

FIFTH EDITION

Sample FRM® Review Test CD Included

Financial Risk Manager Handbook

- Learn the essentials of managing market, credit, operational, and liquidity risk
 - Learn the essentials of investment management and hedge fund risk
 - Learn about structured products, futures, options, and other derivative instruments
 - Identify regulatory and legal issues
- Ideal for self-instruction and in-house training in financial risk management
 - ▶ The official reference book for GARP's FRM® certification program



PHILIPPE JORION



RISK MANAGEMENT LIBRARY

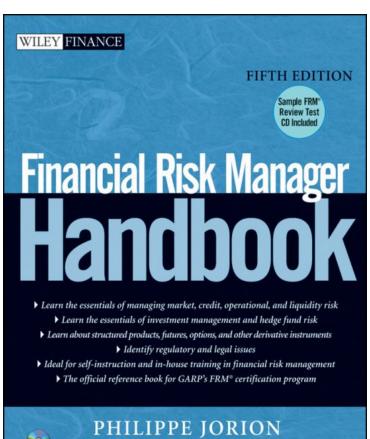






Table of Contents

<u>Title Page</u>
<u>Copyright Page</u>
<u>Preface</u>
About the Author
About GARP
FINANCIAL RISK MANAGER (FRM®) CERTIFICATION
OTHER GARP CERTIFICATIONS
GARP DIGITAL LIBRARY
GARP EVENTS AND NETWORKING
<u>Introduction</u>

PART One - Quantitative Analysis

CHAPTER 1 - Bond Fundamentals

1.1 DISCOUNTING, PRESENT, AND FUTURE VALUE
1.2 PRICE-YIELD RELATIONSHIP
1.3 BOND PRICE DERIVATIVES
1.4 IMPORTANT FORMULAS
1.5 ANSWERS TO CHAPTER EXAMPLES
APPENDIX: APPLICATIONS OF INFINITE SERIES

CHAPTER 2 - Fundamentals of Probability

2.1 CHARACTERIZING RANDOM VARIABLES
2.2 MULTIVARIATE DISTRIBUTION FUNCTIONS
2.3 FUNCTIONS OF RANDOM VARIABLES
2.4 IMPORTANT DISTRIBUTION FUNCTIONS

2.5 LIMIT DISTRIBUTIONS

- 2.6 IMPORTANT FORMULAS
- 2.7 ANSWERS TO CHAPTER EXAMPLES
- APPENDIX: REVIEW OF MATRIX MULTIPLICATION

CHAPTER 3 - Fundamentals of Statistics

- 3.1 REAL DATA
- 3.2 PARAMETER ESTIMATION
- 3.3 REGRESSION ANALYSIS
- 3.4 IMPORTANT FORMULAS
- 3.5 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 4 - Monte Carlo Methods

- 4.1 SIMULATIONS WITH ONE RANDOM VARIABLE
- 4.2 IMPLEMENTING SIMULATIONS
- 4.3 MULTIPLE SOURCES OF RISK
- **4.4 IMPORTANT FORMULAS**
- 4.5 ANSWERS TO CHAPTER EXAMPLES

PART Two - Capital Markets

CHAPTER 5 - Introduction to Derivatives

- 5.1 OVERVIEW OF DERIVATIVES MARKETS
- **5.2 FORWARD CONTRACTS**
- **5.3 FUTURES CONTRACTS**
- **5.4 SWAP CONTRACTS**
- 5.5 IMPORTANT FORMULAS
- 5.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 6 - Options

	-					_		10	_	
h		<i>(</i>)	וט	17	M	U	'N	// N	ь.	
U.	.1	U	ГΙ	IU	יוי	ΙГ.	$\boldsymbol{\vdash}$	ıv		ıJ

- **6.2 OPTION PREMIUMS**
- **6.3 VALUING OPTIONS**
- **6.4 OTHER OPTION CONTRACTS**
- 6.5 VALUING OPTIONS BY NUMERICAL METHODS
- **6.6 IMPORTANT FORMULAS**
- **6.7 ANSWERS TO CHAPTER EXAMPLES**

CHAPTER 7 - Fixed-Income Securities

- 7.1 OVERVIEW OF DEBT MARKETS
- 7.2 FIXED-INCOME SECURITIES
- 7.3 ANALYSIS OF FIXED-INCOME SECURITIES
- 7.4 SPOT AND FORWARD RATES
- 7.5 PREPAYMENT
- 7.6 SECURITIZATION
- 7.7 IMPORTANT FORMULAS
- 7.8 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 8 - Fixed-Income Derivatives

- 8.1 FORWARD CONTRACTS
- 8.2 FUTURES
- 8.3 SWAPS
- 8.4 OPTIONS
- **8.5 IMPORTANT FORMULAS**
- 8.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 9 - Equity, Currency, and Commodity Markets

- 9.1 EQUITIES
- 9.2 CONVERTIBLE BONDS AND WARRANTS
- 9.3 EQUITY DERIVATIVES
- 9.4 CURRENCY MARKETS
- 9.5 CURRENCY SWAPS

9.6 COMMODITIES

- 9.7 IMPORTANT FORMULAS
- 9.8 ANSWERS TO CHAPTER EXAMPLES

PART Three - Market Risk Management

CHAPTER 10 - Introduction to Market Risk

- **10.2 DOWNSIDE RISK MEASURES**
- 10.3 CASH FLOW AT RISK
- **10.4 VAR PARAMETERS**
- 10.5 COMPONENTS OF RISK MEASUREMENT SYSTEMS
- 10.6 STRESS-TESTING
- 10.7 IMPORTANT FORMULAS
- 10.8 ANSWERS TO CHAPTER EXAMPLES
- APPENDIX: DESIRABLE PROPERTIES FOR RISK MEASURES

CHAPTER 11 - Sources of Market Risk

- 11.1 SOURCES OF LOSS: A DECOMPOSITION
- 11.2 CURRENCY RISK
- 11.3 FIXED-INCOME RISK
- 11.4 EQUITY RISK
- 11.5 COMMODITY RISK
- 11.6 RISK SIMPLIFICATION
- 11.7 IMPORTANT FORMULAS
- 11.8 ANSWERS TO CHAPTER EXAMPLES
- APPENDIX: SIMPLIFICATION OF THE COVARIANCE MATRIX

CHAPTER 12 - Hedging Linear Risk

12.1 INTRODUCTION TO FUTURES HEDGING

12.2 OPTIMAL HEDGING

12.3 APPLICATIONS OF OPTIMAL HEDGING 12.4 IMPORTANT FORMULAS 12.5 ANSWERS TO CHAPTER EXAMPLES
CHAPTER 13 - Nonlinear Risk: Options
13.1 EVALUATING OPTIONS 13.2 OPTION "GREEKS" 13.3 DYNAMIC HEDGING 13.4 IMPORTANT FORMULAS 13.5 ANSWERS TO CHAPTER EXAMPLES
CHAPTER 14 - Modeling Risk Factors

CH.

14.1 NORMAL AND LOGNORMAL DISTRIBUTIONS 14.2 FAT TAILS 14.3 TIME-VARIATION IN RISK 14.4 IMPORTANT FORMULAS

14.5 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 15 - VAR Methods

15.1 VAR: LOCAL VERSUS FULL VALUATION 15.2 VAR METHODS: OVERVIEW 15.3 LIMITATIONS OF VAR SYSTEMS 15.4 EXAMPLE 15.5 IMPORTANT FORMULAS

PART Four - Investment Risk Management

15.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 16 - Portfolio Management

16.1 INSTITUTIONAL INVESTORS

16.2 PERFORMANCE MEASUREMENT
16.3 RISK BUDGETING
16.4 IMPORTANT FORMULAS
16.5 ANSWERS TO CHAPTER EXAMPLES
CHAPTER 17 - Hedge Fund Risk Management
<u></u>
17.1 THE HEDGE FUND INDUSTRY
17.2 LEVERAGE, LONG, AND SHORT POSITIONS
17.3 HEDGE FUNDS: MARKET RISKS
17.4 HEDGE FUNDS: SPECIFIC RISKS
17.5 DEALING WITH HEDGE FUND RISKS
17.6 IMPORTANT FORMULAS
17.7 ANSWERS TO CHAPTER EXAMPLES
TYTY THE TO CHAIN TEN EXAMINED
PART Five - Credit Risk Management
CHAPTER 18 - Introduction to Credit Risk
18.1 SETTLEMENT RISK
18.2 OVERVIEW OF CREDIT RISK
18.3 MEASURING CREDIT RISK
18.4 CREDIT RISK DIVERSIFICATION
18.5 IMPORTANT FORMULAS
18.6 ANSWERS TO CHAPTER EXAMPLES
CHAPTER 19 - Measuring Actuarial Default Risk
19.1 CREDIT EVENT
19.2 DEFAULT RATES
19.3 RECOVERY RATES
19.4 ASSESSING CORPORATE AND SOVEREIGN RATING
19.5 IMPORTANT FORMULAS
19.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 20 - Measuring Default Risk from Market Prices

20.1 CORPORATE BOND F	PRICES
-----------------------	--------

- **20.2 EQUITY PRICES**
- **20.3 IMPORTANT FORMULAS**
- 20.4 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 21 - Credit Exposure

21.1 CREDIT EXPOSURE BY INSTRUMENT

- 21.2 DISTRIBUTION OF CREDIT EXPOSURE
- 21.3 EXPOSURE MODIFIERS
- 21.4 CREDIT RISK MODIFIERS
- 21.5 IMPORTANT FORMULAS
- 21.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 22 - Credit Derivatives and Structured Products

- 22.1 INTRODUCTION
- **22.2 CREDIT DEFAULT SWAPS**
- 22.3 OTHER CONTRACTS
- 22.4 STRUCTURED PRODUCTS
- 22.5 CDO MARKET
- 22.6 CONCLUSIONS
- 22.7 IMPORTANT FORMULAS
- 22.8 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 23 - Managing Credit Risk

- 23.1 MEASURING THE DISTRIBUTION OF CREDIT LOSSES
- 23.2 MEASURING EXPECTED CREDIT LOSS
- 23.3 MEASURING CREDIT VAR
- 23.4 PORTFOLIO CREDIT RISK MODELS
- 23.5 CONCLUSIONS

23.6 IMPORTANT FORMULAS 23.7 ANSWERS TO CHAPTER EXAMPLES

<u>PART Six - Legal, Operational, and Integrated Risk</u> <u>Management</u>

CHAPTER 24 - Operational Risk

24.1 THE I	MPORTANCE (OF C	DPERATIONAL	_ RISK
------------	-------------	------	-------------	--------

24.2 IDENTIFYING OPERATIONAL RISK

24.3 ASSESSING OPERATIONAL RISK

24.4 MANAGING OPERATIONAL RISK

24.5 ANSWERS TO CHAPTER EXAMPLES

APPENDIX: CAUSAL NETWORKS

CHAPTER 25 - Liquidity Risk

25.1 SOURCES OF LIQUIDITY RISK

25.2 ASSET LIQUIDITY RISK

25.3 FUNDING LIQUIDITY RISK

25.4 MANAGING LIQUIDITY RISK

25.5 IMPORTANT FORMULAS

25.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 26 - Firm-Wide Risk Management

26.1 INTEGRATED RISK MANAGEMENT

26.2 BEST PRACTICES REPORTS

26.3 ORGANIZATIONAL STRUCTURE

26.4 CONTROLLING TRADERS

26.5 RISK-ADJUSTED PERFORMANCE AND RAROC

26.6 IMPORTANT FORMULAS

26.7 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 27 - Legal Issues

27.1 LEGAL RISKS WITH DERIVATIVES
27.2 NETTING
27.3 ISDA MASTER NETTING AGREEMENT
27.4 GLOSSARY
27.5 ANSWERS TO CHAPTER EXAMPLES

PART Seven - Regulation and Compliance

CHAPTER 28 - Regulation of Financial Institutions

28.1 DEFINITION OF FINANCIAL INSTITUTIONS
28.2 SYSTEMIC RISK
28.3 REGULATION OF COMMERCIAL BANKS
28.4 REGULATION OF SECURITIES HOUSES
28.5 TOOLS AND OBJECTIVES OF REGULATION
28.6 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 29 - The Basel Accord

29.1 STEPS IN THE BASEL ACCORD
29.2 THE 1988 BASEL ACCORD
29.3 ILLUSTRATION
29.4 THE NEW BASEL ACCORD
29.5 CONCLUSIONS
29.6 IMPORTANT FORMULAS
29.7 ANSWERS TO CHAPTER EXAMPLES

CHAPTER 30 - The Basel Market Risk Charge

```
30.1 THE STANDARDIZED METHOD
30.2 THE INTERNAL MODELS APPROACH
30.3 STRESS TESTING
```

30.4 BACKTESTING 30.5 IMPORTANT FORMULAS 30.6 ANSWERS TO CHAPTER EXAMPLES

About the CD-ROM Index

Founded in 1807, John Wiley & Sons is the oldest independent publishing company in the United States. With offices in North America, Europe, Australia, and Asia, Wiley is globally committed to developing and marketing print and electronic products and services for our customers' professional and personal knowledge and understanding.

The Wiley Finance series contains books written specifically for finance and investment professionals as well as sophisticated individual investors and their financial advisors. Book topics range from portfolio management to e-commerce, risk management, financial engineering, valuation, and financial instrument analysis, as well as much more.

For a list of available titles, visit our Web site at www.WileyFinance.com.

Financial Risk Manager Handbook

Fifth Edition

PHILIPPE JORION GARP



Copyright © 2009 by Philippe Jorion, except for FRM sample questions, which are copyright 1997-2009 by GARP. The FRM designation is a GARP trademark. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

Designations used by companies to distinguish their products are often claimed as trademarks. In all instances where John Wiley & Sons, Inc. is aware of a claim, the product names appear in initial capital or all capital letters. Readers, however, should contact the appropriate companies for more complete information regarding trademarks and registration.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 646-8600, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at https://www.wiley.com/go/permissions.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Jorion, Philippe, 1955-Financial risk manager handbook / Philippe Jorion - 5th ed. p. cm. - (Wiley finance series) Includes index.

eISBN: 978-0-470-52199-1

1. Financial risk management. 2. Risk management. 3. Corporate-Finance. I.

Title. HD61.J67 2009 332.64'5-dc22 2009008330

Preface

The Financial Risk Manager Handbook provides the core body of knowledge for financial risk managers. Risk management has evolved rapidly over the past decade and has become an indispensable function in many institutions.

This *Handbook* was originally written to provide support for candidates taking the FRM examination administered by GARP. As such, it reviews a wide variety of practical topics in a consistent and systematic fashion. It covers quantitative methods and capital markets, as well as market, credit, operational, and integrated risk management. It also discusses regulatory and legal issues essential to risk professionals.

This edition has been thoroughly updated to reflect recent developments in financial markets. The unprecedented losses incurred by many institutions have raised questions about risk management practices. These issues are now addressed in various parts of the book, which also include lessons from recent regulatory reports. The securitization process and structured credit products are critically examined. A new chapter on liquidity risk has been added, given the importance of this risk during the recent crisis. Finally, this *Handbook* incorporates the latest questions from the FRM examinations.

Modern risk management systems cut across the entire organization. This breadth is reflected in the subjects covered in this *Handbook*. The book was designed to be self-contained, but only for readers who already have some exposure to financial markets. To reap maximum benefit from this book, readers should have taken the equivalent of an MBA-level class on investments.

Finally, I want to acknowledge the help received in writing this *Handbook*. In particular, I thank the numerous readers who shared comments on previous editions. Any comment or suggestion for improvement will be welcome. This feedback will help us to maintain the high quality of the FRM designation.

Philippe Jorion *February 2009*

About the Author

Philippe Jorion is a Professor of Finance at the Paul Merage School of Business at the University of California at Irvine. He has also taught at Columbia University, Northwestern University, the University of Chicago, and the University of British Columbia. He holds an M.B.A. and a Ph.D. from the University of Chicago and a degree in engineering from the University of Brussels. He is also a managing director at Pacific Alternative Asset Management Company (PAAMCO), a global fund of hedge funds.

Dr. Jorion is the author of more than 90 publications directed to academics and practitioners on the topics of risk management and international finance. He has also written a number of books, including *Big Bets Gone Bad: Derivatives and Bankruptcy in Orange County*, the first account of the largest municipal failure in U.S. history, and *Value at Risk: The New Benchmark for Managing Financial Risk*, which is aimed at finance practitioners and has become an industry standard.

Philippe Jorion is a frequent speaker at academic and professional conferences. He is on the editorial board of a number of finance journals and was editor in chief of the *Journal of Risk*.

About GARP

Founded in 1996, the Global Association of Risk Professionals (GARP) is the leading not-for-profit association for world-class financial risk certification, education, and training with close to 100,000 members representing 167 countries. With deep expertise and a strong reputation, GARP sets global standards and creates risk management programs valued worldwide. All GARP programs are developed with input from experts around the world to ensure that concepts and content reflect globally accepted practices.

GARP is dedicated to advancing the risk profession. For more information about GARP, please visit www.garp.com.

FINANCIAL RISK MANAGER (FRM®) CERTIFICATION

The benchmark FRM designation is the globally accepted risk management certification for financial risk professionals. The FRM objectively measures competency in the risk management profession based on globally accepted standards. With a compound annual growth rate of 25 percent over the past seven years, the FRM program has experienced significant growth in every financial center around the world. Now 16,000+ individuals hold the FRM designation in over 90 countries. In addition, organizations with five or more FRM registrants grew from 105 in 2003 to 424 in 2008, further demonstrating the FRM program's global acceptance.

The FRM Continuing Professional Education (CPE) program, to be offered starting in 2009 exclusively for certified FRM holders, provides the perspective and framework needed to further develop competencies in the ever-evolving field of risk management.

For more information about the FRM program, please visit www.garp.com/frmexam.

OTHER GARP CERTIFICATIONS

International Certificate in Banking Risk and Regulation (ICBRR)

The ICBRR allows individuals to expand their knowledge and understanding of the various risks, regulations, and supervisory requirements banks must face in today's economy, with emphasis on the Basel II Accord. This certificate is ideal for employees who are not professional risk managers but who have a strong need to understand risk concepts. The ICBRR program is designed for employees in nonrisk departments such as internal audit, accounting, information technology (IT), legal, compliance, and sales, acknowledging that everyone in the organization is a risk manager!

Certificate in Energy Risk Management

The Certificate in Energy Risk Management provides individuals with a comprehensive and cross-product understanding of the physical and financial marketplaces

relating to crude oil, natural gas, liquefied natural gas, and electricity/power. This program is valuable for anyone working in or servicing the energy field and requiring an understanding of the physical and financial markets, how they interrelate, and the risks involved. This program will launch in 2Q 2009.

Certificate in Risk Management for Islamic Financial Institutions

This certificate is under development by a practice oversight committee of Islamic finance experts from around the globe. The program will cover the risk management methodologies specific to Sharia'a-compliant financial products and will be the only one of its kind anywhere in the world.

GARP DIGITAL LIBRARY

As the world's largest digital library dedicated to financial risk management, the GARP Digital Library (GDL) is the hub for risk management education and research material. The library's unique iReadings™ allow users to download individual chapters of books, saving both time and money. There are over 1,000 readings available from 12 different publishers. The GDL collection offers readings to meet the needs of anyone interested in risk management.

For more information, please visit www.garpdigitallibrary.org.

GARP EVENTS AND NETWORKING

GARP hosts major conventions throughout the world, where risk professionals come together to share knowledge, network, and learn from leading experts in the field. Conventions are bookended with interactive workshops that provide practical insights and case studies presented by the industry's leading practitioners.

GARP regional chapters provide an opportunity for financial risk professionals to network and share new trends and discoveries in risk management. Each one of our 52 chapters holds several meetings each year, in some locations more often, focusing on issues of importance to the risk management community, either globally or locally.

Introduction

GARP's formal mission is to be the leading professional association for financial risk managers, managed by and for its members and dedicated to the advancement of the risk profession through education, training, and the promotion of best practices globally. As a part of delivering on that mission, GARP has again teamed with Philippe Jorion to produce the fifth edition of the *Financial Risk Manager Handbook*.

The Handbook follows GARP's FRM Committee's published FRM Study Guide, which sets forth primary topics and subtopics covered in the FRM exam. The topics are selected by the FRM Committee as being representative of the theories and concepts utilized by risk management professionals as they address current issues.

Over the years the Study Guide has taken on an importance far exceeding its initial intent of providing guidance for FRM candidates. The Study Guide is now being used by universities, educators, and executives around the world to develop graduate-level business and finance courses, as a reference list for purchasing new readings for personal and professional libraries, as an objective outline to assess the risk management qualifications of an employee or a job applicant, and as guidance on the important trends currently affecting the financial risk management profession.

Given the expanded and dramatically growing recognition of the financial risk management profession globally, the *Handbook* has similarly assumed a natural and advanced role beyond its original purpose. It has now become the primary reference manual for risk professionals,

academicians. and executives around the Professional risk managers must be well versed in a wide variety of risk-related concepts and theories, and must also keep themselves up-to-date with a rapidly changing marketplace. The *Handbook* is designed to allow them to do iust that. It provides a financial risk management practitioner with the latest thinking and approaches to financial risk-related issues. It also provides coverage of advanced topics with questions and tutorials to enhance the reader's learning experience.

This fifth edition of the *Handbook* includes revised coverage of the primary topic areas covered by the FRM examination. Importantly, this edition also includes the latest lessons from the recent credit crisis, as well as new and more recent sample FRM questions.

The *Handbook* continues to keep pace with the dynamic financial risk profession while simultaneously offering serious risk professionals an excellent and cost-effective tool to keep abreast of the latest issues affecting the global risk management community.

Developing credibility and global acceptance for a professional certification program is a lengthy and complicated process. When GARP first administered its FRM exam in 1997, the concept of a professional risk manager and a global certification relating to that person's skill set was more theory than reality. That has now completely changed, as the number of current FRM holders exceeds 16,000.

The FRM is now the benchmark for a financial risk manager anywhere in the world. Professional risk managers having earned the FRM credential are globally recognized as having achieved a level of professional competency and a demonstrated ability to dynamically measure and manage financial risk in a real-world setting in accordance with global standards.

GARP is proud to continue to make this *Handbook* available to financial risk professionals around the world. Philippe Jorion, a preeminent risk management professional, has again compiled an exceptional reference book. Supplemented by an interactive test question CD, this *Handbook* is a requirement for any risk professional's library.

Global Association of Risk Professionals *February 2009*

PART One

Quantitative Analysis

CHAPTER 1

Bond Fundamentals

Risk management starts with the pricing of assets. The simplest assets to study are regular, fixed-coupon bonds. Because their cash flows are predetermined, we can translate their stream of cash flows into a present value by discounting at a fixed interest rate. Thus the valuation of bonds involves understanding compounded interest, discounting, as well as the relationship between present values and interest rates.

Risk management goes one step further than pricing, however. It examines potential changes in the price of assets as the interest rate changes. In this chapter, we assume that there is a single interest rate, or yield, that is used to price the bond. This will be our fundamental risk factor. This chapter describes the relationship between bond prices and yields and presents indispensable tools for the management of fixed-income portfolios.

This chapter starts our coverage of quantitative analysis by discussing bond fundamentals. Section 1.1 reviews the concepts of discounting, present values, and future values. Section 1.2 then plunges into the price-yield relationship. It shows how the Taylor expansion rule can be used to relate movements in bond prices to those in yields. This Taylor expansion rule, however, covers much more than bonds. It is a building block of risk measurement methods based on local valuation, as we shall see later. Section 1.3 then presents an economic interpretation of duration and convexity.

The reader should be forewarned that this chapter, like many others in this handbook, is rather compact. This chapter provides a quick review of bond fundamentals with particular attention to risk measurement applications. By the end of this chapter, however, the reader should be able to answer advanced FRM questions on bond mathematics.

1.1 DISCOUNTING, PRESENT, AND FUTURE VALUE

An investor considers a zero-coupon bond that pays \$100 in 10 years. Assume that the investment is guaranteed by the U.S. government, and that there is no credit risk. So, this is a default-free bond, which is exposed to market risk only. Because the payment occurs at a future date, the current value of the investment is surely less than an up-front payment of \$100.

To value the payment, we need a **discounting factor**. This is also the **interest rate**, or more simply the **yield**. Define C_t as the cash flow at time t and the discounting factor as y. We define T as the number of periods until maturity, e.g., number of years, also known as **tenor**. The **present value** (PV) of the bond can be computed as

(1.1)

$$PV = \frac{C_T}{(1+y)^T}$$

For instance, a payment of $C_T = \$100$ in 10 years discounted at 6 percent is only worth \$55.84 now. So, all else fixed, the market value of zero-coupon bonds decreases

with longer maturities. Also, keeping T fixed, the value of the bond decreases as the yield increases.

Conversely, we can compute the **future value** (FV) of the bond as

(1.2)

$$FV = PV \times (1 + \gamma)^T$$

For instance, an investment now worth PV = \$100 growing at 6 percent will have a future value of FV = \$179.08 in 10 years.

Here, the yield has a useful interpretation, which is that of an **internal rate of return** on the bond, or annual growth rate. It is easier to deal with rates of returns than with dollar values. Rates of return, when expressed in percentage terms and on an annual basis, are directly comparable across assets. An annualized yield is sometimes defined as the **effective annual rate (EAR)**.

It is important to note that the interest rate should be stated along with the method used for compounding. Annual compounding is very common. Other conventions exist, however. For instance, the U.S. Treasury market uses semiannual compounding. Define in this case y^S as the rate based on semiannual compounding. To maintain comparability, it is expressed in annualized form, i.e., after multiplication by 2. The number of periods, or semesters, is now 2T. The formula for finding y^S is

(1.3)

$$PV = \frac{C_T}{(1 + v^S/2)^{2T}}$$