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Zonotopes

From Guaranteed State-estimation to Control

Vu Tuan Hieu Le, Cristina Stoica
Teodoro Alamo, Eduardo F. Camacho
and Didier Dumur

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Notations

$\mathbb{R}$	Set of real numbers
$\mathbb{R}^+$	Set of strictly positive real numbers
$\mathbb{R}^n$	Set of n-dimensional real vector
$\mathbf{B}^n$	Unitary box in $\mathbb{R}^n$
α	General notation for a scalar
$\mathbf{a}$	General notation for a vector
A	General notation for a matrix
A^T	Transpose of matrix A
A^{-1}	Inverse of matrix A
$det(A)$	Determinant of matrix A
$tr(A)$	Trace of matrix A
$Im(A)$	Image of matrix A
$A \succ 0$	General notation for strictly positive definite matrix A
$A \succeq 0$	General notation for positive definite matrix A
$A \prec 0$	General notation for strictly negative definite matrix A
$A \preceq 0$	General notation for negative definite matrix A
I_n	Identity matrix in $\mathbb{R}^{n \times n}$
O_n	Zeros matrix in $\mathbb{R}^{n \times n}$
$diag(a_1,\ldots,a_n)$	Diagonal matrix of dimension n
$\mathcal{Z} = \mathcal{Z}(\mathbf{p};H) = \mathbf{p} \oplus H\mathbf{B}^m$	General notation of a m-zonotope
$rs(H)$	Round-sum of matrix H
$\blacksquare\mathcal{Z}$	Approximation of zonotope $\mathcal{Z}$ by a box
$\blacklozenge\mathcal{Z}$	Approximation of zonotope $\mathcal{Z}$ by a parallelotope
$\diamond\mathcal{Z}$	Zonotope inclusion
$\lvert \cdot \rvert$	Absolute value
$\lVert \cdot \rVert_\infty$	Infinity norm
$\lVert \cdot \rVert_P$	P-norm
$\lVert \cdot \rVert_F$	Frobenius norm
$\in$	It belongs to
$\subset$	Subset

$\cap$	Intersection
$\oplus$	Minkowski sum
$\ominus$	Pontryagin difference
$M(S)$	Image of a set S
$d(\mathcal{X},\mathcal{Y})$	Distance between the sets $\mathcal{X}$ and $\mathcal{Y}$ (also called “normal” distance)
$d_H(\mathcal{X},\mathcal{Y})$	Hausdorff distance between the sets $\mathcal{X}$ and $\mathcal{Y}$
$\binom{n}{m}$	n combination of m elements
$n!$	Factorial of n
$\mathbf{y}_{k/i}$	The ith row of vector $\mathbf{y}_k$
$conv(\cdot)$	Convex hull
$\omega \sim N(0, Q)$	Random variable ω having zero means, normal distribution and covariance matrix Q

Acronyms

BMI	Bilinear Matrix Inequality
CARIMA	Controlled Auto-Regressive Integrated Moving Average
CRHPC	Constrained Receding Horizon Predictive Control
DMC	Dynamic Matrix Control
EHAC	Extended Horizon Adaptive Control
EPSAC	Extended Prediction Self-Adaptive Control
ESO	Equivalent Single-Output
ESOCE	Equivalent Single-Output with Coupling Effect
EVP	Eigenvalue Problem
GPC	Generalized Predictive Control
LMI	Linear Matrix Inequality
LQ	Linear Quadratic control
LQR	Linear Quadratic Regulator
LTI	Linear Time Invariant
MAC	Model Algorithmic Control
MIMO	Multi-Input Multi-Output
MPC	Model Predictive Control
MPHC	Model Predictive Heuristic Control
MPT	Multi-Parametric Toolbox
MURHAC	Multi-predictor Receding Horizon Adaptive Control
MUSMAR	Multi-step Multivariable Adaptive Control
PAZI	Polytope and Zonotope Intersection
PFC	Predictive Functional Control
PMI	Polynomial Matrix Inequality
QP	Quadratic Programming
SISO	Single-Input Single-Output
SOS	Sum of Squares
SVD	Singular Value Decomposition
TMPC	Tube-based Model Predictive Control
UPC	Unified Predictive Control

Introduction

This book stands at a crossroad between two major axes in automatic control: state estimation and robust control, applied to uncertain discrete-time linear time invariant systems. The goal is to take into account disturbances, measurement noises and constraints in order to build a zonotopic guaranteed state estimation and an output feedback control which can guarantee the feasibility and the stability of the closed-loop system in this specific context. Part of the results proposed in this book were developed and published in the PhD thesis of Hieu Le [LE 12c], under the supervision of the co-authors of this book.

In the literature, there are two main approaches for describing uncertainties, disturbances and noises acting on a dynamic system:

– *Stochastic approach,* which assumes that the disturbances, noises and parameter uncertainties are unknown but the probability distributions are known.

– *Deterministic approach,* which assumes that disturbances, noises and parameter uncertainties are unknown but bounded by some convex sets. The main advantage of the deterministic approach is that disturbances and noises are assumed to be bounded and this is often simpler to verify than the criterion on the probability distribution. This is the main reason why many authors [WIT 68, SCH 68, BER 71] have chosen the deterministic approach to model the disturbances and the noises affecting the system behavior. Based on this remark, the deterministic approach has been chosen in this book to model possible uncertainties, disturbances and/or measurement noises.

Because of the presence of measurement noises, the system state, which is necessary for building the control law, is not always available. In this case, the implementation of a state estimator is necessary. This state estimation problem can be solved by different methods, such as a Luenberger observer [LUE 64], functional observer [MUR 73], moving horizon estimation [GRI 90] and set-membership estimation [WIT 68, SCH 68, BER 71]. Owing to its ability to deal with uncertainties and disturbances, the set-membership estimation method has been chosen in this book. This approach has been applied to the problem of state estimation of uncertain systems since the 1960s [WIT 68, SCH 68, COM 03, ALA 05]. The set-membership estimation allows us to obtain a set containing the real system state that is consistent with the disturbances and measurement noises. With the development of robust control theory, the set-membership estimation technique is shown to be suitable in dealing with unknown but bounded uncertainties, disturbances and measurement noises. If constraints are added to the previous problem, then a predictive control feature should be added. This results in using robust predictive control strategies based on set-membership estimation in order to answer the proposed problem. In particular, zonotopic sets will be used due to their flexibility and low-complexity.

This book builds upon previous results on the zonotopic set-membership state estimation [COM 03, ALA 05] and the output feedback Tube-based Model Predictive Control [MAY 09]. The aim of the state estimation problem is to obtain a small estimation set which contains the real state. The method proposed by [COM 03] computes a zonotopic outer approximation of the set of states based on a Singular Value Decomposition of a matrix [STR 05], which offers good accuracy of the estimation. In [ALA 05], the authors proposed a method to compute the zonotopic guaranteed state estimation based on two optimization problems. The first solution is based on the minimization of the volume of a zonotope and offers a high-accuracy estimation with a complex computation, while the second solution considers the minimization of the segments of the zonotope and proposes a simple computation but with a deterioration of the estimation accuracy. For these reasons, the goal of this book is to propose a method which permits the improvement of the estimation accuracy while keeping a low complexity level. Moreover, this zonotopic set-membership estimation is proposed to replace the Luenberger observer in the output feedback Tube-based Model Predictive Control [MAY 09]. This association allows us to improve the performance of the closed-loop system as will be shown in the following chapters.

This book is organized as follows:

– *Chapter 1:* the goal of this chapter is to answer the question on how to represent uncertainties, disturbances and noises in the deterministic approach. The chapter starts with a short description of the deterministic approach in which the disturbances and the noises are assumed to be bounded by known convex sets. Afterwards, some basic definitions and operations necessary for manipulating different sets are presented. The next section consists of presenting a list of the most popular families of sets which are used in the literature to bound uncertainties. Because of the advantages of zonotopes, the family of zonotopic sets is further chosen to bound the disturbances and measurement noises.

– *Chapter 2:* in this chapter, an overview of existing estimation techniques is proposed to solve the problem of state estimation for systems subject to unknown but bounded disturbances and measurement noises. Zonotopic-based guaranteed set-membership estimation techniques are further detailed. Two main classes of approaches are addressed: Singular Value Decomposition-based methods [COM 03] and optimization-based methods. The following optimization-based methods are recalled: minimization of the segments of a zonotope [ALA 05] (offering low computation complexity), minimization of the volume of a zonotope [ALA 05] (offering good accuracy of the estimation) and the minimization of the *P*-radius of a zonotope [LE 11a] (offering a trade-off between the low computation complexity and the accuracy of the state estimation). Moreover, the *P*-radius criterion allows us to guarantee the non-increasing property of the guaranteed state estimation at each time instant.

– *Chapter 3:* this chapter details the *P*-radius-based zonotopic set-membership estimation for both Single-Output systems and Multi-Output systems. The Single-Output system solution consists of a zonotopic outer approximation of the intersection between a zonotope and a strip, solved using matrix inequality optimization techniques.

The case of Multi-Output systems leads to two different classes of solutions. The first class is the direct application proposed for the Single-Output systems for each output of the Multi-Output system leading to a conservative result. Several approaches belonging to this first class are developed and compared (the Equivalent Single-Output approach and Equivalent Single-Output with Coupling Effect and Polynomial Matrix Inequality approach). The second class based on the zonotopic outer approximation of the intersection between a polytope and a zonotope allows us to improve the accuracy of the estimation while considering all the output measurements at the same time.

– *Chapter 4:* the problem of robust predictive control is discussed in this chapter, in the context of zonotopic set-membership estimation. Based on the zonotopic set-membership estimation discussed in Chapter 3, a feedback predictive control based on a tube of trajectories is proposed for the case of linear discrete-time invariant systems with bounded disturbances and measurement noises, subject to constraints. This chapter also proposes an application of the developed approaches to control a magnetic levitation system. The first step consists of describing and modeling this nonlinear unstable continuous-time system subject to bounded disturbances, measurement noises and constraints. The proposed model is linearized around an equilibrium point and discretized for a given sampling time. Based on this model, the Tube-based Model Predictive Control associated with the zonotopic set-membership estimation is used to stabilize this system around the equilibrium point.

– *Conclusion and Perspectives:* the final chapter summarizes the work developed in this book and proposes several directions for future developments.

Chapter 1

Uncertainty Representation Based on Set Theory

Real systems are often complex due to several factors: the system's nature (e.g. mechanical, electrical and chemical systems), interactions between its different components (e.g. multivariable systems), and its different behavior in a dynamic environment (e.g. influence of disturbances, noises and uncertainties). All these aspects have to be considered when modeling a given system, sometimes leading to a complicated model. In the context of control systems, a mathematical model is frequently used to describe the system behavior. On the one hand, the accuracy of the mathematical model is important to analyze and design control strategies for the considered system; on the other hand, in the context of industrial applications, it is suitable to use unsophisticated controllers designed using a simple model. In this context, a trade-off must be found: the system model should be simple but precise enough to characterize the dynamical behavior of the original system. Thus, the simple/simplified mathematical model cannot represent the real system exactly due to a lack of knowledge of, or unreliable information about, the system. To validate this model, some uncertainties can be added to the mathematical model. Frequently, perturbations influencing the real system have to be taken into account in the mathematical model in order to ensure a similar behavior of the real system and the mathematical model. The importance of uncertainties in system design has been