrmass and M = Rotrmass + Housmass. Note that XM/mu is approximately X/u in this case, since $m/M = 0.98$ (the housing mass is negligible). Figure 1-12 shows that system C response starts out at zero and damping in system C reduces the peak amplitude of vibration response and *raises* the critical speed. At very high frequencies the vibration amplitude approaches a limiting value determined by the amount of unbalance. Increasing the housing mass will reduce this value.

Table 1-3 System C parameters for Fig. 1-12

	Data	Units
Input		
Rotrmass	50	lb
Housmass	1	lb
Kstiff	28,000	lb/in
Cdamp	8	lb-sec/in
Cdamp2	16	lb-sec/in
Cdamp3	32	lb-sec/in
Unbalance	0.0015	in
Freqstart	0	rpm
Freqstop	11000	rpm
Npoint	101	use 101
Output		
Resonance	4398.29	rpm
Zeta	0.065798	none
Zeta2	0.131597	none
Zeta3	0.263193	none
Totalmass	51	lb
Massratio	0.980392	none

SIX TECHNIQUES FOR SOLVING VIBRATION PROBLEMS WITH FORCED EXCITATION

When vibration measurements from the real system are compared and identified with the theoretical response from the appropriate model (A, B, or C) one of the following techniques for reducing the vibration will often become apparent.

1. *Identify and reduce the excitation source.* This most obvious solution is also the one least likely to be possible in systems of type A or type B, but it should be investigated first. In rotating machinery (system C), this technique is implemented by balancing the rotating parts. Balancing will be effective only when the vibration frequency is equal to the speed of a rotating part or its integer harmonics, and this fact is the corollary of a diagnostic rule: *Frequency components in a measured spectrum that are synchronous with a rotating speed or one of its harmonics are often caused by rotating imbalance.* In a reciprocating machine (Fig. 1-1), balancing the 2nd harmonic often requires a separate unbalanced *balance*

shaft rotating at twice crankshaft speed to cancel out the inertia forces.

2. *Tune the natural frequency to a value further away from the frequency of excitation to avoid resonance.* A study of the frequency response curves for any of the systems A, B, or C reveals that the vibratory excitation is highly magnified at frequencies near the natural frequency. This magnification factor R, or Q factor as it is sometimes called, can typically range from 5 to 50 or more depending on the amount of damping. The excitation frequency can seldom be changed, but the natural frequency can sometimes be easily changed by changing the modal stiffness. This is one place where intelligent construction of the analytical model becomes important, since the modal stiffness may be made up of several real stiffnesses in parallel or in series. In parallel combinations the very low stiffnesses have little effect in determining the modal stiffness, while in series combinations the very high stiffnesses have little effect. The tuning method is effective only when the excitation frequency is constant or when it only varies over a narrow range.
3. *Isolate the modal mass from the vibratory excitation by making the modal stiffness very low.* Notice that all the response curves show a very low response to the vibratory excitation at frequencies much higher than the natural frequency (far to the right on the response curves). Once again, the excitation frequency usually cannot be changed but the natural frequency can be brought far down by a very soft modal stiffness, thus placing the system response far to the right of resonance on the response curve. This method is particularly effective in systems of type B. A typical application is isolating an electronics box from a vibrating vehicle frame.
4. *Add damping to the system.* Damping is added by incorporating mechanisms that dissipate vibratory energy into heat. When they work, damping mechanisms produce forces that act in opposition to the vibratory velocity. Contrary to popular belief, however, adding damping indiscriminately does not always reduce vibration. Damping does work well whenever operation is near resonance (and this is the operating condition most likely to cause a problem). At frequencies away from resonance damping has very little effect, except to increase the forces transmitted to ground at high frequencies far above resonance. In a system B application where isolation is used, damping added between the modal mass and the vibrating support will actually increase the vibration of the mass at high frequencies. In a system C (rotating machinery) application with rolling element bearings, adding damping to the bearing supports will increase the