

Suspension Geometry and Computation

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This work is dedicated to

Aythe

*the beautiful goddess of truth, hence also of science and mathematics, and of good
computer programs.*

Her holy book is the book of nature.

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Preface

The motor car is over one hundred years old. The suspension is an important part of its design, and there have been many research papers and several books on the topic. However, suspension analysis and design are so complex and bring together so many fields of study that they seem inexhaustible. Certainly, the number of different suspension designs that have been used over the years is considerable, and each one was right for a particular vehicle in that designer's opinion. This breadth of the field, at least, is a justification for another book. It is not that the existing ones are not good, but that new perspectives are possible, and often valuable, and there are new things to say. Here, the focus is on the most fundamental aspect of all, the geometry of the road, the vehicle and the suspension, the basic measurement which is the foundation of all subsequent dynamic analysis.

The process of modern engineering has been deeply affected by the computer, not always for the better because analytical solutions may be neglected with loss of insight to the problem. In this case the solution of complex three-dimensional geometrical problems is greatly facilitated by true coordinate geometry solutions or by iteration, methods which are discussed here in some detail.

In principle, geometry is not really conceptually difficult. Wrestling with actual problems shows otherwise. Analytical geometry, particularly on the computer with its many digits of precision, mercilessly shows up any approximations and errors, and, surprisingly, often reveals incomplete understanding of deep principles.

New material presented in detail here includes relationships between bump, heave and roll coefficients (Table 8.10.1), detailed analysis of linear and non-linear bump steer, design methods for determining wishbone arm lengths and angles, methods of two-dimensional and three-dimensional solutions of suspension-related geometry, and details of numerical iterative methods applied to three-dimensional suspensions, with examples.

As in my previous work, I have tried to present the basic core of theory and practice, so that the book will be of lasting value. I would be delighted to hear from readers who wish to suggest any improvements to presentation or coverage.

John C. Dixon.

1

Introduction and History

1.1 Introduction

To understand vehicle performance and cornering, it is essential to have an in-depth understanding of the basic geometric properties of roads and suspensions, including characteristics such as bump steer, roll steer, the various kinds of roll centre, and the relationships between them.

Of course, the vehicle is mainly a device for moving passengers or other payload from A to B, although in some cases, such as a passenger car tour, a motor race or rally, it is used for the interest of the movement itself. The route depends on the terrain, and is the basic challenge to be overcome. Therefore road characteristics are examined in detail in Chapter 2. This includes the road undulations giving ride quality problems, and road lateral curvature giving handling requirements. These give rise to the need for suspension, and lead to definite requirements for suspension geometry optimisation.

Chapter 3 analyses the geometry of road profiles, essential to the analysis of ride quality and handling on rough roads. Chapter 4 covers suspension geometry as required for ride analysis. Chapter 6 deals with steering geometry. Chapters 6–9 study the geometry of suspensions as required for handling analysis, including bump steer, roll steer, camber, roll centres, compliance steer, etc., in general terms.

Subsequent chapters deal with the properties of the main particular types of suspension, using the methods introduced in the earlier chapters. Then the computational methods required for solution of suspension geometry problems are studied, including two- and three-dimensional coordinate geometry, and numerical iteration.

This chapter gives an overview of suspensions in qualitative terms, with illustrations to show the main types. It is possible to show only a sample of the innumerable designs that have been used.

1.2 Early Steering History

The first common wheeled vehicles were probably single-axle hand carts with the wheels rotating independently on the axle, this being the simplest possible method, allowing variations of direction without any steering mechanism. This is also the basis of the lightweight horse-drawn chariot, already important many thousands of years ago for its military applications. Sporting use also goes back to antiquity, as illustrated in films such as *Ben Hur* with the famous chariot race. Suspension, such as it was, must have been important for use on rough ground, for some degree of comfort, and also to minimise the stress of the structure, and was based on general compliance rather than the inclusion of special spring members. The axle can be made long and allowed to bend vertically and longitudinally to ride the bumps. Another important factor in riding over rough roads is to use large wheels.

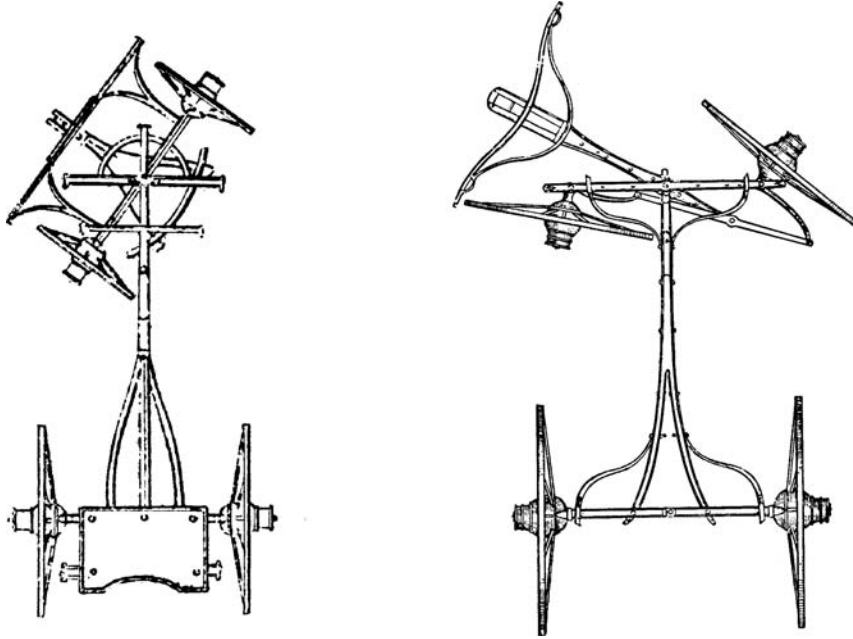


Figure 1.2.1 Steering: (a) basic cart steering by rotating the whole axle; (b) Langensperger's independent steering of 1816.

For more mundane transport of goods, a heavier low-speed two-axle cart was desirable, and this requires some form of steering mechanism. Initially this was achieved by the simple means of allowing the entire front axle to rotate, as shown in Figure 1.2.1(a).

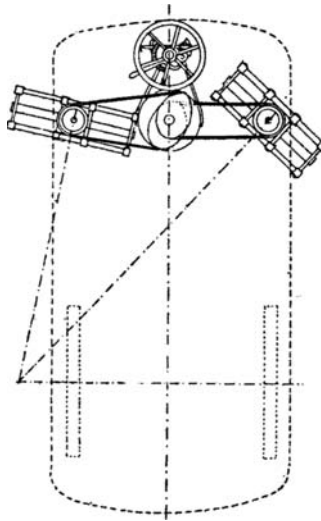


Figure 1.2.2 Ackermann steering effect achieved by two cams on L'Obeissante, designed by Amedée Bollée in 1873.

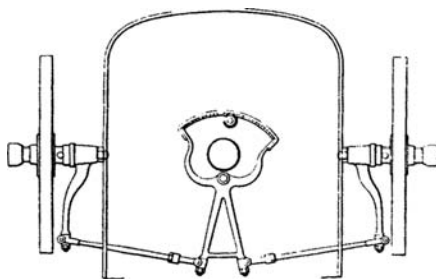


Figure 1.2.3 Ackermann steering effect achieved with parallel steering arms, by using angled drive points at the inner end of the track rods: ‘La Mancelle’, 1878.

Steering by the movement of the whole axle gives good geometric positioning, with easy low-speed manoeuvring, but the movement of the axle takes up useful space. To overcome this, the next stage was to steer the wheels independently, each turning about an axis close to the wheel. The first steps in this direction were taken by Erasmus Darwin (1731–1802), who had built a carriage for his doctor’s practice, allowing larger-diameter wheels of great help on the rough roads. However, if the two wheels are steered through the same angle then they must slip sideways somewhat during cornering, which greatly increases the resistance to motion in tight turns. This is very obvious when a parallel-steered cart is being moved by hand. To solve this, the two wheels must be steered through different angles, as in Figure 1.2.1(b). The origin of this notion may be due to Erasmus Darwin himself in 1758, or to Richard Edgeworth, who produced the earliest known drawing of such a system. Later, in 1816 Langensperger obtained a German patent for such a concept, and in 1817 Rudolf Ackermann, acting as Langensperger’s agent, obtained a British patent. The name Ackermann has since then been firmly attached to this steering design. The first application of this steering to a motor vehicle, rather than hand or horse-drawn carts, was by Edward Butler. The simplest way to achieve the desired geometry is to angle the steering arms inwards in the straight-ahead position, and to link them by a tie rod (also known as a track rod), as was done by Langensperger. However, there are certainly other methods, as demonstrated by French engineer Amedée Bollée in 1873, Figure 1.2.2, possibly allowing a greater range of action, that is, a smaller minimum turning circle.

The ‘La Mancelle’ vehicle of 1878 (the name refers to a person or thing from Le Mans) achieved the required results with parallel steering arms and a central triangular member, Figure 1.2.3. In 1893 Benz obtained a German patent for the same system, Figure 1.2.4. This shows tiller control of the steering, the common method of the time. In 1897 Benz introduced the steering wheel, a much superior system to the tiller, for cars. This was rapidly adopted by all manufacturers. For comparison, it is interesting to note that dinghies use tillers, where it is suitable, being convenient and economic, but ships use a large wheel, and aircraft use a joystick for pitch and roll, although sometimes they have a partial wheel on top of a joystick with only fore–aft stick movement.

1.3 Leaf-Spring Axles

Early stage coaches required suspension of some kind. With the limited technology of the period, simple wrought-iron beam springs were the practical method, and these were made in several layers to obtain the required combination of compliance with strength. These multiple-leaf springs became known simply as leaf springs. To increase the compliance, a pair of leaf springs were mounted back-to-back. They were curved, and so then known, imprecisely, as elliptical springs, or elliptics for short. Single ones were called

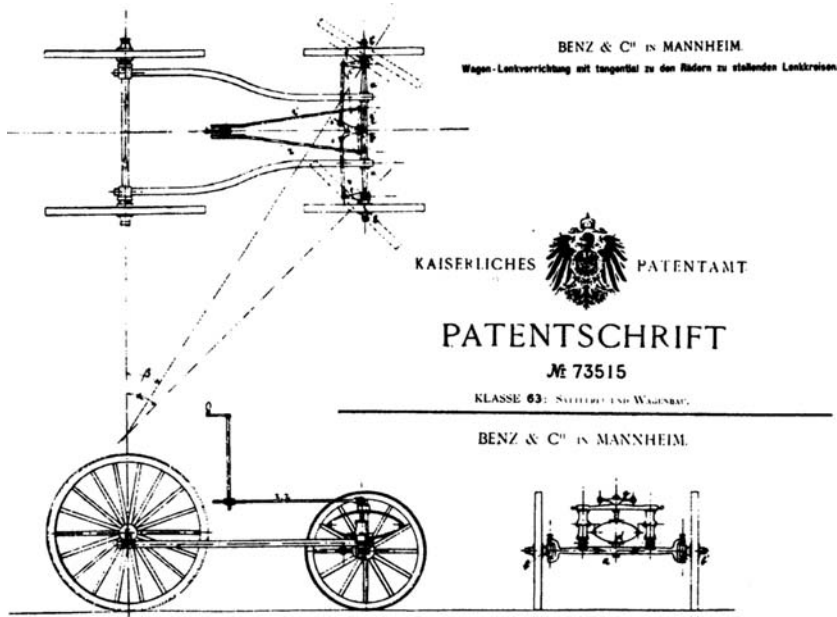


Figure 1.2.4 German patent of 1893 by Benz for a mechanism to achieve the Ackermann steering effect, the same mechanism as La Mancelle.

semi-elliptics. In the very earliest days of motoring, these were carried over from the stage coaches as the one practical form of suspension, as may be seen in Figure 1.3.1.

The leaf spring was developed in numerous variations over the next 50 years, for example as in Figure 1.3.2. With improving quality of steels in the early twentieth century, despite the increasing average

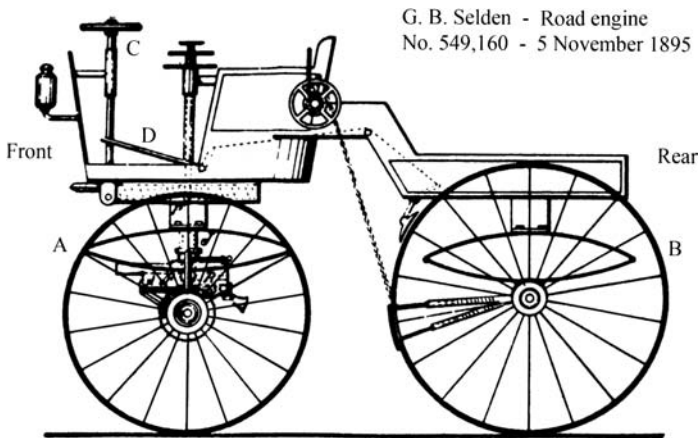


Figure 1.3.1 Selden's 1895 patent showing the use of fully-elliptic leaf springs at the front A and rear B. The steering wheel is C and the foot brake D.

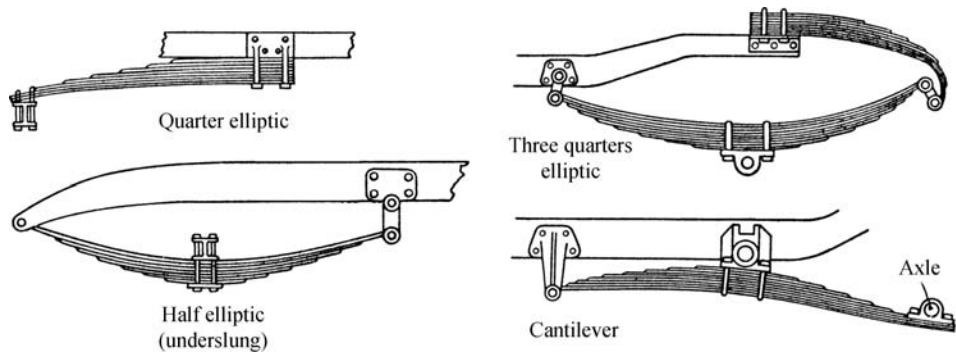


Figure 1.3.2 Some examples of the variation of leaf springs in the early days. As is apparent here, the adjective ‘elliptical’ is used only loosely.

weight of motor cars, the simpler semi-elliptic leaf springs became sufficient, and became widely standardised in principle, although with many detailed variations, not least in the mounting systems, position of the shackle, which is necessary to permit length variation, and so on. The complete vehicle of Figure 1.3.3 shows representative applications at the front and rear, the front having a single compression shackle, the rear two tension shackles. A very real advantage of the leaf spring in the early days was that the spring provides lateral and longitudinal location of the axle in addition to the springing compliance action. However, as engine power and speeds increased, the poor location geometry of the leaf spring became an increasing problem, particularly at the front, where the steering system caused many problems in bump and roll. To minimise these difficulties, the suspension was made stiff, which caused poor ride quality.

Figures 1.3.4 and 1.3.5 show representative examples of the application of the leaf spring at the rear of normal configuration motor cars of the 1950s and 1960s, using a single compression shackle.

Greatly improved production machinery by the 1930s made possible the mass production of good quality coil springs, which progressively replaced the leaf spring for passenger cars. However, leaf-spring use on passenger cars continued through into the 1970s, and even then it functioned competitively, at the rear at least, Figure 1.3.6. The leaf spring is still widely used for heavily loaded axles on trucks and military vehicles, and has some advantages for use in remote areas where only basic maintenance is possible, so leaf-spring geometry problems are still of real practical interest.

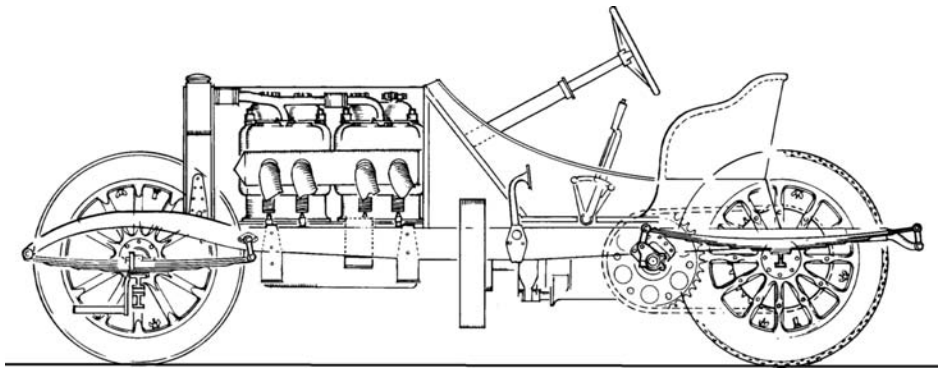


Figure 1.3.3 Grand Prix car of 1908, with application of semi-elliptic leaf springs at the front and rear (Mercedes-Benz).

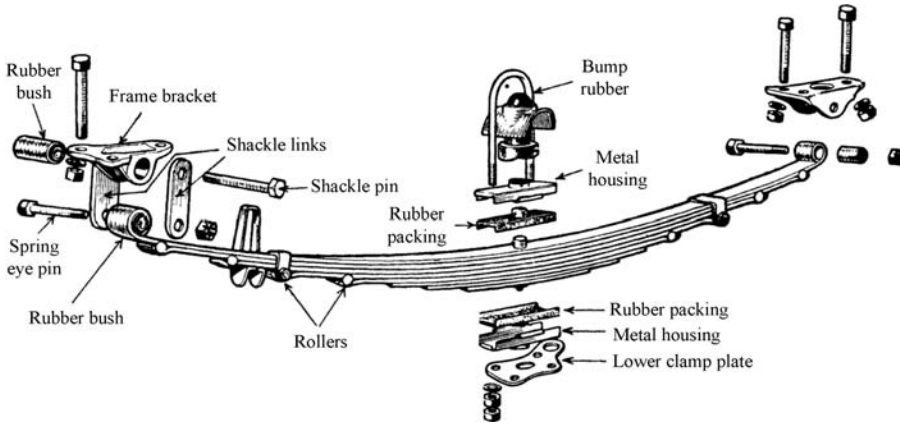


Figure 1.3.4 A representative rear leaf-spring assembly (Vauxhall).

At the front, the leaf spring was much less satisfactory, because of the steering geometry difficulties (bump steer, roll steer, brake wind-up steering effects, and shimmy vibration problems). Figure 1.3.7 shows a representative layout of the typical passenger car rigid-front-axle system up to about 1933. In bump, the axle arc of movement is centred at the front of the spring, but the steering arm arc is centred at

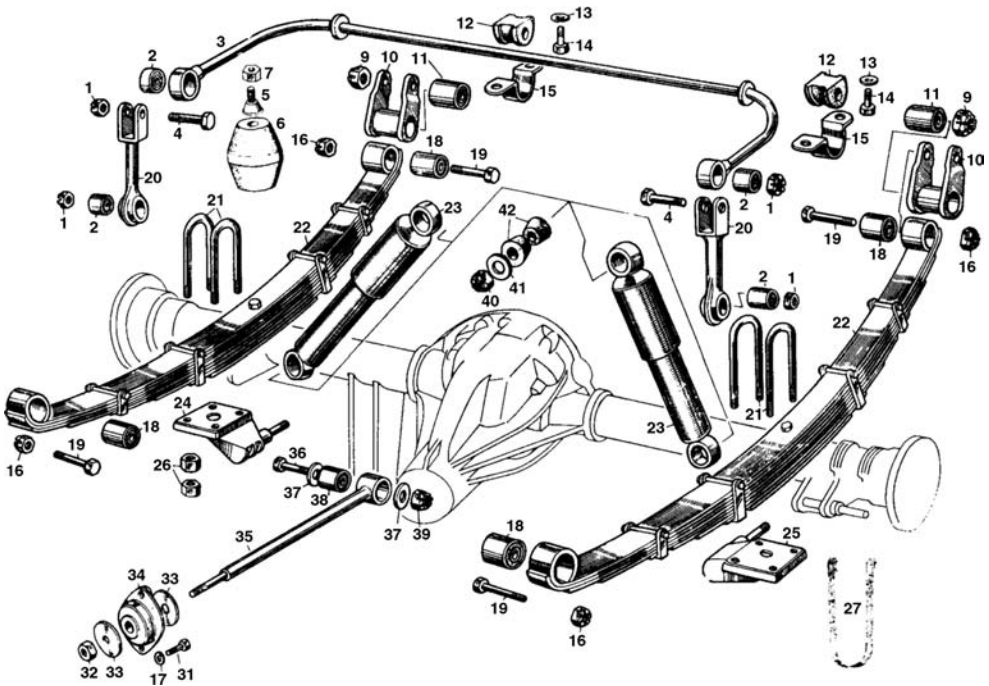


Figure 1.3.5 A 1964 live rigid rear axle with leaf springs, anti-roll bar and telescopic dampers. The axle clamps on top of the springs (Maserati).

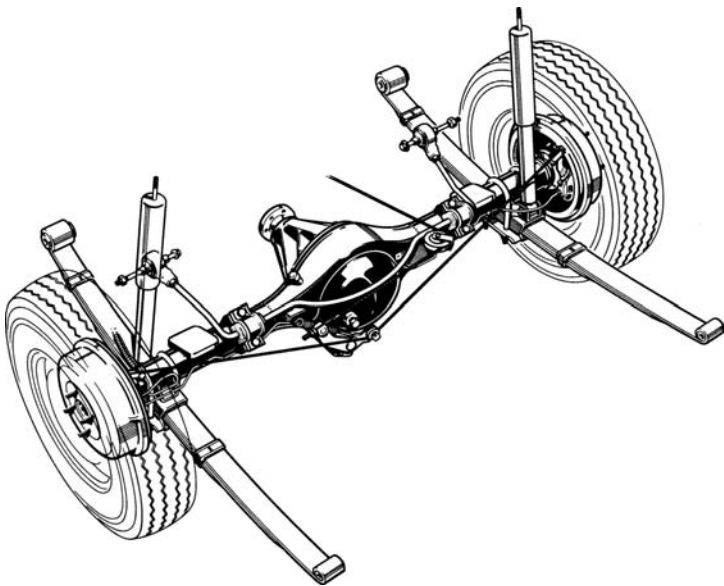


Figure 1.3.6 Amongst the last of the passenger car leaf-spring rear axles used by a major manufacturer was that of the Ford Capri. Road testers at the time found this system in no way inferior to more modern designs.

the steering box. These conflicting arcs give a large and problematic bump steer effect. The large bump steer angle change also contributed to the shimmy problems by causing gyroscopic precession moments on the wheels. Figure 1.3.8 shows an improved system with a transverse connection.

Truck and van steering with a leaf spring generally has the steering box ahead of the axle, to give the maximum payload space, as seen in Figure 1.3.9. In bump, the arc of motion of the steering arm and the axle on the spring are in much better agreement than with the rear box arrangement of Figure 1.3.7, so bump steer is reduced. Also, the springs are likely to be much stiffer, with reduced range of suspension movement, generally reducing the geometric problems.

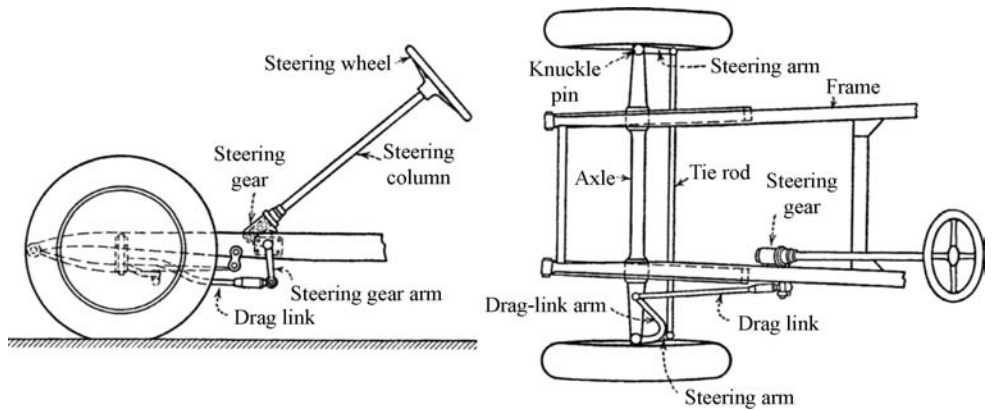


Figure 1.3.7 Classical application of the rigid axle at the front of a passenger car, the normal design up to 1933. Steering geometry was a major problem because of the variability of rigid axle movements.

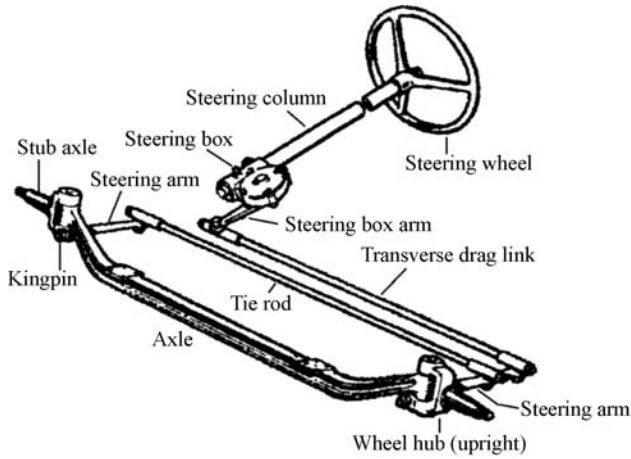


Figure 1.3.8 Alternative application of the rigid axle at the front of a passenger car, with a transverse steering link between the steering box on the sprung mass and the axle, reducing bump steer problems.

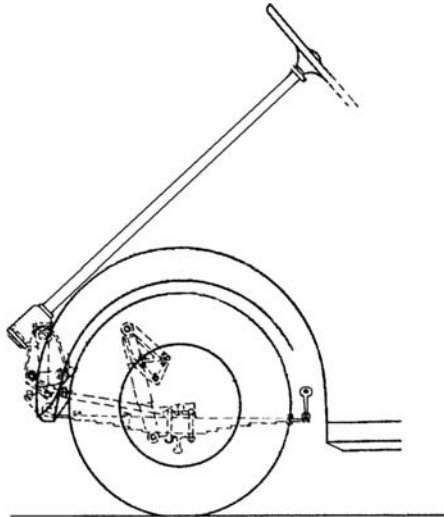


Figure 1.3.9 Van or truck steering typically has a much steeper steering column with a steering box forward of the axle, as here. The steering geometry problems are different in detail, but may be less overall because a stiffer suspension is more acceptable.

1.4 Transverse Leaf Springs

Leaf springs were not used only in longitudinal alignment. There have been many applications with transverse leaf springs. In some cases, these were axles or wheel uprights located by separate links, to overcome the geometry problems, with the leaf spring providing only limited location service, or only the springing action. Some transverse leaf examples are given in Figures 1.4.1–1.4.4

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