MATHEMATICAL METHODS IN SCIENCE AND ENGINEERING

Ş. SELÇUK BAYIN

Middle East Technical University
Ankara, Turkey

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Preface

Courses on mathematical methods of physics are among the essential courses for graduate programs in physics, which are also offered by most engineering departments. Considering that the audience in these courses comes from all subdisciplines of physics and engineering, the content and the level of mathematical formalism has to be chosen very carefully. Recently the growing interest in interdisciplinary studies has brought scientists together from physics, chemistry, biology, economy, and finance and has increased the demand for these courses in which upper-level mathematical techniques are taught. It is for this reason that the mathematics departments, who once overlooked these courses, are now themselves designing and offering them.

Most of the available books for these courses are written with theoretical physicists in mind and thus are somewhat insensitive to the needs of this new multidisciplinary audience. Besides, these books should not only be tuned to the existing practical needs of this multidisciplinary audience but should also play a lead role in the development of new interdisciplinary science by introducing new techniques to students and researchers.

About the Book

We give a coherent treatment of the selected topics with a style that makes advanced mathematical tools accessible to a multidisciplinary audience. The book is written in a modular way so that each chapter is actually a review of its subject and can be read independently. This makes the book very useful as a reference for scientists. We emphasize physical motivation and the multidisciplinary nature of the methods discussed.

The entire book contains enough material for a three-semester course meeting three hours a week. However, the modular structure of the book gives enough flexibility to adopt the book for several different advanced undergraduate and graduate-level courses. Chapter 1 is a philosophical prelude about physics, mathematics, and mind for the interested reader. It is not a part of the curriculum for courses on mathematical methods of physics. Chapters 2-8, 12, 13 and 15-19 have been used for a two-semester compulsory graduate course meeting three hours a week. Chapters 16-20 can be used for an introductory graduate course on Green's functions. For an upper-level undergraduate course on special functions, colleagues have used Chapters 1-8. Chapter 14 on fractional calculus can be expanded into a one-term elective course supported by projects given to students. Chapters 2-11 can be used in an introductory graduate course, with emphasis given to Chapters 8-11 on Sturm-Liouville theory, factorization method, coordinate transformations, general tensors, continuous groups, Lie algebras, and representations.

Students are expected to be familiar with the topics generally covered during the first three years of the science and engineering undergraduate curriculum. These basically comprise the contents of the books *Advanced Calculus* by Kaplan, *Introductory Complex Analysis* by Brown and Churchill, and *Differential Equations* by Ross, or the contents of books like *Mathematical Methods in Physical Sciences* by Boas, *Mathematical Methods: for Students of Physics and Related Fields* by Hassani, and *Essential Mathematical Methods for Physicists* by Arfken and Weber. Chapters (10 and 11) on coordinates, tensors, and groups assume that the student has already seen orthogonal transformations and various coordinate systems. These are usually covered during the third year of the undergraduate physics curriculum at the level of *Classical Mechanics* by Marion or *Theoretical Mechanics* by Bradbury. For the sections on special relativity (in Chapter 10) we assume that the student is familiar with basic special relativity, which is usually covered during the third year of undergraduate curriculum in modern physics courses with text books like *Concepts of Modern Physics* by Beiser.

Three very interesting chapters on the method of factorization, fractional calculus, and path integrals are included for the first time in a text book on mathematical methods. These three chapters are also extensive reviews of these subjects for beginning researchers and advanced graduate students.

Summary of the Book

In Chapter 1 we start with a philosophical prelude about physics, mathematics, and mind.

In Chapters 2-6 we present a detailed discussion of the most frequently encountered special functions in science and engineering. This is also very timely, because during the first year of graduate programs these functions are used extensively. We emphasize the fact that certain second-order partial differential equations are encountered in many different areas of science, thus allowing one to use similar techniques. First we approach these partial differential equations by the method of separation of variables and reduce them to a set of ordinary differential equations. They are then solved by the method of series, and the special functions are constructed by imposing appropriate boundary conditions. Each chapter is devoted to a particular special function, where it is discussed in detail. Chapter 7 introduces hypergeometric equation and its solutions. They are very useful in parametric representations of the commonly encountered second-order differential equations and their solutions. Finally our discussion of special functions climaxes with Chapter 8, where a systematic treatment of their common properties is given in terms of the Sturm-Liouville theory. The subject is now approached as an eigenvalue problem for second-order linear differential operators.

Chapter 9 is one of the special chapters of the book. It is a natural extension of the chapter on Sturm-Liouville theory and approaches second-order differential equations of physics and engineering from the viewpoint of the theory of factorization. After a detailed analysis of the basic theory we discuss specific cases. Spherical harmonics, Laguerre polynomials, Hermite polynomials, Gegenbauer polynomials, and Bessel functions are revisited and studied in detail with the factorization method. This method is not only an interesting approach to solving Sturm-Liouville systems, but also has deep connections with the symmetries of the system.

Chapter 10 presents an extensive treatment of coordinates, their transformations, and tensors. We start with the Cartesian coordinates, their transformations, and Cartesian tensors. The discussion is then extended to general coordinate transformations and general tensors. We also discuss Minkowski spacetime, coordinate transformations in spacetime, and four-tensors in detail. We also write Maxwell's equations and Newton's dynamical theory in covariant form and discuss their transformation properties in spacetime.

In Chapter 11 we discuss continuous groups, Lie algebras, and group representations. Applications to the rotation group, special unitary group, and homogeneous Lorentz group are discussed in detail. An advanced treatment of spherical harmonics is given in terms of the rotation group and its representations. We also discuss symmetry of differential equations and extension (prolongation) of generators.

Chapters 12 and 13 deal with complex analysis. We discuss the theory of analytic functions, mappings, and conformal and Schwarz-Christoffel transformations with interesting examples like the fringe effects of a parallel plate capacitor and fluid flow around an obstacle. We also discuss complex integrals, series, and analytic continuation along with the methods of evaluating some definite integrals.

Chapter 14 introduces the basics of fractional calculus. After introducing the experimental motivation for why we need fractional derivatives and integrals, we give a unified representation of the derivative and integral and extend it to fractional orders. Equivalency of different definitions, examples, properties, and techniques with fractional derivatives are discussed. We conclude with examples from Brownian motion and the Fokker-Planck equation. This is an emerging field with enormous potential and with applications to physics, chemistry, biology, engineering, and finance. For beginning researchers and instructors who want to add something new and interesting to their course, this self-contained chapter is an excellent place to start.

Chapter 15 contains a comprehensive discussion of infinite series: tests of convergence, properties, power series, and uniform convergence along with the methods of evaluating sums of infinite series. An interesting section on divergent series in physics is added with a discussion of the Casimir effect.

Chapter 16 treats integral transforms. We start with the general definition, and then the two most commonly used integral transforms, Fourier and Laplace transforms, are discussed in detail with their various applications and techniques.

Chapter 17 is on variational analysis. Cases with different numbers of dependent and independent variables are discussed. Problems with constraints, variational techniques in eigenvalue problems, and the Rayleigh-Ritz method are among other interesting topics covered.

In Chapter 18 we introduce integral equations. We start with their classification and their relation to differential equations and vice versa. We continue with the methods of solving integral equations and conclude with the eigenvalue problem for integral operators, that is, the Hilbert-Schmidt theory.

In Chapter 19 (and 20) we present Green's functions, and this is the second climax of this book, where everything discussed so far is used and their connections seen. We start with the time-independent Green's functions in one dimension and continue with three-dimensional Green's functions. We discuss their applications to electromagnetic theory and the Schrödinger equation. Next we discuss first-order time-dependent Green's functions with applications to diffusion problems and the time-dependent Schrödinger equation. We introduce the propagator interpretation and the compounding of propagators. We conclude this section with second-order time-dependent Green's functions, and their application to the wave equation and discuss advanced and retarded solutions.

Chapter 20 is an extensive discussion of path integrals and their relation to Green's functions. During the past decade or so path integrals have found wide range of applications among many different fields ranging from physics to finance. We start with the Brownian motion, which is considered a prototype of many different processes in physics, chemistry, biology, finance etc. We discuss the Wiener path integral approach to Brownian motion. After the Feynman-Kac formula is introduced, the perturbative solution of the Bloch equation is given. Next an interpretation of $V(x)$ in the Bloch equation is given, and we continue with the methods of evaluating path integrals. We also discuss the Feynman path integral formulation of quantum mechanics along with the phase space approach to Feynman path integrals.

Story of the Book

Since 1989, I have been teaching the graduate level 'Methods of Mathematical Physics I & II' courses at the Middle East Technical University in Ankara. Chapters 2-8 with 12 and 13 have been used for the first part and Chapters 15-19 for the second part of this course, which meets three hours a week. Whenever possible I prefer to introduce mathematical techniques through physical applications. Examples are often used to extend discussions of specific techniques rather than as mere exercises. Topics are introduced in a logical sequence and discussed thoroughly. Each sequence climaxes with a part where the material of the previous chapters is unified in terms of a general theory, as in Chapter 8 (and 9) on the Sturm-Liouville theory, or with a part that utilizes the gains of the previous chapters, as in Chapter 19 (and 20) on Green's functions. Chapter 9 is on factorization method, which is a natural extension of our discussion on the Sturm-Liouville theory. It also presents a different and advanced treatment of special functions. Similarly, Chapter 20 on path integrals is a natural extension of our chapter on Green's functions. Chapters 10 and 11 on coordinates, tensors, and continuous groups have been located after Chapter 9 on the Sturm-Liouville theory and the factorization method. Chapters 12 and 13 are on complex techniques, and they are self-contained. Chapter 14 on fractional calculus can either be integrated into the curriculum of the mathematical methods of physics courses or used independently.

During my lectures and first reading of the book I recommend that readers view equations as statements and concentrate on the logical structure of the discussions. Later, when they go through the derivations, technical details become understood, alternate approaches appear, and some of the questions are answered. Sufficient numbers of problems are given at the back of each chapter. They are carefully selected and should be considered an integral part of the learning process.

In a vast area like mathematical methods in science and engineering, there is always room for new approaches, new applications, and new topics. In fact, the number of books, old and new, written on this subject shows how dynamic this field is. Naturally this book carries an imprint of my style and lectures. Because the main aim of this book is pedagogy, occasionally I have followed other books when their approaches made perfect sense to me. Sometimes I indicated this in the text itself, but a complete list is given at the back. Readers of this book will hopefully be well prepared for advanced graduate studies in many areas of physics. In particular, as we use the same terminology and style, they should be ready for full-term graduate courses based on the books: *The Fractional Calculus* by Oldham and Spanier and *Path Inte-*

grals in Physics, Volumes I and II by Chaichian and Demichev, or they could jump into the advanced sections of these books, which have become standard references in their fields.

I recommend that students familiarize themselves with the existing literature. Except for an isolated number of instances I have avoided giving references within the text. The references at the end should be a good first step in the process of meeting the literature. In addition to the references at the back, there are also three websites that are invaluable to students and researchers: For original research, http://lanl.arxiv.org/ and the two online encyclopedias: http://en.wikipedia.org and http://scienceworld.wolfram.com/ are very useful. For our chapters on special functions these online encyclopedias are extremely helpful with graphs and additional information.

A precursor of this book (Chapters 1-8, 12, 13, and 15-19) was published in Turkish in 2000. With the addition of two new chapters on fractional calculus and path integrals, the revised and expanded version appeared in 2004 as 440 pages and became a widely used text among the Turkish universities. The positive feedback from the Turkish versions helped me to prepare this book with a minimum number of errors and glitches. For news and communications about the book we will use the website http://www.physics.metu.edu.tr/~ bayin, which will also contain some relevant links of interest to readers.

S. BAYIN

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Ankara/TURKEY
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I would like to pay tribute to all the scientists and mathematicians whose works contributed to the subjects discussed in this book. I would also like to compliment the authors of the existing books on mathematical methods of physics. I appreciate the time and dedication that went into writing them. Most of them existed even before I was a graduate student. I have benefitted from them greatly. I am indebted to Prof. K.T. Hecht of the University of Michigan, whose excellent lectures and clear style had a great influence on me. I am grateful to Prof. P.G.L. Leach for sharing his wisdom with me and for meticulously reading Chapters 1 and 9 with 14 and 20. I also thank Prof. N. K. Pak for many interesting and stimulating discussions, encouragement, and critical reading of the chapter on path integrals. I thank Wiley for the support by a grant during the preparation of the camera ready copy. My special thanks go to my editors at Wiley, Steve Quigley, Susanne Steitz, and Danielle Lacourciere for sharing my excitement and their utmost care in bringing this book into existence.

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