

# **GLOBAL POSITIONING SYSTEMS, INERTIAL NAVIGATION, AND INTEGRATION**

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**SECOND EDITION**

**MOHINDER S. GREWAL  
LAWRENCE R. WEILL  
ANGUS P. ANDREWS**



**WILEY-INTERSCIENCE**  
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**GLOBAL  
POSITIONING SYSTEMS,  
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M. S. G. dedicates this book to the memory of his parents, Livlin Kaur and Sardar Sahib Sardar Karam Singh Grewal.

L. R. W. dedicates his work to his late mother, Christine R. Weill, for her love and encouragement in pursuing his chosen profession.

A. P. A. dedicates his work to his wife Jeri, without whom it could not have been done.





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# PREFACE TO THE SECOND EDITION

This book is intended for people who need to combine global navigation satellite systems (GNSSs), inertial navigation systems (INSs), and Kalman filters. Our objective is to give our readers a working familiarity with both the *theoretical* and *practical* aspects of these subjects. For that purpose we have included “real-world” problems from practice as illustrative examples. We also cover the more practical aspects of implementation: how to represent problems in a mathematical model, analyze performance as a function of model parameters, implement the mechanization equations in numerically stable algorithms, assess its computational requirements, test the validity of results, and monitor performance in operation with sensor data from GNSS and INS. These important attributes, often overlooked in theoretical treatments, are essential for effective application of theory to real-world problems.

The accompanying CD-ROM contains MATLAB m-files to demonstrate the workings of the Kalman filter algorithms with GNSS and INS data sets, so that the reader can better discover how the Kalman filter works by observing it in action with GNSS and INS. The implementation of GNSS, INS, and Kalman filtering on computers also illuminates some of the practical considerations of finite-wordlength arithmetic and the need for alternative algorithms to preserve the accuracy of the results. Students who wish to apply what they learn, must experience all the workings and failings of Kalman Filtering—and learn to recognize the differences.

The book is organized for use as a text for an introductory course in GNSS technology at the senior level or as a first-year graduate-level course in GNSS, INS, and Kalman filtering theory and application. It could also be used for self-instruction or review by practicing engineers and scientists in these fields.

This second edition includes some significant changes in GNSS/INS technology since 2001, and we have taken advantage of this opportunity to incorporate

many of the improvements suggested by reviewers and readers. Changes in this second edition include the following:

1. New signal structures for GPS, GLONASS, and Galileo
2. New developments in augmentation systems for satellite navigation, including
  - (a) Wide-area differential GPS (WADGPS)
  - (b) Local-area differential GPS (LADGPS)
  - (c) Space-based augmentation systems (SBASs)
  - (d) Ground-based augmentation systems (GBASs)
3. Recent improvements in multipath mitigation techniques, and new clock steering algorithms
4. A new chapter on satellite system integrity monitoring
5. More thorough coverage of INS technology, including development of error models and simulations in MATLAB for demonstrating system performance
6. A new chapter on GNSS/INS integration, including MATLAB simulations of different levels of tight/loose coupling

The CD-ROM enclosed with the second edition has given us the opportunity to incorporate more background material as files. The chapters have been reorganized to incorporate the new material.

Chapter 1 informally introduces the general subject matter through its history of development and application. Chapters 2–7 cover the basic theory of GNSS and present material for a senior-level class in geomatics, electrical engineering, systems engineering, and computer science.

Chapters 8–10 cover GNSS and INS integration using Kalman filtering. These chapters could be covered in a graduate-level course in electrical, computer, and systems engineering. Chapter 8 gives the basics of Kalman filtering: linear optimal filters, predictors, nonlinear estimation by “extended” Kalman filters, and algorithms for MATLAB implementation. Applications of these techniques to the identification of unknown parameters of systems are given as examples. Chapter 9 is a presentation of the mathematical models necessary for INS implementation and error analysis. Chapter 10 deals with GNSS/INS integration methods, including MATLAB implementations of simulated trajectories to demonstrate performance.

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# ACRONYMS AND ABBREVIATIONS

A/D	Analog-to-digital (conversion)
ADC	Analog-to-digital converter
ADR	Accumulated delta range
ADS	Automatic dependent surveillance
AGC	Automatic gain control
AHRS	Attitude and heading reference system
AIC	Akaike information-theoretic criterion
AIRS	Advanced inertial reference sphere
ALF	Atmospheric loss factor
ALS	Autonomous landing system
altBOC	Alternate binary offset carrier
AODE	Age of data word, ephemeris
AOR-E	Atlantic Ocean Region East (WAAS)
AOR-W	Atlantic Ocean Region West (WAAS)
AR	Autoregressive
ARMA	Autoregressive moving average
ASD	Amplitude spectral density
ASIC	Application-specific integrated circuit
ASQF	Application-Specific Qualification Facility (EGNOS)
A-S	Antispoofing
ATC	Air traffic control
BOC	Binary offset carrier
BPSK	Binary phase-shift keying
C/A	Coarse acquisition (channel or code)
C&V	Correction and verification (WAAS)
CDM	Code-division multiplexing

CDMA	Code-division multiple access
CEP	Circle error probable
CNMP	Code noise and multipath
CONUS	Conterminous United States, also Continental United States
CORS	Continuously operating reference station
COSPAS	Acronym from transliterated Russian title “Cosmicheskaya Sistyema Poiska Avariynich Sudov,” meaning “Space System for the Search of Vessels in Distress”
CPS	Chips per second
CRC	Cyclic redundancy check
CWAAS	Canadian WAAS
DGNSS	Differential GNSS
DGPS	Differential GPS
DME	Distance measurement equipment
DOD	Department of Defense (USA)
DOP	Dilution of precision
ECEF	Earth-centered, earth-fixed (coordinates)
ECI	Earth-centered inertial (coordinates)
EGNOS	European (also Geostationary) Navigation Overlay System
EIRP	Effective isotropic radiated power
EMA	Electromagnetic accelerator
EMA	Electromagnetic accelerometer
ENU	East–north–up (coordinates)
ESA	European Space Agency
ESG	Electrostatic gyroscope
ESGN	Electrically Supported Gyro Navigation (System; USA)
EU	European Union
EWAN	EGNOS Wide-Area (communication) Network (EGNOS)
FAA	Federal Aviation Administration (USA)
FEC	Forward error correction
FLL	Frequency-lock loop
FM	Frequency modulation
FOG	Fiberoptic gyroscope
FPE	Final prediction error (Akaike’s)
FSLF	Free-space loss factor
FT	Feet
GAGAN	GPS & GEO Augmented Navigation (India)
GBAS	Ground-based augmentation system
GCCS	GEO communication and control segment
GDOP	Geometric dilution of precision



GEO	Geostationary earth orbit
GES	GPS Earth Station COMSAT
GIC	GPS Integrity Channel
GIPSY	GPS Infrared Positioning System
GIS	Geographic information system(s)
GIVE	Grid ionosphere vertical error
GLONASS	Global Orbiting Navigation Satellite System
GNSS	Global navigation satellite system
GOA	GIPSY/OASIS analysis
GPS	Global Positioning System
GUS	GEO uplink subsystem
GUST	GEO uplink subsystem type 1
HDOP	Horizontal dilution of precision
HMI	Hazardously misleading information
HOW	Handover word
HRG	Hemispheric resonator gyroscope
ICAO	International Civil Aviation Organization
ICC	Ionospheric correction computation
IDV	Independent Data Verification (of WAAS)
IF	Intermediate frequency
IFOG	Integrating or interferometric Fiberoptic gyroscope
IGP	Ionospheric grid point (for WAAS)
IGS	International GNSS Service
ILS	Instrument landing system
IMU	Inertial measurement unit
Inmarsat	International Mobile (originally “Maritime”) Satellite Organization
INS	Inertial navigation system
IODC	Issue of data, clock
IODE	Issue of data, ephemeris
IONO	Ionosphere, Ionospheric
IOT	In-orbit test
IRU	Inertial reference unit
ISA	Inertial sensor assembly
ITRF	International Terrestrial Reference Frame
JPALS	Joint precision approach and landing system
JTIDS	Joint Tactical Information Distribution System
LAAS	Local-Area Augmentation System
LADGPS	Local-area differential GPS
LD	Location determination
LEM	Lunar Excursion module
LHCP	Left-hand circularly polarized
LORAN	Long-range navigation
LOS	Line of sight
LPV	Lateral positioning with vertical guidance

LSB	Least significant bit
LTP	Local tangent plane
M	Meter
MBOC	Modified BOC
MCC	Mission/Master Control Center (EGNOS)
MCPS	Million Chips Per Second
MEDLL	Multipath-estimating delay-lock loop
MEMS	Microelectromechanical system(s)
ML	Maximum likelihood
MLE	Maximum-likelihood estimate (or estimator)
MMSE	Minimum mean-squared error (estimator)
MMT	Multipath mitigation technology
MOPS	Minimum Operational Performance Standards
MSAS	MTSAT Satellite-based Augmentation System (Japan)
MTSAT	Multifunctional Transport Satellite (Japan)
MVUE	Minimum-variance unbiased estimator
MWG	Momentum wheel gyroscope
NAS	National Airspace System
NAVSTAR	Navigation system with time and ranging
NCO	Numerically controlled oscillator
NED	North-east-down (coordinates)
NGS	National Geodetic Survey (USA)
NLES	Navigation Land Earth Station(s) (EGNOS)
NPA	Nonprecision approach
NSRS	National Spatial Reference System
NSTB	National Satellite Test Bed
OASIS	Orbit analysis simulation software
OBAD	Old but active data
OD	Orbit determination
OPUS	Online Positioning User Service (of NGS)
OS	Open service (of Galileo)
PA	Precision approach
PACF	Performance Assessment and Checkout Facility (EGNOS)
P-code	Precision code
pdf	portable document format
PDOP	Position dilution of precision
PI	Proportional and integral (controller)
PID	Process Input Data (of WAAS); Proportional, integral, and differential (control)
PIGA	Pulse integrating gyroscopic accelerometer
PLL	Phase-lock loop
PLRS	Position Location and Reporting System (U.S. Army)
PN	Pseudorandom noise
POR	Pacific Ocean Region

PPS	Precise Positioning Service
PPS	Pulse(s) per second
PR	Pseudorange
PRN	Pseudorandom noise or pseudorandom number (=SVN for GPS)
PRS	Public Regulated service (of Galileo)
PSD	Power spectral density
RAG	Receiver antenna gain (relative to isotropic)
RAIM	Receiver autonomous integrity monitoring
RF	Radiofrequency
RHCP	Right-hand circularly polarized
RIMS	Ranging and Integrity Monitoring Station(s) (EGNOS)
RINEX	Receiver independent exchange format (for GPS data)
RLG	Ring laser gyroscope
RMA	Reliability, maintainability, availability
RMS	Root-mean-squared; reference monitoring station
RPY	Roll–pitch–yaw (coordinates)
RTCA	Radio Technical Commission for Aeronautics
RTCM	Radio Technical Commission for Maritime Service
RTOS	Real-time operating system
RVCG	Rotational vibratory coriolis gyroscope
s	second
SA	Selective availability (also abbreviated “S/A”)
SAR	Search and Rescue (service; of Galileo)
SARP	Standards and Recommended Practices (Japan)
SARSAT	Search and rescue satellite–aided tracking
SAW	Surface acoustic wave
SBAS	Space-based augmentation system
SBIRLEO	Space-based infrared low earth orbit
SCOUT	Scripps coordinate update tool
SCP	Satellite Correction Processing (of WAAS)
SF	Scale Factor
SIS	Signal in space
SM	Solar magnetic
SNAS	Satellite Navigation Augmentation System (China)
SNR	Signal-to-noise ratio
SOL	Safety of Life Service (of Galileo)
SPS	Standard Positioning Service
STF	Signal Task Force (of Galileo)
SV	Space vehicle
SVN	Space vehicle number (= PRN for GPS)
TCS	Terrestrial communications subsystem (for WAAS)
TCXO	Temperature-compensated Xtal (crystal) oscillator
TDOA	Time difference of arrival
TDOP	Time dilution of precision

TEC	Total electron content
TECU	Total electron content units
TLM	Telemetry word
TOA	Time of arrival
TOW	Time of week
TTA	Time to alarm
TTFF	Time to first fix
UDRE	User differential range error
USERE	User-equivalent range error
URE	User range error
USAF	United States Air Force
USN	United States Navy
UTC	Universal Time, Coordinated (or Coordinated Universal Time)
UTM	Universal Transverse Mercator
VAL	Vertical alert limit
VCG	Vibratory coriolis gyroscope
VDOP	Vertical dilution of precision
VHF	Very high frequency (30–300 MHz)
VOR	VHF Omnidirectional Range (radionavigation aid)
VRW	Velocity Random Walk
WAAS	Wide-Area Augmentation System (U.S.)
WADGPS	Wide-area differential GPS
WGS	World Geodetic System
WMS	Wide-area Master Station
WN	Week number
WNT	WAAS network time
WRE	Wide-area reference equipment
WRS	Wide-area Reference Station
ZLG	Zero-Lock Gyroscope (“Zero Lock Gyro” and “ZLG” are trademarks of Northrop Grumman Corp.)

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# 1

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## INTRODUCTION

There are five basic forms of navigation:

1. *Pilotage*, which essentially relies on recognizing landmarks to know where you are and how you are oriented. It is older than humankind.
2. *Dead reckoning*, which relies on knowing where you started from, plus some form of heading information and some estimate of speed.
3. *Celestial navigation*, using time and the angles between local vertical and known celestial objects (e.g., sun, moon, planets, stars) to estimate orientation, latitude, and longitude [186].
4. *Radio navigation*, which relies on radiofrequency sources with known locations (including global navigation satellite systems satellites).
5. *Inertial navigation*, which relies on knowing your initial position, velocity, and attitude and thereafter measuring your attitude rates and accelerations. It is the only form of navigation that does not rely on external references.

These forms of navigation can be used in combination as well [18, 26, 214]. The subject of this book is a combination of the fourth and fifth forms of navigation using Kalman filtering.

### 1.1 GNSS/INS INTEGRATION OVERVIEW

Kalman filtering exploits a powerful synergism between the *global navigation satellite systems* (GNSSs) and an *inertial navigation system* (INS). This synergism is possible, in part, because the INS and GNSS have very complementary

error characteristics. Short-term position errors from the INS are relatively small, but they degrade without bound over time. GNSS position errors, on the other hand, are not as good over the short term, but they do not degrade with time. The Kalman filter is able to take advantage of these characteristics to provide a common, integrated navigation implementation with performance superior to that of either subsystem (GNSS or INS). By using statistical information about the errors in both systems, it is able to combine a system with tens of meters position uncertainty (GNSS) with another system whose position uncertainty degrades at kilometers per hour (INS) and achieve bounded position uncertainties in the order of centimeters [with differential GNSS (DGNSS)] to meters.

A key function performed by the Kalman filter is the statistical combination of GNSS and INS information to track drifting parameters of the sensors in the INS. As a result, the INS can provide enhanced inertial navigation accuracy during periods when GNSS signals may be lost, and the improved position and velocity estimates from the INS can then be used to cause GNSS signal reacquisition to occur much sooner when the GNSS signal becomes available again.

This level of integration necessarily penetrates deeply into each of these subsystems, in that it makes use of partial results that are not ordinarily accessible to users. To take full advantage of the offered integration potential, we must delve into technical details of the designs of both types of systems.

## 1.2 GNSS OVERVIEW

There are currently three global navigation satellite systems (GNSSs) operating or being developed.

### 1.2.1 GPS

The Global Positioning System (GPS) is part of a satellite-based navigation system developed by the U.S. Department of Defense under its NAVSTAR satellite program [82, 84, 89–94, 151–153].

**1.2.1.1 GPS Orbits** The fully operational GPS includes 24 or more (28 in March 2006) active satellites approximately uniformly dispersed around six circular orbits with four or more satellites each. The orbits are inclined at an angle of  $55^\circ$  relative to the equator and are separated from each other by multiples of  $60^\circ$  right ascension. The orbits are nongeostationary and approximately circular, with radii of 26,560 km and orbital periods of one-half sidereal day ( $\approx 11.967$  h). Theoretically, three or more GPS satellites will always be visible from most points on the earth's surface, and four or more GPS satellites can be used to determine an observer's position anywhere on the earth's surface 24 h per day.

**1.2.1.2 GPS Signals** Each GPS satellite carries a cesium and/or rubidium atomic clock to provide timing information for the signals transmitted by the satellites. Internal clock correction is provided for each satellite clock. Each GPS satellite transmits two spread spectrum, L-band carrier signals—an  $L_1$  signal with carrier frequency  $f_1 = 1575.42$  MHz and an  $L_2$  signal with carrier frequency  $f_2 = 1227.6$