Introduction to Electrical Power Systems

Mohamed E. El-Hawary

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PREFACE

This book is written primarily as an introduction to the basics of electrical power systems. It is intended as a general introduction to the area for students in all engineering disciplines, as well as being useful as a reference and self-study guide for those professionals who wish to have a succinct introduction to this important area. The coverage of the book is designed to allow its use in a number of ways including service courses taught to non-electrical majors. The organization and details of the material in this book enables maximum flexibility for the instructor to select topics to include in courses within the modern engineering curriculum.

The book does not assume a level of mathematical sophistication beyond that acquired through completion of undergraduate courses in basic physics and introductory electric circuits. Emphasis is placed on providing an improved appreciation of the operational characteristics of the electrical apparatus discussed, on the basis of linear mathematical models. Almost every key concept is illustrated through the use of in-text examples that are worked out in detail to enforce the reader's understanding. The text coverage includes some usage of MATLAB™ to solve fundamental problems of evaluating basic performance characteristics to assist in understanding power system devices.

The first chapter in this book offers a historical perspective on the t are development of electric power systems. While this topic is not an integral part of conventional coverage in texts and courses in this area, this chapter should provide interesting insights into the influence of these developments on present day civilization. It is through an appreciation of the past developments and achievements that we can understand our present and forge ahead with future advances.

Much of the material in Chapters 2 to 7 is derived from the more detailed expose' found in my earlier book on "Electrical Power System Design and Analysis. The present treatment deals with fundamental topics to be covered in introductory courses in electric power systems. Emphasis is given to practical aspects such as the main performance characteristics of the apparatus discussed and system applications.

In Chapter 2, a review of fundamentals of electric circuits that are necessary for the studies to follow. Chapter 3 discusses the basics of synchronous machines for steady state operations. This is followed by treatments of the transformer in Chapter 4 and power transmission lines in Chapter 5. Unlike the coverage of the more detailed "Electric Power Systems: Design and Analysis," Chapter 6 of the present book deals with Induction Motors both polyphase and single phase machines. In addition, Chapter 7 combines both fault analysis and protection.

The importance of computer control in power system operations is highlighted in Chapter 8 where we discuss the structure and functions involved in a modern energy control center. The chapter includes a brief introduction to functions performed in the electric energy control centre. Naturally some functions that are discussed in detail in "Electric Power Systems: Design and Analysis" such as Power Flow, Stability, optimal operation of power systems, are discussed briefly in this chapter.

Chapter 9 is new to this book, and offers a brief discussion of the Present and Future of Electric Energy Systems. Of specific interest is a discussion of the challenges facing the system designer and operator in view of blackout events that took place during the first decade of the 21st Century.

I have attempted to make this book as self-containing as possible. As a result, the reader will find that many background topics such as the per unit system and three-phase circuits are included in the text's main body as opposed to the recent trend toward including many appendices dealing with these topics. In studying and teaching electrical power systems it has been my experience that a problem solving approach is most effective in exploring this rich area.

A textbook such as this could not have been written without the continuing input of the many students who have gone through many versions of its material as it was developed. I wish to thank the members of the many classes to whom I was privileged to teach this fascinating subject. I wish to acknowledge the able work of Elizabeth Sanford of Dalhousie University in putting this manuscript in a better form than I was able to produce.

As has always been the case, the patience and understanding of my wife, Dr. Ferial El-Hawary, made this project another joy to look forward to completing. It goes without saying that the younger generation of our family deserves a greater share of my appreciation for their continuous understanding.

MOHAMED E. EL-HAWARY

Halifax, Nova Scotia
May 1, 2008

Chapter 1

INTRODUCTION

This chapter has three objectives. We first offer a brief perspective on the development of electric power systems. This is not intended to be a detailed historical review, but rather it uses historical landmarks as a background to highlight the features and structure of the modern power systems, which are discussed in Section 1.2. The chapter concludes with an outline of the book.

1.1 A BRIEF HISTORY OF ELECTRIC POWER SYSTEMS

The electric power industry shapes and contributes to the welfare, progress, and technological advances of humanity. The growth of electric energy consumption in the world has been phenomenal. In the United States, for example, electric energy sales have grown to well over 400 times in the period between the turn of the century and the early 1970s. This growth rate was 50 times as much as the growth rate in all other energy forms used during the same period. The installed kW capacity per capita in the U.S. is estimated to be close to 3 kW.

Edison Electric Illuminating Company of New York inaugurated the Pearl Street Station in 1881. The station had a capacity of four 250-hp boilers supplying steam to six engine-dynamo sets. Edison's system used a 110-V dc underground distribution network with copper conductors insulated with a jute wrapping. In 1882, the first water wheel-driven generator was installed in Appleton, Wisconsin. The *low voltage of the circuits* limited the service area of a central station, and consequently, central stations proliferated throughout metropolitan areas.

The invention of the transformer, then known as the "inductorium," made ac systems possible. The first practical ac distribution system in the U.S. was installed by W. Stanley at Great Barrington, Massachusetts, in 1866 for Westinghouse, who acquired the American rights to the transformer from its British inventors Gaulard and Gibbs. Early ac distribution utilized 1000-V overhead lines. The Nikola Tesla invention of the induction motor in 1888 helped replace dc motors and hastened the advance in use of ac systems.

The first American single-phase ac system was installed in Oregon in 1889. Southern California Edison Company established the first three phase 2.3 kV system in 1893.

By 1895, Philadelphia had about twenty electric companies with distribution systems operating at 100-V and 500-V two-wire dc and 220-V three-wire dc, single-phase, two-phase, and three-phase ac, with frequencies of 60, 66, 125, and 133 cycles per second, and feeders at 1000-1200 V and 2000-2400 V.

Introduction to Electrical Power Engineering. By Mohamed E. El-Hawary

The subsequent consolidation of electric companies enables the realization of economies of scale in generating facilities, the introduction of equipment standardization, and the utilization of the load diversity between areas. Generating unit sizes of up to 1300 MW are in service, an era that was started by the 1973 Cumberland Station of the Tennessee Valley Authority.

Underground distribution at voltages up to 5 kV was made possible by the development of rubber-base insulated cables and paper-insulated, lead-covered cables in the early 1900s. Since then, higher distribution voltages have been necessitated by load growth that would otherwise overload low-voltage circuits and by the requirement to transmit large blocks of power over great distances. Common distribution voltages presently are in 5-, 15-, 25-, 35-, and 69-kV voltage classes.

The growth in size of power plants and in the higher voltage equipment was accompanied by interconnections of the generating facilities. These interconnections decreased the probability of service interruptions, made the utilization of the most economical units possible, and decreased the total reserve capacity required to meet equipment-forced outages. This was accompanied by use of sophisticated analysis tools such as the network analyzer. Central control of the interconnected systems was introduced for reasons of economy and safety. The advent of the load dispatcher heralded the dawn of power systems engineering, an exciting area that strives to provide the best system to meet the load requirements reliably, safely, and economically, utilizing state-of-the-art computer facilities.

Extra higher voltage (EHV) has become dominant in electric power transmission over great distances. By 1896, an 11-kv three-phase line was transmitting 10 MW from Niagara Falls to Buffalo over a distance of 20 miles. Today, transmission voltages of 230 kV, 287 kV, 345 kV, 500 kV, 735 kV, and 765 kV are commonplace, with the first 1100-kV line already energized in the early 1990s. The trend is motivated by economy of scale due to the higher transmission capacities possible, more efficient use of right-of-way, lower transmission losses, and reduced environmental impact.

In 1954, the Swedish State Power Board energized the 60-mile, 100-kV dc submarine cable utilizing U. Lamm's Mercury Arc valves at the sending and receiving ends of the world's first high-voltage direct current (HVDC) link connecting the Baltic island of Gotland and the Swedish mainland. Currently, numerous installations with voltages up to 800-kV dc are in operation around the world.

In North America, the majority of electricity generation is produced by investor-owned utilities with a certain portion done by federally and provincially (in Canada) owned entities. In the United States, the Federal Energy Regulatory Commission (FERC) regulates the wholesale pricing of electricity and terms and conditions of service.

The North American transmission system is interconnected into a large power grid known as the North American Power Systems Interconnection. The grid is divided into several pools. The pools consist of several neighboring utilities which operate jointly to schedule generation in a cost-effective manner. A privately regulated organization called the North American Electric Reliability Council (NERC) is responsible for maintaining system standards and reliability. NERC works cooperatively with every provider and distributor of power to ensure reliability. NERC coordinates its efforts with FERC as well as other organizations such as the Edison Electric Institute (EEI). NERC currently has four distinct electrically separated areas. These areas are the Electric Reliability Council of Texas (ERCOT), the Western States Coordination Council (WSCC), the Eastern Interconnect, which includes all the states and provinces of Canada east of the Rocky Mountains (excluding Texas), and Hydro-Quebec. These electrically separate areas exchange with each other but are not synchronized electrically.

The electric power industry in the United States has undergone fundamental changes since the deregulation of the telecommunication, gas, and other industries. The generation business is rapidly becoming market-driven. The power industry was characterized by larger, vertically integrated entities. The advent of open transmission access has resulted in wholesale and retail markets. Utilities may be divided into power generation, transmission, and retail segments. Generating companies (GENCO) sell directly to an independent system operator (ISO). The ISO is responsible for the operation of the grid and matching demand and generation dealing with transmission companies as well (TRANSCO). This scenario is not the only possibility, as the power industry continues to evolve to create a more competitive environment for electricity markets to promote greater efficiency. The industry now faces new challenges and problems associated with the interaction of power system entities in their efforts to make crucial technical decisions while striving to achieve the highest level of human welfare.

1.2 THE STRUCTURE OF THE POWER SYSTEM

An interconnected power system is a complex enterprise that may be subdivided into the following major subsystems:

- Generation Subsystem
- Transmission and Subtransmission Subsystem
- Distribution Subsystem
- Utilization Subsystem

Generation Subsystem

This includes generators and transformers.

Generators – An essential component of power systems is the three-phase ac generator known as synchronous generator or alternator. Synchronous generators have two synchronously rotating fields: One field is produced by the rotor driven at synchronous speed and excited by dc current. The other field is produced in the stator windings by the three-phase armature currents. The dc current for the rotor windings is provided by excitation systems. In the older units, the exciters are dc generators mounted on the same shaft, providing excitation through slip rings. Current systems use ac generators with rotating rectifiers, known as *brushless* excitation systems. The excitation system maintains generator voltage and controls the reactive power flow. Because they lack the commutator, ac generators can generate high power at high voltage, typically 30 kV.

The source of the mechanical power, commonly known as the prime mover, may be hydraulic turbines, steam turbines whose energy comes from the burning of coal, gas and nuclear fuel, gas turbines, or occasionally internal combustion engines burning oil.

Steam turbines operate at relatively high speeds of 3600 or 1800 rpm. The generators to which they are coupled are cylindrical rotor, two-pole for 3600 rpm, or four-pole for 1800 rpm operation. Hydraulic turbines, particularly those operating with a low pressure, operate at low speed. Their generators are usually a salient type rotor with many poles. In a power station, several generators are operated in parallel in the power grid to provide the total power needed. They are connected at a common point called a *bus*.

With concerns for the environment and conservation of fossil fuels, many alternate sources are considered for employing the untapped energy sources of the sun and the earth for generation of power. Some alternate sources used are solar power, geothermal power, wind power, tidal power, and biomass.

Transformers – The transformer transfers power with very high efficiency from one level of voltage to another. The power transferred to the secondary is almost the same as the primary, except for losses in the transformer. Using a step-up transformer will reduce losses in the line, which makes the transmission of power over long distances possible.

Insulation requirements and other practical design problems limit the generated voltage to low values, usually 30 kV. Thus, step-up transformers are used for transmission of power. At the receiving end of the transmission lines step-down transformers are used to reduce the voltage to suitable values for distribution or utilization. The electricity in an electric power system may undergo four or five transformations between generator and consumers.

Transmission and Subtransmission Subsystem

An overhead transmission network transfers electric power from generating units to the distribution system which ultimately supplies the load. Transmission lines also interconnect neighboring utilities which allow the economic dispatch of power within regions during normal conditions, and the transfer of power between regions during emergencies.

Standard transmission voltages are established in the United States by the American National Standards Institute (ANSI). Transmission voltage lines operating at more than 60 kV are standardized at 69 kV, 115 kV, 138 kV, 161 kV, 230 kV, 345 kV, 500 kV, and 765 kV line-to-line. Transmission voltages above 230 kV are usually referred to as extra-high voltage (EHV).

High voltage transmission lines are terminated in substations, which are called *high-voltage substations*, *receiving substations*, or *primary substations*. The function of some substations is switching circuits in and out of service; they are referred to as switching stations. At the primary substations, the voltage is stepped down to a value more suitable for the next part of the flow toward the load. Very large industrial customers may be served directly from the transmission system.

The portion of the transmission system that connects the high-voltage substations through step-down transformers to the distribution substations is called the subtransmission network. There is no clear distinction between transmission and subtransmission voltage levels. Typically, the subtransmission voltage level ranges from 69 to 138 kV. Some large industrial customers may be served from the subtransmission system. Capacitor banks and reactor banks are usually installed in the substations for maintaining the transmission line voltage.

Distribution Subsystem

The distribution system connects the distribution substations to the consumers' service-entrance equipment. The primary distribution lines range from 4 to 34.5 kV and supply the load in a well-defined geographical area. Some small industrial customers are served directly by the primary feeders.

The secondary distribution network reduces the voltage for utilization by commercial and residential consumers. Lines and cables not exceeding a few hundred feet in length then deliver power to the individual consumers. The secondary distribution serves most of the customers at levels of 240/120 V, single-phase, three-wire; 208Y/120 V, three-phase, four-wire; or 480Y/277 V, three-phase, four-wire. The power for a typical home is derived from a transformer that reduces the primary feeder voltage to 240/120 V using a three-wire line.

Distribution systems utilize both *overhead* and *underground* conductors. The growth of underground distribution has been extremely rapid and as much as 70 percent of new residential construction in North America is via underground systems.

Load Subsystems

Power systems loads are divided into industrial, commercial, and residential. Industrial loads are composite loads, and induction motors form a high proportion of these loads. These composite loads are functions of voltage and frequency and form a major part of the system load. Commercial and residential loads consist largely of lighting, heating, and cooking. These loads are independent of frequency and consume negligibly small reactive power.

The load varies throughout the day, and power must be available to consumers on demand. The daily-load curve of a utility is a composite of demands made by various classes of users. The greatest value of load during a 24-hr period is called the peak or *maximum demand*. To assess the usefulness of the generating plant the *load factor* is defined. The load factor is the ratio of average load over a designated period of time to the peak load occurring in that period. Load factors may be given for a day, a month, or a year. The yearly, or annual load factor is the most useful since a year represents a full cycle of time. The daily load factor is

$$\text{Daily L.F.} = \frac{\text{average load}}{\text{peak load}} \tag{1.1}$$

Multiplying the numerator and denominator of (1.1) by a time period of 24 hr, we obtain

$$\text{Daily L.F.} = \frac{\text{average load} \times 24\text{ hr}}{\text{peak load} \times 24\text{ hr}} = \frac{\text{energy consumed during 24 hr}}{\text{peak load} \times 24\text{ hr}} \tag{1.2}$$

The annual load factor is

$$\text{Annual L.F.} = \frac{\text{total annual energy}}{\text{peak load} \times 8760\text{ hr}} \tag{1.3}$$

Generally there is diversity in the peak load between different classes of loads, which improves the overall system load factor. In order for a power plant to operate economically, it must have a high system load factor. Typical system load factors are in the range of 55 to 70 percent. Load-forecasting at all levels is an important function in the operation, operational planning, and planning of an electric power system. Other devices and systems are required for the satisfactory operation and protection of a power system. Some of the protective devices directly connected to the circuits are called *switchgear*. They include instrument transformers, circuit breakers, disconnect switches, fuses and lightning arresters. These devices are necessary to deenergize either for normal operation or on the occurrence of faults. The associated control equipment and protective relays are placed on *switchboards* in *control houses*.

For reliable and economical operation of the power system it is necessary to monitor the entire system in a so-called control center. The modern control center is called the *energy control center* (ECC). Energy control centers are equipped with on-line computers performing all signal processing through the remote acquisition system. Computers work in a hierarchical structure to properly coordinate different functional requirements in normal as well as emergency conditions. Every energy control center contains control consoles which consist of a visual display unit (VDU), keyboard, and light pen. Computers may give alarms as advance warnings to the operators (dispatchers) when deviation from the normal state occurs. The dispatcher makes decisions and executes them with the aid of a computer. Simulation tools and software packages are implemented for efficient operation and reliable control of the system. In addition, SCADA, an acronym for "supervisory control and data acquisition," systems are auxiliaries to the energy control center.

1.3 OUTLINE OF THE TEXT

Chapter 2 lays the foundations for the development in the rest of the book. The intention of the discussion offered here is to provide a brief review of fundamentals including power concepts, three-phase systems, principles of electromagnetism, and electromechanical energy conversion. Chapter 3 treats the synchronous machine from an operational modeling point of view. Emphasis here is on performance characteristics of importance to the electric power specialist. Chapter 4 provides a comprehensive treatment of transformers. This is followed, in Chapter 5, by a brief coverage of induction motors including the fractional horsepower category.

Chapter 6 is concerned with transmission lines starting from parameter evaluation for different circuit and conductor configurations. Various transmission line performance modeling approaches are covered.

Faults on electric energy systems are considered in Chapter 7. Here we start with the transient phenomenon of a symmetrical short circuit, followed by a treatment of unbalanced and balanced faults. Realizing the crucial part that system protection plays in maintaining service integrity is the basis for the remainder of this chapter. Here an introduction to this important area is given.

Chapter 8 is concerned with the Energy Control Center, its structure, and role in the operation of a modern power system. We outline the objectives and aims of many of the decision support functions adopted in these significant "smarts" of the power system. In chapter 9, we focus our attention on a number of technological challenges that face power system planners and operators. The major North East blackout of 8/14 in 2003 is discussed along with possible mitigation measures. Wherever relevant, we introduce MATLAB™ scripts that allow the student to automate many of the computational details. This feature is deemed important for this textbook's coverage.

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Chapter 2

BASICS OF ELECTRIC ENERGY SYSTEM THEORY

2.1 INTRODUCTION

This chapter lays the groundwork for the study of electric energy systems. We develop some basic tools involving fundamental concepts, definitions, and procedures. The chapter can be considered as simply a review of topics utilized throughout this work. We start by introducing the principal electrical quantities.

2.2 CONCEPTS OF POWER IN ALTERNATING CURRENT SYSTEMS

Generally, the electric power systems specialist is more concerned with electric power in the circuit rather than the currents. The value of instantaneous power flowing into an element is the product of voltage across and current through it. It seems, then, reasonable to exchange the current for power without losing any information. In treating sinusoidal steady-state behavior of circuits, some further definitions are necessary. To illustrate the concepts, we use a cosine representation of the waveforms.

Consider the impedance element $Z = Z\angle\phi$. For a sinusoidal voltage, $v(t)$ given by

$$v(t) = V_m \cos \omega t$$

The instantaneous current in the circuit is

$$i(t) = I_m \cos(\omega t - \phi)$$

Here

$$I_m = V_m / |Z|$$

The instantaneous power into the element is given by

$$p(t) = v(t)i(t) = V_m I_m[\cos(\omega t)\cos(\omega t - \phi)]$$

This reduces to

$$p(t) = \frac{V_m I_m}{2}[\cos\phi + \cos(2\omega t - \phi)]$$

Since the average of cos $(2\omega t - \phi)$ is zero, through 1 cycle, this term therefore contributes nothing to the average of p, and the average power p_{av} is given by

$$p_{av} = \frac{V_m I_m}{2} \cos\phi \tag{2.1}$$

Using the effective (rms) values of voltage and current and substituting $V_m = \sqrt{2}(V_{rms})$, and $I_m = \sqrt{2}(I_{rms})$, we get

$$p_{av} = V_{rms} I_{rms} \cos\phi \tag{2.2}$$

The power entering any network is the product of the effective values of terminal voltage and current and the cosine of the phase angle ϕ, which is, called the *power factor* (PF). This applies to sinusoidal voltages and currents only. When reactance and resistance are present, a component of the current in the circuit is engaged in conveying the energy that is periodically stored in and discharged from the reactance. This stored energy, being shuttled to and from the magnetic field of an inductance or the electric field of a capacitance, adds to the current in the circuit but does not add to the average power.

The value of the average power consumed in a circuit is called *active power*, and the power that supplies the stored energy in reactive elements is called *reactive power*. Active power is P, and the reactive power, designated Q, are thus*

$$P = VI \cos\phi \tag{2.3}$$

$$Q = VI \sin\phi \tag{2.4}$$

In both equations, V and I are rms values of terminal voltage and current, and ϕ is the phase angle by which the current lags the voltage.

To emphasize that the Q represents the nonactive power, it is measured in reactive voltampere units (var).

---

* If we write the instantaneous power as

$$p(t) = V_{rms} I_{rms} [\cos\phi(1 + \cos 2\omega t)] + V_{rms} I_{rms} \sin\phi \sin 2\omega t$$

then it is seen that

$$p(t) = P(1 + \cos 2\omega t) + Q \sin 2\omega t$$

Thus P and Q are the average power and the amplitude of the pulsating power, respectively.

Figure 2.1 shows the time variation of the various variables discussed.

Assume that V, $V\cos\phi$, and $V\sin\phi$, all shown in Figure 2.2, are each multiplied by I, the rms values of current. When the components of voltage $V\cos\phi$ and $V\sin\phi$ are multiplied by current, they become P and Q, respectively. Similarly, if I, $I\cos\phi$, and $I\sin\phi$ are each multiplied by V, they become VI, P, and Q, respectively. This defines a power triangle.

We now define a quantity called the *complex* or *apparent power*, designated S, of which P and Q are components. By definition,

$$\begin{aligned} S &= P + jQ \\ &= VI(\cos\phi + j\sin\phi) \end{aligned}$$

Using Euler's identity, we thus have

$$S = VIe^{j\phi}$$

or

$$S = VI\angle\phi$$

It is clear that an equivalent definition of complex or apparent power is

$$S = VI^* \tag{2.5}$$

We can write the complex power in two alternative forms by using the relationships

$$V = ZI \qquad \text{and} \qquad I = YV$$

This leads to

$$S = ZII^* = Z|I|^2 \tag{2.6}$$

or

$$S = VY^*V^* = Y^*|V|^2 \tag{2.7}$$

Consider the series circuit shown in Figure 2.3. The applied voltage to the overall arrangement is equal to the sum of the voltage drops:

$$V = I(Z_1 + Z_2 + \ldots + Z_n)$$

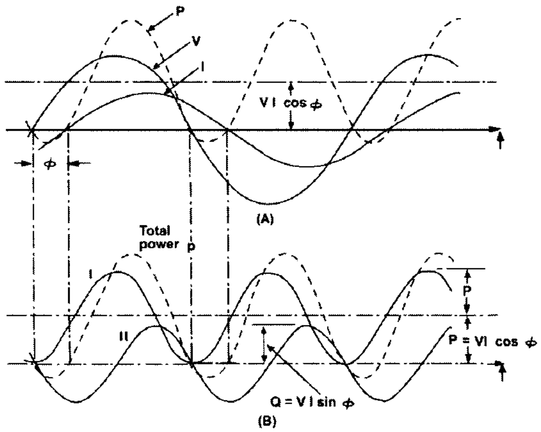

Figure 2.1 Voltage, Current, and Power in a Single-Phase Circuit.

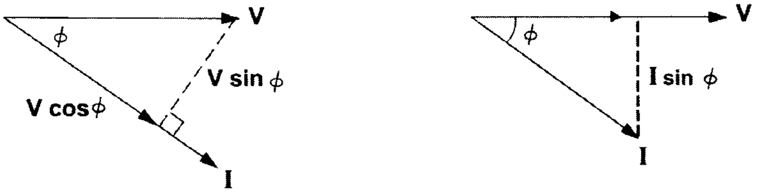

Figure 2.2 Phasor Diagrams Leading to Power Triangles.

Multiplying both sides of this relation by I^* results in

$$S = \sum_{i=1}^{n} S_i \tag{2.8}$$

with the individual element's complex power.

$$S_i = |I|^2 Z_i \tag{2.9}$$

Equation (2.8) is known as the summation rule for complex powers. The rule also applies to parallel circuits.

The phasor diagram shown in Figure 2.2 can be converted into complex power diagrams by simply following the definitions relating complex power to voltage and current.

Consider an inductive circuit in which the current lags the voltage by the angle ϕ. The conjugate of the current will be in the first quadrant in the complex plane as shown in Figure 2.4(a). Multiplying the phasors by **V**, we obtain the complex power diagram shown in Figure 2.4(b). Inspection of the diagram as well as the previous development leads to a relation for the power factor of the circuit:

$$\cos\phi = \frac{P}{|S|}$$

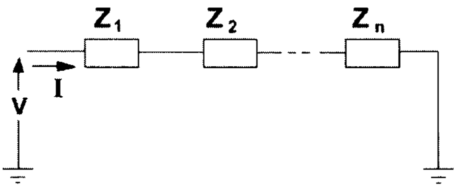

Figure 2.3 Series Circuit.

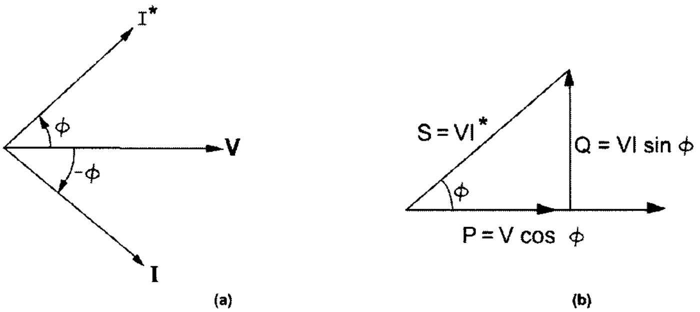

Figure 2.4 Complex Power Diagram

Example 2.1

Consider the circuit composed of a series *R-L* branch in parallel with capacitance with the following parameters:

$$R = 0.5 \text{ ohms}$$
$$X_L = 0.8 \text{ ohms}$$
$$B_c = 0.6 \text{ siemens}$$

Assume that:

$$V = 200\angle 0 \text{ V}$$

Calculate the input current and the active, reactive, and apparent power into the circuit.

Solution

The current into the *R-L* branch is given by

$$I_Z = \frac{200}{0.5 + j0.8} = 212\angle -57.99^\circ \text{ A}$$

The power factor (PF) of the *R-L* branch is

$$\begin{aligned} \mathrm{PF}_Z &= \cos\phi_Z = \cos 57.99^\circ \\ &= 0.53 \end{aligned}$$

The current into the capacitance is

$$I_c = j(0.6)(200) = 120\angle 90^\circ \text{ A}$$

The input current I_t is

$$\begin{aligned} I_t &= I_c + I_Z \\ &= 212\angle -57.99^\circ + 120\angle 90^\circ \\ &= 127.28\angle -28.01^\circ \end{aligned}$$

The power factor (PF) of the overall circuit is

$$\mathrm{PF}_t = \cos\phi_t = \cos 28.01^\circ = 0.88$$

Note that the magnitude of I_t is less than that of I_Z, and that $\cos\phi_t$ is higher than $\cos\phi_Z$. This is the effect of the capacitor, and its action is called *power factor correction* in power system terminology.

The apparent power into the circuit is

$$\begin{aligned} S_t &= VI_t^* \\ &= (200\angle 0)(127.28)\angle 28.01^\circ \\ &= 25{,}456.00\angle 28.01^\circ \text{ VA} \end{aligned}$$

In rectangular coordinates we get

$$S_t = 22{,}471.92 + j11{,}955.04$$

Thus, the active and reactive powers are

$$\begin{aligned} P_t &= 22{,}471.92 \text{ W} \\ Q_t &= 11{,}955.04 \text{ var} \end{aligned}$$

The major portion of all electric power presently used in generation, transmission, and distribution uses balanced three-phase systems. Three-phase operation makes more efficient use of generator copper and iron. Power flow in single-phase circuits was shown in the previous section to be pulsating. This drawback is not present in a three-phase system. Also, three-phase motors start more conveniently and, having constant torque, run more satisfactorily than single-phase motors. However, the complications of additional phases are not compensated for by the slight increase of operating efficiency when polyphase systems other than three-phase are used.

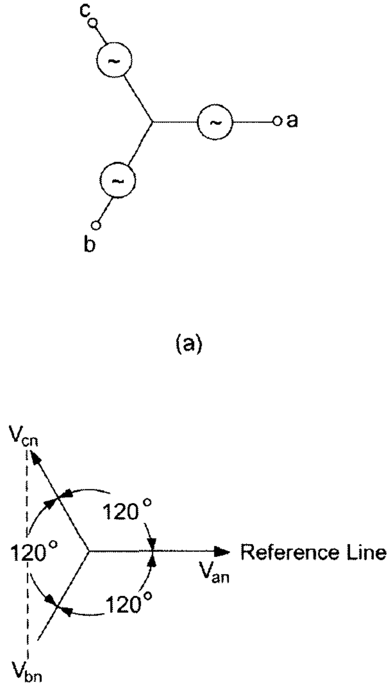

Figure 2.5 A Y-Connected Three-Phase System and the Corresponding Phasor Diagram.

A balanced three-phase voltage system consists of three single-phase voltages with the same magnitude and frequency but time-displaced from one another by 120°. Figure 2.5(a) shows a schematic representation where the three single-phase voltage sources appear in a Y connection; a Δ configuration is also possible. A phasor diagram showing each of the phase voltages is also given in Figure 2.5(b).

Phase Sequence

As the phasors rotate at the angular frequency ω with respect to the reference line in the counterclockwise (positive) direction, the positive maximum value first occurs for phase *a* and then in succession for phases *b* and *c*. Stated in a different way, to an observer in the phasor space, the voltage of phase *a* arrives first followed by that of *b* and then that of *c*. The three-phase voltage of Figure 2.5 is then said to have the phase sequence *abc* (*order* or *phase* *sequence* or *rotation* are all synonymous terms). This is important for applications such as three-phase induction motors, where the phase sequence determines whether the motor turns clockwise or counterclockwise.

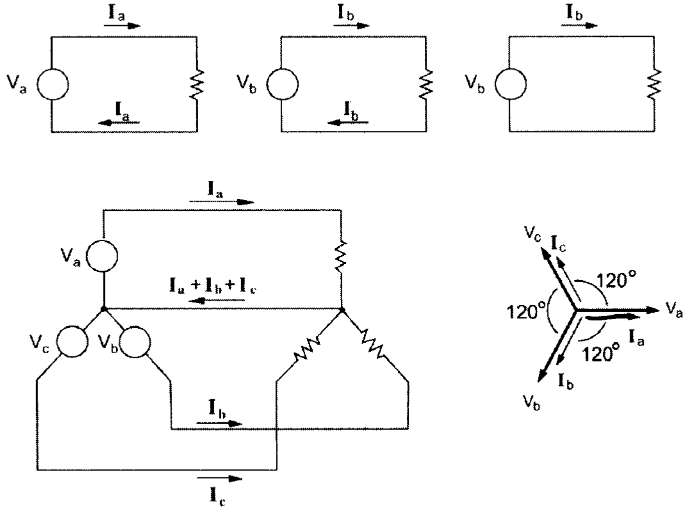

Figure 2.6 A Three-Phase System.

With very few exceptions, synchronous generators (commonly referred to as *alternators*) are three-phase machines. To produce a set of three voltages phase-displaced by 120 electrical degrees in time, it follows that we must use a minimum of three coils phase-displaced 120 electrical degrees in space.

It is convenient to consider representing each coil as a separate generator. An immediate extension of the single-phase circuits discussed above would be to carry the power from the three generators along six wires. However, instead of having a return wire from each load to each generator, a single wire is used for the return of all three. The current in the return wire will be $I_a + I_b + I_c$; and for a balanced load, these will cancel out. If the load is unbalanced, the value of the return current will still be small compared to either I_a, I_b, or I_c. Thus the return wire could be made smaller than the other three. This connection is known as a four-wire three-phase system. It is desirable for safety and system protection to have a connection from the electrical system to the ground (equipotential surface at zero potential). A logical point for grounding is the generator neutral point.

Current and Voltage Relations

Balanced three-phase systems can be studied using techniques developed for single-phase circuits. The arrangement of the three single-phase voltages into a Y or a Δ configuration requires some modification in dealing with the overall system.

Y Connection

With reference to Figure 2.7, the common terminal n is called the *neutral* or *star* (Y) *point.* The voltages appearing between any two of the line terminals a, b, and c have different relationships in magnitude and phase to the voltages appearing between any one line terminal and the neutral point n. The set of voltages V_{ab}, V_{bc}, and V_{ca} are called the *line voltages*, and the set of voltages V_{an}, V_{bn}, and V_{cn} are referred to as the *phase voltages.* Analysis of phasor diagrams provides the required relationships.

The effective values of the phase voltages are shown in Figure 2.7 as V_{an}, V_{bn}, and V_{cn}. Each has the same magnitude, and each is displaced 120° from the other two phasors.

Observe that the voltage existing from a to b is equal to the voltage from a to n (i.e., V_{an}) plus the voltage from n to b.

For a balanced system, each phase voltage has the same magnitude, and we define

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p \tag{2.10}$$

where V_p denotes the effective magnitude of the phase voltage.

We can show that

$$\begin{aligned} V_{ab} &= V_p(1 - 1\angle -120^\circ) \\ &= \sqrt{3}V_p\angle 30^\circ \end{aligned} \tag{2.11}$$

Similarly, we obtain

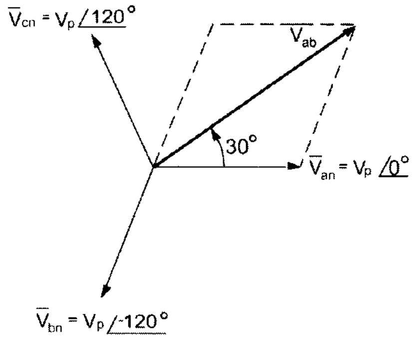

Figure 2.7 Illustrating the Phase and Magnitude Relations Between the Phase and Line Voltage of a Y Connection.

$$V_{bc} = \sqrt{3}V_p\angle -90^\circ \tag{2.12}$$

$$V_{ca} = \sqrt{3}V_p\angle 150^\circ \tag{2.13}$$

The line voltages constitute a balanced three-phase voltage system whose magnitudes are $\sqrt{3}$ times the phase voltages. Thus, we write

$$V_L = \sqrt{3}V_p \tag{2.14}$$

A current flowing out of a line terminal *a* (or *b* or *c*) is the same as that flowing through the phase source voltage appearing between terminals *n* and *a* (or *n* and *b*, or *n* and *c*). We can thus conclude that for a *Y*-connected three-phase source, the line current equals the phase current. Thus,

$$I_L = I_p \tag{2.15}$$

Here I_L denotes the effective value of the line current and I_p denotes the effective value for the phase current.

Δ Connection

Consider the case when the three single-phase sources are rearranged to form a three-phase Δ connection as shown in Figure 2.8. The line and phase voltages have the same magnitude:

$$|V_L| = |V_P| \tag{2.16}$$

The phase and line currents, however, are not identical, and the relationship