



Structural Dynamics in Industry

Alain Girard
and Nicolas Roy

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Foreword

When Alain Girard and Nicolas Roy told me of their intention of writing a book on structural dynamics in industry, I was instantly won over by their initiative.

In fact, experience has shown me that the essential theory-practice connection is the subject of so few works that I could only support the initiative of two experts wanting to impart their knowledge in a pragmatic way.

Having had a chance to appreciate the relevance of their analyses and expertise, I am convinced that the reader will find in this book concrete answers to multiple problems, and at the very least the elements that will enable them to make progress notably in their methodology.

I wish a long life to this book which illustrates so well our INTESPACE slogan “Environmental intelligence” with “The Intelligence of Structural Dynamics in Industry” that it brings.

Jean-Louis MARCÉ
President General Manager

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Preface

Structural dynamics has become increasingly important in different fields such as automotive, aeronautics or space, where the need for performance is ever-increasing. This expansion was encouraged by the capability of computers to simulate phenomena through increasingly large models. However, to control these phenomena, we must first understand them.

This book sets out to explain basic notions with a methodic approach to help improve this comprehension and to deal with industrial structures without becoming too specific. The developments are outlined with minimum mathematics and are often illustrated by simple examples before moving on to cases taken from industrial reality.

This book is addressed to the student or the researcher wanting to organize and clarify their knowledge in this field, as well as to the engineer concerned with bringing a practical response to problems they have to solve.

This field is vast; consequently, certain related aspects are limited to a relatively quick presentation and readers are referred to more detailed works, in order to refocus on the most fundamental principles. This is also the case, to differing degrees, with numerical analysis, finite elements, fluid-structure coupling, high frequency, non-linear, fatigue or experimental techniques.

The content is the result of the authors' experience acquired through their involvement with research and industrial activities mainly at the Centre National d'Etudes Spatiales and INTESPACE Test Center, as well as from teaching, specifically at the engineering schools SUPAÉRO, ENSICA and internships.

The authors wish to thank all who have contributed in one way or another to the achievement of this work, and in particular:

- Jean-Louis Marcé, President General Manager of INTESPACE, and Louis-Patrice Bugeat, Head of Systems and Expertise Department , for providing a favorable environment for their work;

- all the colleagues who, through discussions, developments or implementations, have helped the subject move forward, especially Jacky Chatelain and Paul-Eric Dupuis;

- Jean-François Imbert, former Structures Department Manager at CNES then Assistant Director for Studies and Engineering of INTESPACE, who has provided the impetus and the guidance for these activities with proper judgment;

- Jean-Noël Bricout, from CNES, who gave the manuscript a meticulous re-reading;

- and finally, all departments from the different companies that have encouraged research in this field pushing back the limits: in particular, ALCATEL/Cannes, ASTRIUM/Toulouse, BMW AG Dept EK-213, CNES/Toulouse, DGA/DCE/Toulon, EDF/DER/Clamart, ESA/ESTEC/Noordwijk, PSA PEUGEOT CITROEN, SNECMA PROPULSION SOLIDE/Le Haillan.

Introduction

The title of this book is derived from three key words that should be explained in order to clarify the context:

– *Structures*: structures are referred to here as supports for objects of any nature, for example a mast, bridge, building, car body, a plane's fuselage, etc. We encounter them in all fields and at all levels since the supported objects can in turn serve as support for smaller objects. A launcher then has a structure supporting a payload which in turn has a structure supporting equipment with a structure supporting circuit boards, etc. In all these structures, the main mission is to resist their environment.

– *Dynamic*: structural dynamics is the study of structures subjected to a mechanical environment which depends on time and leading to a movement. We can compare dynamics to statics where the environment does not depend on time, such as gravity or constant pressure. We can also consider statics as a particular aspect of dynamics where the frequency of movement is zero. In this regard, these two disciplines connect in the study of structures. In statics, the idea is mainly to verify that movements or stresses remain within acceptable limits. In dynamics, we must first study the movement which, in certain conditions, can lead to much higher levels than those from a static behavior. Prediction of these phenomena must be controlled in order to understand, remedy and optimize.

– *Industrial*: structures considered will be industrial in the sense that they will not be limited to academic cases. The idea is of course not to oppose these two domains, but to make them complementary. A simple example will enable us to understand and make basic calculations which could be sufficient in certain cases. The complex application processed by a computer will illustrate possibilities in concrete cases. Another aspect of industrial structures is that prediction can rest on two types of activities: analysis and test. To ignore one in favor of the other is dangerous: only relying on analysis is not reliable enough, and relying only on

testing only shows a partial view of the situation. These two worlds must be perfectly interlinked in the development of a structure, with correlation and mathematical model updating phases in relation to tests.

Structural dynamics in industry will be addressed as methodically as possible starting with basic notions, in order to pave the way for handling more complex cases.

The starting point is a dynamic environment imposed on the structure. This excitation may take on very different forms that we must categorize in order to complete an analysis adapted to the situation. We can make the following distinctions:

- concerning the excitation transmission type: this transmission can be:

- mechanical, i.e. using the solid path: the structure is excited by contact with a moving adjacent structure, leading to localized or distributed forces, such as a car body excited by motor vibrations;

- acoustic, using the airborne path or, more generally, that of any fluid: the structure is excited by a pressure field exerted on all exposed surfaces, for example, the same body excited by aerodynamic turbulence. This case involves two completely different environments, structure and fluid, each requiring specific techniques. These two transmissions often coexist: besides the previous example, a typical case is the satellite on its launcher, excited by its mechanical interface and its surfaces exposed to acoustic field under the fairing. This environment is called vibroacoustics;

- concerning the type of time functions: it may be:

- sinusoidal: this particular case is very important because of the movement amplifications that it can generate. It also represents a basic case which helps to solve the general case that we will see later;

- transient: actually, any real excitation can be qualified as transient, because everything has a beginning and an end. However, we generally reserve the word “transient” for a relatively short excitation. This can be from a few milliseconds for pyrotechnic events, for example, to several seconds for seismic events. For very short durations, we talk about shock. For a relatively longer duration, the deterministic analysis can become awkward, in which case a possible solution is illustrated by the following characteristic;

- random (or stochastic): this is the majority of actual excitations resulting from various and generally independent causes. It will be advantageous to work with them in a probabilistic manner by statistical analysis, in order to bring out the movement’s major properties;

– Concerning the type of frequencies involved: this aspect is particularly important. Any function of time, under certain conditions, may be decomposed into a sum of sinusoids: the decomposition given by its Fourier transform. We then have a description of the excitation in the field of frequencies and its content can be:

- low frequency; or
- broadband (low frequency + high frequency);

in relation to the structure involved. A low frequency excitation will generate relatively simple shaped responses on the structure, characterized by wavelengths similar to its size, making analysis of movement easier, contrary to a higher frequency. To be more precise, low frequency is the domain which only involves a limited number of normal modes, stationary shapes related to frequencies that we will explain in more detail later and which enable efficient analysis. At high frequency, these normal modes are numerous, complex and less significant individually, which requires the use of appropriate analysis techniques. This is the case with vibroacoustic environments frequently generating broadband random vibrations, and shock environments where phenomena are often more propagative than stationary.

Concerning analysis, we mainly focus here on low frequency. High frequency techniques come from a slightly different world, which is more difficult to master and which will not be explained here. However, the different functions of time will all be addressed since they all end up using the same solution approach. Finally, concerning types of transmission, acoustic excitation will be considered as long as it can be represented by a mechanical excitation by transforming pressures into known forces: fluid-structure coupling will therefore not be discussed as such.

Next, excited by its dynamic environment, a structure responds. There again, analysis will depend on several factors:

– concerning the nature of structural behavior: it can be linear or proportional to excitation, or non-linear for different reasons. In this last case, the analysis is much more difficult, unless we can linearize it. In the linear case, equations of motion can be easily integrated but in a rather inefficient way;

– at low frequency, as indicated above, the notion of normal mode enables an efficient analysis technique called mode superposition. Each mode, in fact, behaves as a “spring-mass” system, or single-degree-of-freedom system, i.e. the simplest dynamic system, whose state only depends on one parameter. It is therefore doubly interesting to start by analyzing this system: its simplicity makes it possible to understand basic phenomena well and it prepares for more complex systems, at N degrees-of-freedom, with the help of their normal modes. In addition, it can be used as a structure of reference for environmental characterization;

- concerning the domain of analysis: we can work by remaining in the time domain, or we can move to the frequency domain. These two worlds communicate with the help of Fourier transform and its inverse. In the frequency domain, relations between excitations and responses are commonly called frequency response functions (FRF) and are easily handled, which has many advantages;

- concerning structure representation with a mathematical model: this can be continuous or discrete. Any real structure is continuous (at the macroscopic level) and the equations controlling its movement are initially partial differential equations. The difficulty in resolving them analytically in the case of complex structures might lead us to discretize them, or in other words to describe their state by a finite number of parameters, or degrees of freedom, for example by the method of finite elements. In this case, equations are expressed using matrices, and the size of matrices directly depends on the discretization involved. The continuous approach applies to simple cases which can be used as a reference when needed, for example, to evaluate discretization errors;

- if the structure is modular, or in other words if it is made up of distinct parts connected together by simple interfaces, we could be tempted to first analyze each part before assembling them: this technique, called substructuring, has several advantages from a time of calculation point of view but also with organization. It can be executed at different levels by matrix assembly, in particular with FRF for frequency response synthesis and for normal modes for modal synthesis;

- if there is room for maneuver in the definition of structure, we may want to optimize it on dynamic criteria, for example, maximizing natural frequencies or minimizing certain responses to a given excitation. In addition, if test results are available, they must be compared to results from the model and the model must be modified by an operation called updating to make it more representative by minimizing the gap between results. These two problems use the same optimization techniques.

We mainly focus here on linear analysis; non-linear analysis being presented in an introductory chapter. The modal approach to discrete systems will be discussed in more detail to better use its advantages for low frequency, once we have examined the single-degree-of-freedom system, which is the keystone of this analysis. Continuous systems will follow to provide some references, and then complex modes to take certain phenomena into consideration in a more effective manner. In light of these developments, substructuring, optimization and updating in relation to tests will be addressed, bringing responses adapted to generally expressed needs in the industry. We will complete these analysis techniques with a description of the different test techniques, with the goal of identifying dynamic properties or of simulating an environment.

The different types of analysis that we have identified are summarized in Figure 1 and will be developed in the following chapters:

1) General points on linear analysis techniques, explaining the previous comments, which should be read first to provide an overview of the subject.

2) The single-degree-of-freedom system, the keystone of this analysis: equations on movement and resolution. Responses, response spectrums for environment characterization.

3) N -degree-of-freedom systems: equations of motion. Determination of structural matrices. The finite element method. Resolution by direct integration.

4) Modal approach to discrete systems: real and complex modes. Mode superposition techniques with real modes for efficient calculation of low frequency responses.

5) Modal effective parameters: introduced in the previous chapter, representing essential information for understanding the phenomena and mastering the behavior.

6) Continuous systems, the limiting case of discrete systems: analysis techniques. Processing simple cases which can serve as references.

7) Complex modes approach, for a better consideration of certain phenomena: dissipative systems, gyroscopic effects, general cases which can be handled by mode superposition.

8) Substructuring by modal synthesis: representation of each substructure by its normal modes in a given configuration. Formulation based on selected configurations.

9) Substructuring by frequency response synthesis: representation of each substructure by its FRF. Complementarity with modal synthesis.

10) An introduction to nonlinear analysis: sources of nonlinearities and simple examples. The case of the single-degree-of-freedom system. Some analysis approaches for multiple-degree-of-freedom systems.

11) Testing techniques: tests in structure development. Identification of dynamic properties. Environment simulation.

12) Model updating and optimization: test/analysis comparison, model reanalysis and updating. Optimization techniques.

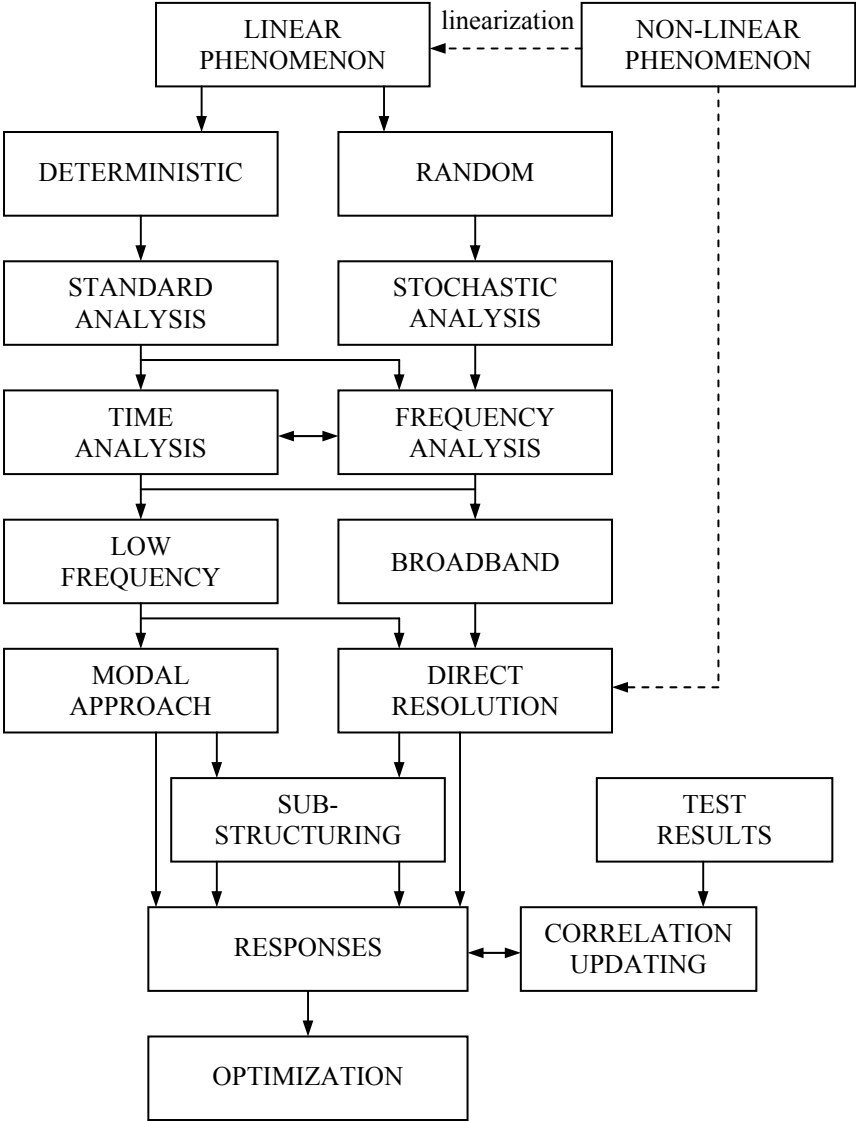


Figure 1. Major types of analyses

Glossary

Abbreviations

DOF	Degree-of-freedom
FRF	Frequency response function
PSD	Power spectral density

Convention for matrices

In order to efficiently develop a formulation implying a certain formalism with matrices, we will use notations where subscripts play an important role: first mnemonics (whenever possible), second, avoiding multiplicity of matrix notations, and finally coherence in matrix products by subscript ordering.

Matrices introduced in the different chapters represent properties relative to DOF, or in a broad sense, parameters defining the state of the structure involved.

By general convention, $\mathbf{X}_{ij}$ designates a matrix of dimension (n, p) where rows are relative to n DOF i and columns to p DOF j .

We should note that this convention implies the relation $\mathbf{X}_{ji} = \mathbf{X}_{ij}^T$ (transposed matrix) since it comes down to permuting rows and columns. In particular, matrix $\mathbf{X}_{ii}$ (which is not necessarily diagonal) is necessarily symmetric. These properties are verified here as long as the principle of reciprocity applies. Otherwise, in the presence of rotating parts destroying certain symmetries for example, precautions will be taken to remove ambiguities without losing the advantages of this notation.

A subscript related to all DOF i and j will logically be named $(i+j)$: for example, we will write $\mathbf{X}_{(i+j)k}$ or $\mathbf{X}_{(i+j)(i+j)}$.

The previous convention must be completed by another convention to be able to distinguish rows, columns or terms of a matrix $\mathbf{X}_{ij}$.

By convention, an underlined subscript has a fixed value.

Therefore: $\mathbf{X}_{i\underline{j}}$ represents row $\underline{i}$ in matrix $\mathbf{X}_{ij}$,
 $\mathbf{X}_{i\underline{j}}$ represents column $\underline{j}$ in matrix $\mathbf{X}_{ij}$,
 $\mathbf{X}_{\underline{ij}}$ represents term $\underline{ij}$ in matrix $\mathbf{X}_{ij}$.

In certain cases, matrix subscripts are followed by other information after a comma to explain affiliation:

Therefore: $\mathbf{X}_{ij,\underline{k}}$ represents a matrix $\mathbf{X}_{ij}$ relative to mode $\underline{k}$,
 $\mathbf{X}_{ij,res}$ represents a matrix $\mathbf{X}_{ij}$ “res” (of residual value: term explained when appropriate).

These conventions are not customary, particularly when they replace the notation T for a transpose of a matrix. Product $(\mathbf{X}_{ij})^T \mathbf{Y}_i$ is simply written as $\mathbf{X}_{ji} \mathbf{Y}_i$ where the sequencing of subscripts is clearly visible. Experience has shown that any reading problems disappear after an adaptation period.

Notations

Below is a list of the main notations used in this book. Some have more than one meaning, but have different symbols or are used in different contexts. Other notations can appear in certain chapters for particular points: they are explained locally to remove any ambiguity.

Scalars or matrices

A	Accelerance (acceleration/force), dynamic amplification H or T , interpolation function
A, B	Matrices in space $2N$
a, b	Generalized parameters in space $2N$
C, c	Linear viscous damping, coefficient

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A

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