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Shouting Gao

Cloud-Resolving Modeling of Convective Processes

Second Edition

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Second Edition

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Preface

Clouds and cloud systems and their interactions with larger scales of motion, radiation, and the Earth's surface are extremely important parts of weather and climate systems. Their treatment in weather forecast and climate models is a significant source of errors and uncertainty. As computer power increases, it is beginning to be possible to explicitly resolve cloud and precipitation processes in these models, presenting opportunities for improving precipitation forecasts and larger-scale phenomena such as tropical cyclones which depend critically on cloud and precipitation physics.

This book by Professor Shouting Gao of the Institute of Atmospheric Physics in Beijing and Dr. Xiaofan Li of NOAA's National Environmental Satellite, Data, and Information Service (NESDIS) presents an update and review of results of high-resolution, mostly two-dimensional models of clouds and precipitation and their interactions with larger scales of motion and the Earth's surface. It provides a thorough description of cloud and precipitation physics, including basic governing equations and related physics, such as phase changes of water, radiation, and mixing. Model results are compared with observations from the 1992–1993 Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) experiment. The importance of the ocean to tropical convective systems is clearly shown here in the numerical results of simulations with their air–sea coupled modeling system. While the focus is on tropical convection, the methodology and applicability can be extended to cloud and precipitation processes elsewhere.

The results described in this well-written book form a solid foundation for future high-resolution model weather forecasts and climate simulations that resolve clouds explicitly in three dimensions – a future that I believe has great promise for the understanding and prediction of weather and climate for the great benefit of society.

President, University Corporation for Atmospheric Research Richard Anthes
Boulder, CO, USA
June 2007

Contents

Introduction

The material in this book is based on our recent research work in the latest 10 years. It is the first book that focuses on cloud-resolving modeling of convective processes. Clouds play an important role in linking atmospheric and hydrological processes and have profound impacts on regional and global climate. The better description of clouds and associated cloud-radiation interaction is a key for successful simulations of cloud processes, which require the physical presence of cloud hydrometeors, prognostic cloud equations, and interactive radiative schemes in models. The cloud-resolving models have been developing for four decades toward providing the better understanding of cloud-scale processes associated with the convective development. With the explosive increase of computational powers, the cloud-resolving models, which were once used to develop cloud schemes for general circulation models, have been directly applied to a global domain with a high horizontal resolution (grid mesh is less than 5 km), whose preliminary results are promising.

This book starts with basic equations and physical packages used in cloud-resolving models and coupled ocean–cloud-resolving atmosphere model. The cloud-resolving model discussed in this book is the two-dimensional version of the Goddard Cumulus Ensemble Model. The model simulations are evaluated with available observations during Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE). The book covers many research aspects related to convective development, cloud, and precipitation. The material in this book has been used in a part of a graduate course at the Graduate School of the Chinese Academy of Sciences, Beijing, China. Therefore, this book can be used as a reference and textbook for graduate students and researchers whose research interests are mesoscale, cloud, and precipitation modeling.

This book is comprised of thirteen chapters. Chapter 1 presents governing equations, parameterization schemes of radiation, cloud microphysics, and subgrid-scale turbulence. Two model frameworks imposed by different large-scale forcing are intensively discussed. Chapter 2 describes thermal and vapor budgets, surface rainfall equation, energetics equation, and partitioning of convective and stratiform rainfall, which are frequently applied to the analysis of cloud-resolving model simulation data. The cloud-resolving model simulation data are evaluated with the high-quality observational data from TOGA COARE in terms of thermodynamic states, apparent heat sink and moisture source, surface radiative and latent heat fluxes, and surface rain rate in Chap. 3. Since most of the research work is from two-dimensional cloud-resolving modeling, the similarities and differences between two- and three-dimensional cloud-resolving modeling are discussed.

The surface rainfall equation is introduced to examine the contributions of water vapor and cloud hydrometeors in surface rainfall processes in Chap. 4. The intensive discussions of surface rainfall processes are conducted in raining stratiform, convective, non-raining stratiform, and clear-sky regions, respectively. In Chap. 5, kinematics and propagation of tropical cloud clusters are discussed. Chapter 6 addresses cloud microphysics and radiation. In particular, the depositional growth of snow from cloud ice as an important sink of cloud ice and precipitation–radiation interaction are examined. The vorticity vectors associated with tropical convection are discussed in Chap. 7. The dominant physical processes that are responsible for the diurnal variations of tropical convection including nocturnal and afternoon rainfall peaks and tropical convective and stratiform rainfall are quantitatively identified with analysis of surface rainfall equation in Chap. 8. The precipitation efficiencies and statistical equivalence of efficiencies defined with water vapor and cloud microphysics budgets are addressed in Chap. 9. The coupled ocean–cloud-resolving atmosphere model is developed to study the small-scale effects of precipitation in ocean mixing processes in Chap. 10. Effects of SST, diurnal variation, and cloud radiation on equilibrium states are discussed in Chap. 11. The microwave radiative transfer model with cloud-resolving model simulation data is applied to radiance simulations in Chap. 12. Finally, the future perspective of cloud-resolving modeling, including simplification of prognostic cloud schemes, and its application to the general circulation model and to the global domain are discussed in Chap. 13.

We would like to thank Dr. Richard A. Anthes, president of the University Corporation for Atmospheric Research, who read the book draft and wrote the preface for this book. Our sincere thanks also go to Dr. Wei-Kuo Tao and Dr. David Adamec at NASA/Goddard Space Flight Center (GSFC), Professor Ming-Dah Chou at National Taiwan University, and Professor Minghua Zhang at the State University of New York, Stony Brook, for providing the two-dimensional Goddard Cumulus Ensemble (GCE) Model, ocean mixed-layer model, radiative transfer code used in GCE model, and TOGA COARE forcing data, respectively. We also thank Dr. Hsiao-Ming Hsu at the National Center for Atmospheric Research for his comments; Drs. Fan Ping, Xiaopeng Cui, Yushu Zhou, and Lingkun Ran at the Institute of Atmospheric Physics, Chinese Academy of Sciences, for efficient and productive research collaborations; and Miss Di Li at the University of Maryland, College Park, for editing this book. Xiaofan Li would like to thank Dr. William K.-M. Lau, chief of the Laboratory for Atmospheres at NASA/GSFC, and Professor Chung-Hsiung Sui at the National Central University for their support, encouragement, and academic guidance when he worked at GSFC as a contract research scientist during 1994–2001, Drs. Fuzhong Weng and Quanhua Liu at NOAA/NESDIS/Center for Satellite Applications and Research for providing microwave radiative transfer model, and

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Beijing, China
Camp Springs, MD, USA
Fall 2007

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Xiaofan Li

Introduction for the Second Edition

This book is written based on our research work in the last 17 years. In addition to the thirteen chapters from the first edition, the second edition has five more chapters which discuss the structures of precipitation systems, thermal impacts of doubled carbon dioxide on precipitation, precipitation predictability, and modeling of the depositional growth of ice crystal. The material in this book has been used in a part of a graduate course at the School of Earth Sciences, Zhejiang University, and Graduate School, Chinese Academy of Sciences, Beijing, China.

The second edition of this book is comprised of eighteen chapters. Chapter 1 discusses the equations, physical packages, and basic parameters of the cloud-resolving model. Chapter 2 describes methodologies for analyzing cloud-resolving model simulation data. Chapter 3 presents validation for cloud-resolving modeling of TOGA COARE using available observational data. Chapter 4 introduces surface precipitation budget and discusses the important contribution from cloud hydrometeors to precipitation. Chapters 5 and 6 examine structures of precipitation systems through the analyses of cloud content and budgets. New convective–stratiform rainfall separation schemes are developed based on the cloud content and cloud and precipitation budgets, respectively. The new scheme based on surface precipitation budget can be used to define maximum rainfall as water vapor, hydrometeor convergence, and atmospheric drying or, alternatively, as a 100 % precipitation efficiency.

Chapter 7 analyzes kinematics, propagation, and the merging processes of tropical cloud clusters. Chapter 8 discusses cloud radiative and microphysical processes associated with the production of precipitation. Chapter 9 examines the thermal aspects of the impacts of doubled carbon dioxide on precipitation based on a tropical rainfall event during the winter solstice and a pre-summer torrential event during the summer solstice. In particular, the effects of doubled carbon dioxide on rainfall responses to radiative processes are investigated. Chapter 10 introduces convective, moist, and dynamic vorticity vectors as important diagnostic tools for studying convective development and their linkages to thermal, water vapor, and cloud microphysics. Chapter 11 discusses the diurnal variations of tropical rainfall including nocturnal and afternoon rainfall peaks. Chapter 12 defines precipitation efficiency in the budgets of surface precipitation and cloud and rain microphysics and analyzes the relationship between these precipitation efficiencies and their connections to physical factors and processes.

Chapter 13 examines the small-scale effects of precipitation on the ocean mixing layer using simulation data from a coupled ocean–cloud-resolving atmosphere model. Chapter 14 discusses the effects of sea surface temperature, diurnal variation, and cloud radiative processes on equilibrium states. Chapter 15 addresses the application of cloud-resolving model simulation data to remote sensing with the aid of the microwave radiative transfer model. Chapter 16 studies precipitation predictability and its dependence on temporal and spatial-scale and large-scale forcing. Chapter 17 discusses improving the modeling of depositional growth of ice crystal using a modified parameterization scheme. Finally, Chap. 18 briefly discusses the simplification of prognostic microphysical schemes, coupled large-scale model with cloud-resolving model, and global cloud-resolving model.

We thank Drs. Yushu Zhou and Lingkun Ran at the Institute of Atmospheric Physics, Chinese Academy of Sciences, and Prof. Xinyong Shen at Nanjing University of Information Science and Technology for efficient and productive research collaborations, and Miss Di Li for editing this book. Xiaofan Li also thanks Zhejiang University for its support and his graduate students Wei Huang, Tingting Li, Lingyun Lou, Bin Wang, Jin Xin, Huiyan Xu, and Haoran Zhu for their assistance.

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Hangzhou, China — Xiaofan Li
Beijing, China — Shouting Gao
Spring 2016

Chapter 1
Model and Physics

Abstract In this chapter, the governing equations for cloud-resolving model are derived. The model is imposed by large-scale forcing due to a small model domain. The differences in large-scale forcing between vertical velocity and advection are compared. The large-scale forcing includes vertical velocity and zonal wind in this book. The model has prognostic equations for potential temperature, specific humidity, mixing ratios of five cloud species (cloud water, raindrop, cloud ice, snow, and graupel), and perturbation zonal wind and vertical velocity. The model physical package including cloud microphysical and radiation parameterization schemes and turbulence closure is presented. The model boundary conditions and basic parameters for model setup are discussed. The governing equations for coupled ocean-cloud resolving atmosphere model are presented. The ocean model includes mixed-layer model and ocean circulation model.

Keywords Cloud-resolving model • Governing equations • Cloud microphysics • Radiation • Turbulence closure • Parameterization • Coupled ocean-cloud resolving atmosphere model

Cloud-resolving models differ from general circulation and mesoscale models in two ways. First, cloud-resolving models cannot simulate large-scale circulations due to small model domains whereas general circulation and mesoscale models can simulate large-scale circulations. A large-scale forcing is imposed in the cloud-resolving model. Second, cloud-resolving models with fine spatial resolutions use prognostic cloud microphysical parameterization to simulate cloud and precipitation processes. In contrast, general circulation and mesoscale models use diagnostic cumulus parameterization and/or prognostic cloud microphysical parameterization due to coarse spatial resolutions. Many cloud-resolving models have been developed to study convective responses to the large-scale forcing (Table 1.1). In this chapter, the cloud-resolving model and coupled ocean-cloud resolving atmosphere model will be described in a two-dimensional (2D) framework in terms of governing equations, large-scale forcing, parameterization schemes of cloud microphysics, radiation, subgrid-scale turbulence closure, ocean mixing closure, and boundary conditions.

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Table 1.1 A summary of cloud-resolving models

Model	Dynamic core	Cloud scheme	Turbulence closures	Radiation scheme
Clark (1977, 1979), Clark and Hall (1991), and Clark et al. (1996)	Anelastic and hydrostatic approximations	Water cloud (Kessler 1969), ice cloud (Koenig and Murray 1976)	Smagorinsky (1963), Lilly (1962)	Kiehl et al. (1994)
Soong and Ogura (1980) and Tao and Simpson (1993)	Anelastic approximation and non-hydrostatic form	Water and ice schemes (Lin et al. 1983; Rutledge and Hobbs 1983, 1984; Tao et al. 1989; Krueger et al. 1995)	Klemp and Wilhelmson (1978)	Solar (Chou et al. 1998) and IR infrared (Chou et al. 1991; Chou and Suarez 1994) schemes
Lipps and Hemler (1986, 1988, 1991)	Anelastic and hydrostatic approximations	Water cloud (Kessler 1969)	Klemp and Wilhelmson (1978)	Newtonian damping
Redelsperger and Sommeria (1986)	Deardorff (1972); Sommeria (1976); Redelsperger and Sommeria (1981a, b, 1982)	Water cloud (Kessler 1969)	Subgrid-scale turbulence closure (Redelsperger and Sommeria (1981a, b, 1982)	–
Hu and He (1988), Hu et al. (1998), and Zou (1991)	Nonhydrostatic primitive dynamic core (Zou 1991)	Prognostic equations for hydrometeors and number concentrations (Hu and He 1988)	Second-moment turbulence closure	–
Kruegger (1988)	Anelastic and hydrostatic approximations	Three-phase bulk scheme (Lin et al. 1983; Lord et al. 1984; Krueger et al. 1995)	Third-moment turbulence closure (Krueger 1988)	Harshvardhan et al. (1987)
Model	Dynamic core	Cloud scheme	Turbulence closures	Radiation scheme
Nakajima and Matsuno (1988)	Yamasaki (1975) and Ogura and Phillps (1962)	Water cloud (Kessler 1969)	Velocity deformation and static stability dependent	Horizontal homogeneous cooling
Tripoli (1992)	Nonhydrostatic core with enstrophy conservation	Flatau et al. (1989)	A turbulent kinetic closure for eddy diffusion	Chen and Cotton (1983)
Xu (1992) and Xu and Huang (1994)	Nonhydrostatic primitive dynamic core	Xu and Duan (1999)	Second-moment turbulence closure	–
Ferrier (1994) and Ferrier et al. (1995)	Anelastic approximation and non-hydrostatic form	Double-moment multiple-phase four-class ice scheme (Ferrier 1994)	Klemp and Wilhelmson (1978)	Chou (1984, 1986) and Chou and Kouvaris (1991)
Tompkins and Craig (1998)	Anelastic quasi-Boussineq approximation	Three-phase ice scheme (Swann 1994; Brown and Swann 1997); the water vapor and cloud water are combined into the total-water mixing ratio	First-order subgrid-scale turbulence closure (Shutts and Gray 1994)	Two-stream plane-parallel approximation (Edwards and Slingo 1996; Petch 1998)
Satoh et al. (2005)	Non-hydrostatic formulations in the icosahedral grids covering the global domain	Grabowski (1998)	Mellor Yamada (1974) level-2 closure	Nakajima et al. (2000)

1.1 Governing Equations

The cloud-resolving model was originally developed by Soong and Ogura (1980) and Soong and Tao (1980). This model was significantly improved by Tao and Simpson (1993) at the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC) and was modified by Sui et al. (1994, 1998). The model was named the Goddard cumulus ensemble (GCE) model. The cloud-resolving model used in this book is the 2D modified version of GCE model. The non-hydrostatic governing equations with anelastic approximation can be expressed by

$$\nabla \cdot \mathbf{V} + \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} w = 0, \tag{1.1a}$$

$$\frac{\partial A}{\partial t} = -\nabla \cdot \mathbf{V} A - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} w A + S_A + D_A, \tag{1.1b}$$

$$\frac{\partial B}{\partial t} = -\nabla \cdot \mathbf{V} B - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} \left(w - w_{TB}\right) B + S_B + D_B. \tag{1.1c}$$

Here, $\mathbf{V}$ and w are horizontal wind vector and vertical velocity, respectively; $\overline{\rho}$ is a mean density, which is a function of height only; $A = (\theta, q_v, \mathbf{V}, \mathrm{w})$; $\mathrm{B} = (q_c, q_r, q_i, q_s, q_g)$, θ and q_v are potential temperature and specific humidity, respectively; q_c, q_r, q_i, q_s, q_g are the mixing ratios of cloud water (small cloud droplets), raindrops, cloud ice (small ice crystals), snow (density 0.1 g cm^{-3}), and graupel (density 0.4 g cm^{-3}), respectively; w_{TB} is a terminal velocity that is zero for cloud water and ice; S_A is a source and sink in momentum, temperature, and moisture equations such as pressure gradient force, buoyancy force, condensational heating, and radiative heating; the radiation parameterization schemes will be addressed in Sect. 1.3; S_B is a cloud source and sink that is determined by microphysical processes, which will be discussed in Sect. 1.2; D_A and D_B are dissipation terms related to sub-grid scale turbulence closure, which will be elucidated in Sect. 1.4.

For model calculations by Li et al. (1999), it is convenient to partition (A, $\mathbf{V}$) into area means ($\bar{A}$, $\overline{\mathbf{V}}$) and deviations (A', $\mathbf{V}'$), i.e.,

$$A = \overline{A} + A', \tag{1.2a}$$

$$\mathbf{V} = \overline{\mathbf{V}} + \mathbf{V}'. \tag{1.2b}$$

Applying (1.2) to (1.1b) leads to

$$\begin{aligned} \frac{\partial A}{\partial t} = & -\nabla \cdot \left(\mathbf{V}'A' + \mathbf{V}'\overline{A} + \overline{\mathbf{V}}A'\right) - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} \left(w'A' + w'\overline{A} + \overline{w}A'\right) \\ & + S_A + D_A - \overline{\mathbf{V}} \cdot \nabla \overline{A} - \overline{w} \frac{\partial}{\partial z} \overline{A}. \end{aligned} \tag{1.3}$$

Here, the area-mean continuity equation (1.1a) is used in the derivation of (1.3).

Taking an area mean over (1.3), we get the equation for $\overline{A}$,

$$\frac{\partial \overline{A}}{\partial t} = -\nabla \cdot \overline{\mathbf{V}'A'} - \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}\overline{w'A'} + \overline{S}_A + \overline{D}_A - \overline{\mathbf{V}} \cdot \nabla\overline{A} - \overline{w}\frac{\partial}{\partial z}\overline{A}. \tag{1.4a}$$

Perturbation equation for A' is obtained by subtracting (1.4a) from (1.3):

$$\frac{\partial A'}{\partial t} = -\nabla \cdot \left(\mathbf{V}'\overline{A} + \overline{\mathbf{V}}A'\right) - \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}\left(w'\overline{A} + \overline{w}A'\right) - \nabla \cdot \left(\mathbf{V}'A' - \overline{\mathbf{V}'A'}\right)$$
$$-\frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}\left(w'A' - \overline{w'A'}\right) + S_A - \overline{S}_A + D_A - \overline{D}_A. \tag{1.4b}$$

Environment has an important impact on convective development. When convection develops, associated momentum, heat, and moisture transport upward through convective activity, which in turn modify environment significantly. Environment and convection interact in a nonlinear way (e.g., Chao 1962). Due to a small domain in the cloud-resolving model (e.g., 768 km in a 2D framework), the large-scale circulation cannot be simulated. Thus, the large-scale forcing needs to be imposed in the cloud-resolving model. Soong and Ogura (1980) were the first to develop ways to impose the observed large-scale variables into the cloud-resolving model to examine the convective response to the imposed large-scale forcing. The major forcing is vertical velocity and associated vertical advections. Thus, there are two ways to impose the large-scale forcing into the cloud model. The horizontally uniform and vertically varying vertical velocity can be imposed, as first introduced by Soong and Ogura (1980), or the horizontally uniform total advection of the heat and moisture can be imposed (e.g., Wu et al. 1998). Li et al. (1999) discussed the two model setups intensively.

For the model with the imposed vertical velocity ($\overline{w}^o$), horizontal wind ($\overline{\mathbf{V}}^o$) and the horizontal advection ($-\overline{\mathbf{V}}^o \cdot \nabla\overline{A}^o$) are also imposed. These forcing data denoted by subscript "^{o}" are calculated from the observational data [e.g., Tropical Ocean Global Atmosphere Coupled Ocean–atmosphere Response Experiment (TOGA COARE) in Li et al. (1999) and Global Atmosphere Research Program (GARP) Atlantic Tropical Experiment (GATE) in Grabowski et al. (1996)]. With the assumption that $-\mathbf{V}' \cdot \nabla\overline{A}^o = 0$, the model equations for potential temperature and specific humidity can be expressed by

$$\frac{\partial A}{\partial t} = -\nabla \cdot \mathbf{V}'A' - \overline{\mathbf{V}}^o \cdot \nabla A' - \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}w'A' - \overline{w}^o\frac{\partial}{\partial z}A' - w'\frac{\partial}{\partial z}\overline{A}$$
$$+S_A + D_A - \overline{\mathbf{V}}^o \cdot \nabla\overline{A}^o - \overline{w}^o\frac{\partial}{\partial z}\overline{A}. \tag{1.5}$$

The model consists of (1.1a) and (1.1c), perturbation momentum equation (1.4b), and equations for potential temperature and specific humidity (1.5).

For the model with imposed horizontally uniform total advection of the heat and moisture, horizontal wind is also imposed. With the assumption that

$-\nabla \cdot \overline{\mathbf{V}}^o A' - \partial \overline{\rho} A' \overline{w}^o / \overline{\rho} \partial z = 0$, the model equations for potential temperature and specific humidity can be written by

$$\frac{\partial A}{\partial t} = -\nabla \cdot \mathbf{V}' A - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} w' A + S_A + D_A - \overline{\mathbf{V}}^o \cdot \nabla \overline{A}^o - \overline{w}^o \frac{\partial}{\partial z} \overline{A}^o. \tag{1.6}$$

The model is comprised of (1.1a) and (1.1c), perturbation momentum equation (1.4b), and equations for potential temperature and specific humidity (1.6). Li et al. (1999) found that the terms omitted in (1.5) and (1.6) do not have any impact on the model simulations. The comparison between simulations by the two model setups will be discussed with the TOGA COARE data in chapter three.

The governing equations in the 2D cloud-resolving model can be expressed as follows:

$$\frac{\partial u'}{\partial x} + \frac{1}{\overline{\rho}} \frac{\partial (\overline{\rho} w')}{\partial z} = 0, \tag{1.7a}$$

$$\begin{aligned} \frac{\partial u'}{\partial t} = & -\frac{\partial}{\partial x} \left(2u'\overline{u}^o + u'u'\right) - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} \left(w'\overline{u}^o + \overline{w}^o u' + w'u' - \overline{w'u'}\right) \\ & -c_p \frac{\partial \left(\overline{\theta} \pi'\right)}{\partial x} + D_u - \overline{D}_u, \end{aligned} \tag{1.7b}$$

$$\begin{aligned} \frac{\partial w'}{\partial t} = & -\frac{\partial}{\partial x} \left(u'\overline{w}^o + \overline{u}^o w' + u'w'\right) - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} \left(2w'\overline{w}^o + w'w' - \overline{w'w'}\right) \\ & -c_p \frac{\partial \left(\overline{\theta} \pi'\right)}{\partial z} + g \left(\frac{\theta'}{\theta_o} + 0.61 q_v' - q_l'\right) + D_w - \overline{D}_w, \end{aligned} \tag{1.7c}$$

$$\begin{aligned} \frac{\partial \theta}{\partial t} = & -\frac{\partial (u'\theta')}{\partial x} - \overline{u}^o \frac{\partial \theta'}{\partial x} - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \left(\overline{\rho} w' \theta'\right) - \overline{w}^o \frac{\partial \theta'}{\partial z} - w' \frac{\partial \overline{\theta}}{\partial z} \\ & + \frac{Q_{cn}}{\pi c_p} + \frac{Q_R}{\pi c_p} - \overline{u}^o \frac{\partial \overline{\theta}^o}{\partial x} - \overline{w}^o \frac{\partial \overline{\theta}}{\partial z} + D_\theta, \end{aligned} \tag{1.7d}$$

$$\begin{aligned} \frac{\partial q_v}{\partial t} = & -\frac{\partial \left(u' q_v'\right)}{\partial x} - \overline{u}^o \frac{\partial q_v'}{\partial x} - \overline{w}^o \frac{\partial q_v'}{\partial z} - w' \frac{\partial \overline{q}_v}{\partial z} - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} w' q_v' \\ & -S_{qv} - \overline{u}^o \frac{\partial \overline{q}_v{}^o}{\partial x} - \overline{w}^o \frac{\partial \overline{q}_v}{\partial z} + D_{qv}, \end{aligned} \tag{1.7e}$$

$$\frac{\partial q_c}{\partial t} = -\frac{\partial (u q_c)}{\partial x} - \frac{1}{\overline{\rho}} \frac{\partial (\overline{\rho} w q_c)}{\partial z} + S_{qc} + D_{qc}, \tag{1.7f}$$

$$\frac{\partial q_r}{\partial t} = -\frac{\partial (u q_r)}{\partial x} - \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} \overline{\rho} \left(w - w_{Tr}\right) q_r + S_{qr} + D_{qr}, \tag{1.7g}$$

$$\frac{\partial q_i}{\partial t} = -\frac{\partial (u q_i)}{\partial x} - \frac{1}{\overline{\rho}} \frac{\partial (\overline{\rho} w q_i)}{\partial z} + S_{qi} + D_{qi}, \tag{1.7h}$$

$$\frac{\partial q_s}{\partial t} = -\frac{\partial (uq_s)}{\partial x} - \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}(w - w_{Ts}) q_s + S_{qs} + D_{qs}, \tag{1.7i}$$

$$\frac{\partial q_g}{\partial t} = -\frac{\partial (uq_g)}{\partial x} - \frac{1}{\overline{\rho}}\frac{\partial}{\partial z}\overline{\rho}(w - w_{Tg}) q_g + S_{qg} + D_{qg}, \tag{1.7j}$$

where

$$\begin{aligned} Q_{cn} = {} & L_v (P_{CND} - P_{REVP}) + L_s\Big\{P_{DEP} + (1-\delta_1) P_{SDEP}(T<T_o) \\ & + (1-\delta_1) P_{GDEP}(T<T_o) - P_{MLTS}(T>T_o) - P_{MLTG}(T>T_o)\Big\} \\ & + L_f\Big\{P_{SACW}(T<T_o) + P_{SFW}(T<T_o) + P_{GACW}(T<T_o) \\ & + P_{IACR}(T<T_o) + P_{GACR}(T<T_o) + P_{SACR}(T<T_o) \\ & + P_{GFR}(T<T_o) - P_{RACS}(T>T_o) - P_{SMLT}(T>T_o) \\ & - P_{GMLT}(T>T_o) + P_{IHOM}(T<T_{oo}) - P_{IMLT}(T>T_o) \\ & + P_{IDW}(T_{oo}<T<T_o)\Big\}, \end{aligned} \tag{1.8a}$$

$$\begin{aligned} S_{qv} = {} & P_{CND} - P_{REVP} + P_{DEP} + (1-\delta_1) P_{SDEP}(T<T_o) \\ & + (1-\delta_1) P_{GDEP}(T<T_o) - P_{MLTS}(T>T_o) \\ & - P_{MLTG}(T>T_o), \end{aligned} \tag{1.8b}$$

$$\begin{aligned} S_{qc} = {} & -P_{SACW} - P_{RAUT} - P_{RACW} - P_{SFW}(T<T_o) - P_{GACW} \\ & + P_{CND} - P_{IHOM}(T<T_{oo}) + P_{IMLT}(T>T_o) \\ & - P_{IDW}(T_{oo}<T<T_o), \end{aligned} \tag{1.8c}$$

$$\begin{aligned} S_{qr} = {} & P_{SACW}(T>T_o) + P_{RAUT} + P_{RACW} + P_{GACW}(T>T_o) \\ & - P_{REVP} + P_{RACS}(T>T_o) - P_{IACR}(T<T_o) - P_{GACR}(T<T_o) \\ & - P_{SACR}(T<T_o) - P_{GFR}(T<T_o) + P_{SMLT}(T>T_o) \\ & + P_{GMLT}(T>T_o), \end{aligned} \tag{1.8d}$$

$$\begin{aligned} S_{qi} = {} & -P_{SAUT}(T<T_o) - P_{SACI}(T<T_o) - P_{RACI}(T<T_o) \\ & - P_{SFI}(T<T_o) - P_{GACI}(T<T_o) + P_{IHOM}(T<T_{oo}) \\ & - P_{IMLT}(T>T_o) + P_{IDW}(T_{oo}<T<T_o) + P_{DEP}, \end{aligned} \tag{1.8e}$$

$$\begin{aligned} S_{qs} = {} & P_{SAUT}(T<T_o) + P_{SACI}(T<T_o) + \delta_4 P_{SACW}(T<T_o) \\ & + P_{SFW}(T<T_o) + P_{SFI}(T<T_o) + \delta_3 P_{RACI}(T<T_o) \\ & - P_{RACS}(T>T_o) - P_{GACS} - P_{SMLT}(T>T_o) \\ & - (1-\delta_2) P_{RACS}(T<T_o) + \delta_2 P_{SACR}(T<T_o) \\ & + (1-\delta_1) P_{SDEP}(T<T_o) - P_{MLTS}(T>T_o) \\ & + \delta_3 P_{IACR}(T<T_o) - (1-\delta_4) P_{WACS}(T<T_o), \end{aligned} \tag{1.8f}$$

$$
\begin{aligned}
S_{qg} = & + (1 - \delta_3) P_{RACI} (T < T_o) + P_{GACI} (T < T_o) \\
& + P_{GACW} (T < T_o) + (1 - \delta_4) P_{SACW} (T < T_o) + P_{GACS} \\
& + (1 - \delta_3) P_{IACR} (T < T_o) + P_{GACR} (T < T_o) \\
& + (1 - \delta_2) P_{RACS} (T < T_o) + P_{GFR} (T < T_o) \\
& + (1 - \delta_4) P_{WACS} (T < T_o) - P_{GMLT} (T > T_o) \\
& + (1 - \delta_1) P_{GDEP} (T < T_o) - P_{MLTG} (T > T_o) \\
& + (1 - \delta_2) P_{SACR} (T < T_o),
\end{aligned} \tag{1.8g}
$$

and

$$\delta_1 = 1, \text{ only if } q_c + q_i > 10^{-8}\text{gg}^{-1}, T < T_o, \tag{1.8h}$$

$$\delta_2 = 1, \text{ only if } q_s + q_r < 10^{-4}\text{gg}^{-1}, T < T_o, \tag{1.8i}$$

$$\delta_3 = 1, \text{ only if } q_r > 10^{-4}\text{gg}^{-1}, T < T_o, \tag{1.8j}$$

$$\delta_4 = 1, \text{ only if } q_s \le 10^{-4}\text{gg}^{-1}, q_c > 5 \times 10^{-4}\text{gg}^{-1}, T < T_o. \tag{1.8k}$$

Here, u is zonal wind; θ_o is initial potential temperature; $q_l = q_c + q_r + q_i + q_s + q_g$; T is air temperature, and $T_o = 0$ °C, $T_{oo} = -35$ °C; $\pi = (p/p_o)^\kappa$, $\kappa = R/c_p$; R is the gas constant; c_p is the specific heat of dry air at constant pressure p, and $p_o = 1000$ mb; L_v, L_s, and L_f are latent heat of vaporization, sublimation, and fusion at 0 °C, respectively, and $L_s = L_v + L_f$; Q_R is the radiative heating rate due to convergence of the net flux of solar and infrared radiative fluxes, which will be discussed in Sect. 1.3. The cloud microphysical terms in (1.8b, 1.8c, 1.8d, 1.8e, 1.8f, and 1.8g) are defined in Table 1.2, which will be discussed in Sect. 1.2. The cloud microphysical processes are also summarized in Fig. 1.1.

When the model is integrated over the ocean, a time-invariant or temporally varied horizontally uniform sea surface temperature (*SST*) is imposed in both model setups. Li et al. (2000) developed a coupled ocean-cloud resolving atmosphere model to study the impacts of precipitation and associated salinity stratification in ocean mixed-layer temperature and salinity at small spatial scales. An embedded mixed layer-ocean circulation model was originally developed by Adamec et al. (1981). The mixed-layer equations in the 2D framework are

$$\frac{\partial h_m}{\partial t} = -\frac{\partial u_m h_m}{\partial x} + W_e, \tag{1.9a}$$

$$\frac{\partial u_m}{\partial t} = -\frac{W_e}{h_m} H(W_e)(u_m - u_e) - \frac{\tau_o}{\rho_r h_m}, \tag{1.9b}$$

$$\frac{\partial T_m}{\partial t} = -\frac{W_e}{h_m} H(W_e)(T_m - T_e) - \frac{Q_o + I(0) - I(h_m)}{\rho_r c_w h_m}, \tag{1.9c}$$

Table 1.2 List of microphysical processes and their parameterization schemes

Notation	Description	Scheme
P_{MLTG}	Growth of vapor by evaporation of liquid from graupel surface	RH84
P_{MLTS}	Growth of vapor by evaporation of melting snow	RH83
P_{REVP}	Growth of vapor by evaporation of raindrops	RH83
P_{IMLT}	Growth of cloud water by melting of cloud ice	RH83
P_{CND}	Growth of cloud water by condensation of supersaturated vapor	TSM
P_{GMLT}	Growth of raindrops by melting of graupel	RH84
P_{SMLT}	Growth of raindrops by melting of snow	RH83
P_{RACI}	Growth of raindrops by the accretion of cloud ice	RH84
P_{RACW}	Growth of raindrops by the collection of cloud water	RH83
P_{RACS}	Growth of raindrops by the accretion of snow	RH84
P_{RAUT}	Growth of raindrops by the autoconversion of cloud water	LFO
P_{IDW}	Depositional growth of cloud ice from cloud water	KFLC
P_{IACR}	Growth of cloud ice by the accretion of rain	RH84
P_{IHOM}	Growth of cloud ice by the homogeneous freezing of cloud water	
P_{DEP}	Growth of cloud ice by the deposition of supersaturated vapor	TSM
P_{SAUT}	Growth of snow by the conversion of cloud ice	RH83
P_{SACI}	Growth of snow by the collection of cloud ice	RH83
P_{SACW}	Growth of snow by the accretion of cloud water	RH83
P_{SFW}	Growth of snow by the deposition of cloud water	KFLC
P_{SFI}	Depositional growth of snow from cloud ice	KFLC
P_{SACR}	Growth of snow by the accretion of raindrops	LFO
P_{SDEP}	Growth of snow by the deposition of vapor	RH83
P_{GACI}	Growth of graupel by the collection of cloud ice	RH84
P_{GACR}	Growth of graupel by the accretion of raindrops	RH84
P_{GACS}	Growth of graupel by the accretion of snow	RH84
P_{GACW}	Growth of graupel by the accretion of cloud water	RH84
P_{WACS}	Growth of graupel by the riming of snow	RH84
P_{GDEP}	Growth of graupel by the deposition of vapor	RH84
P_{GFR}	Growth of graupel by the freezing of raindrops	LFO

The schemes are Rutledge and Hobbs (1983; 1984; RH83, RH84), Lin et al. (1983, LFO), Tao et al. (1989, TSM), and Krueger et al. (1995, KFLC)

$$\frac{\partial S_m}{\partial t} = -\frac{W_e}{h_m} H\left(W_e\right)\left(S_m - S_e\right) - \frac{S_m\left(P_s - E_s\right)}{\rho_r c_w h_m}. \tag{1.9d}$$

Here, T_m, S_m, u_m, and h_m are ocean mixed-layer temperature, salinity, zonal current, and depth respectively; T_e and S_e are temperature and salinity of the level just beneath the mixed layer, respectively; H is the heavyside step function in which $H = 1$ as $W_e > 0$ while $H = 0$ as $W_e < 0$; ρ_r is a constant reference sea water density; c_w is the heat capacity of water; $I = I_o\left[re^{-\gamma_1 z} + (1 - r)\, e^{-\gamma_2 z}\right]$, I_o is solar radiation at the ocean surface, γ_1, γ_2 are attenuation parameters for solar radiation penetration,

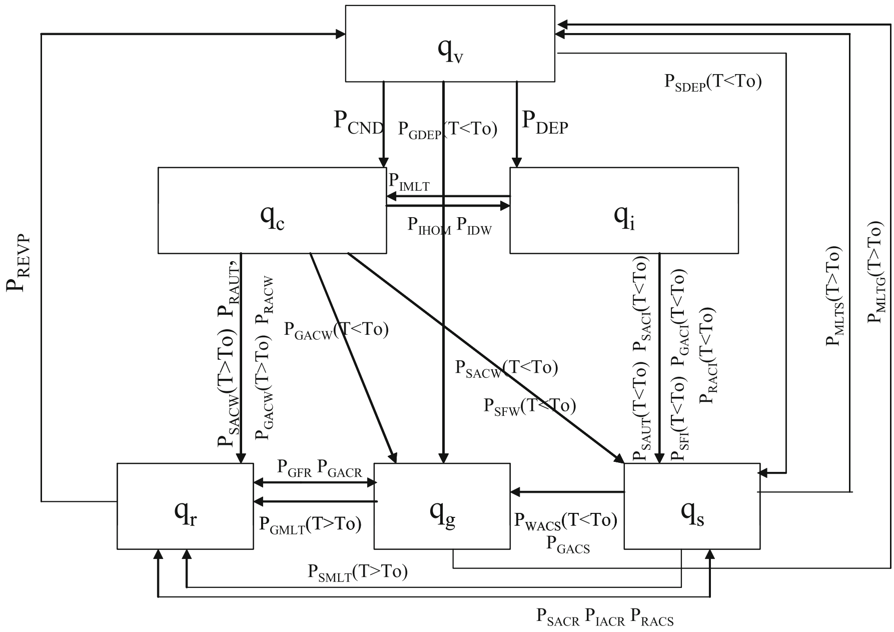

Fig. 1.1 A flow chart of the cloud microphysical schemes

and z is positive downward with $z = 0$ being the ocean surface; Q_o is the sum of longwave radiation, sensible and latent heat at the ocean surface; P_s and E_s denote the rates of precipitation and evaporation at the ocean surface, respectively; W_e is the entrainment velocity at the mixed-layer base, which can be obtained by calculating Kraus-Turner's equation that was originally derived by Niller and Kraus (1977), modified by Sui et al. (1997), and is similar to Garpar (1988),

$$\begin{aligned} & W_e H(W_e)\, h_m g\left[\alpha\left(T_m - T_e\right) - \beta\left(S_m - S_e\right)\right] \\ & \quad = 2 m_s {u_*}^3 - \tfrac{h_m}{2}\left[\left(1 + m_b\right) B_o + \left(1 - m_b\right)\left|B_o\right|\right]. \end{aligned} \tag{1.10}$$

$$\begin{aligned} B_o = \frac{g\alpha}{\rho_r c_w} \{ Q_o + [\, 1 + r e^{-\gamma_1 h_m} + (1 - r)\, e^{-\gamma_2 h_m} - \frac{2r}{\gamma_1 h_m}\left(1 - e^{-\gamma_1 h_m}\right) \\ - \frac{2(1 - r)}{\gamma_2 h_m}\left(1 - e^{-\gamma_2 h_m}\right)] I_o \} + \frac{g}{\rho_r} \beta S_m \left(P_s - E_s\right), \end{aligned} \tag{1.10a}$$

where u_* is a surface friction velocity; α and β describe the logarithmic expansion of ocean water density ρ_r as a function of temperature and salinity, respectively; g is the gravitational acceleration; m_s and m_b are turbulent mixing factors due to wind stirring and convection, respectively.

The 2D model equation for ocean circulations on the equator can be written as

$$\frac{\partial u_1}{\partial t} = -\frac{\partial u_1 u_1}{\partial x} - \frac{\partial w_1 u_1}{\partial z} + A_M \frac{\partial^2 u_1}{\partial x^2} + K_M \frac{\partial^2 u_1}{\partial z^2}, \tag{1.11a}$$

$$\frac{\partial T_1}{\partial t} = -\frac{\partial u_1 T_1}{\partial x} - \frac{\partial w_1 T_1}{\partial z} + A_T \frac{\partial^2 T_1}{\partial x^2} + K_T \frac{\partial^2 T_1}{\partial z^2} - \frac{1}{\rho_r c_w}\frac{\partial I}{\partial z}, \tag{1.11b}$$

$$\frac{\partial S_1}{\partial t} = -\frac{\partial u_1 S_1}{\partial x} - \frac{\partial w_1 S_1}{\partial z} + A_S \frac{\partial^2 S_1}{\partial x^2} + K_S \frac{\partial^2 S_1}{\partial z^2}, \tag{1.11c}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} = 0, \tag{1.11d}$$

$$\frac{\partial p_1}{\partial z} = -\rho_1 g, \tag{1.11e}$$

$$\rho_1 = \rho_r \left[1 - \alpha\left(T_1 - T_r\right) - \beta\left(S_1 - S_r\right)\right], \tag{1.11f}$$

where u_1 and w_1 are zonal and vertical components of ocean current, respectively; T_1 and S_1 are ocean temperature and salinity, respectively; A_M, A_T, and A_S are horizontal momentum, heat and salinity diffusivity coefficients, respectively; K_M, K_T, and K_S are vertical momentum, heat and salinity diffusivity coefficients, respectively; T_r and S_r are the reference temperature and salinity, respectively; the mixed-layer model and the ocean circulation model communicate with each other through the embedding technique proposed by Adamec et al. (1981). The model also includes a convective adjustment scheme that ensures the static stability of the upper ocean.

1.2 Cloud Microphysical Parameterization Schemes

The formulations of cloud microphysical parameterization schemes are documented in this section. Table 1.2 shows the list of microphysical processes and their parameterization schemes. The schemes are Rutledge and Hobbs (1983, 1984), Lin et al. (1983), Tao et al. (1989), and Krueger et al. (1995).

$$P_{MLTG} = \frac{2\pi N_{0G}\left(S-1\right)}{\rho\left(A' + B'\right)}\left[\frac{0.78}{{\lambda_G}^2} + 0.31\left(\frac{\overline{a}\rho}{\mu}\right)^{\frac{1}{2}}\left(\frac{\rho_o}{\rho}\right)^{\frac{1}{4}}\frac{\Gamma\left(\frac{\overline{b}+5}{2}\right)}{{\lambda_G}^{\frac{\overline{b}+5}{2}}}\right], \tag{1.12}$$

where N_{0G} ($=4\times10^6$ m^{-4}) is the intercept value in graupel size distribution; S ($=q_w/q_{ws}$), where q_{ws} is the saturated mixing ratio with respect to water; $\overline{a}\left(= 19.3\ \mathrm{m}^{1-\overline{b}}\mathrm{s}^{-1}\right)$ is the constant in fall-speed relation for graupel; $\overline{b}\,(= 0.37)$

is the fall-speed exponent for graupel; $A' = L_v/K_aT\ (L_vM_w/RT - 1)$; $B' = RT/\chi M_w e_{ws}$ (Pruppacher and Klett 1978); K_a (=2.43 × 10^{-2} J m^{-1} s^{-1} K^{-1}) is the thermal conductivity coefficient of air; M_w (=18.016) is the molecular weight of water; $\chi\left(= 2.26 \times 10^{-5}\mathrm{m^2 s^{-1}}\right)$ is the diffusivity coefficient of water vapor in air; $R\left(= 8.314 \times 10^3 \mathrm{Jkmol^{-1}K^{-1}}\right)$ is the universal gas constant; e_{ws} is the saturation vapor pressure for water; $\lambda_G\left[= \left(\pi\rho_G N_{0G}/\rho q_g\right)^{\frac{1}{4}}\right]$ is the slope of graupel size distribution; $\rho_G\left(= 400\ \mathrm{kgm^{-3}}\right)$ is the density of graupel; $\mu\left(= 1.718 \times 10^{-5}\mathrm{kgm^{-1}s^{-1}}\right)$ is the dynamic viscosity of air; Γ is the Gamma function.

$$P_{MLTS} = \frac{4N_{0S}\,(S-1)}{\rho\,(A'+B')}\left[\frac{0.65}{{\lambda_S}^2} + 0.44\left(\frac{a''\rho}{\mu}\right)^2\left(\frac{\rho_o}{\rho}\right)^{\frac{1}{4}}\frac{\Gamma\left(\frac{b+5}{2}\right)}{{\lambda_S}^{\frac{b+5}{2}}}\right], \qquad (1.13)$$

where $N_{0S}\left(= 4 \times 10^6\mathrm{m^{-4}}\right)$ is the intercept value in snowflake size distribution; $a''\left(= 1.139\ \mathrm{m^{1-b}s^{-1}}\right)$ is the constant in fall-speed relation for snow; $b\,(= 0.11)$ is the fall-speed exponent for snow; $\lambda_S\left[= (\pi\rho_S N_{0S}/\rho q_s)^{\frac{1}{4}}\right]$ is the slope of snow size distribution; $\rho_S\left(= 100\ \mathrm{kgm^{-3}}\right)$ is the density of snow.

$$P_{REVP} = \frac{2\pi N_{0R}\,(S-1)}{\rho\,(A'+B')}\left[\frac{0.78}{{\lambda_R}^2} + 0.31\left(\frac{a'\rho}{\mu}\right)^{\frac{1}{2}}\left(\frac{\rho_o}{\rho}\right)^{\frac{1}{4}}\frac{\Gamma(3)}{{\lambda_R}^3}\right], \qquad (1.14)$$

where $N_{0R}\left(= 8 \times 10^6\mathrm{m^{-4}}\right)$ is the intercept value in raindrop size distribution; $a'\left(= 3 \times 10^3\mathrm{s^{-1}}\right)$ is the constant in linear fall-speed relation for raindrops; $\lambda_R\left[= (\pi\rho_L N_{0R}/\rho q_r)^{\frac{1}{4}}\right]$ is the slope of raindrop size distribution; $\rho_L\left(= 10^3\ \mathrm{kgm^{-3}}\right)$ is the density of raindrops.

$$P_{IMLT} = \frac{q_i}{\Delta t}, \qquad (1.15)$$

where Δt is the time step.

$$P_{CND} = \frac{1}{\Delta t}\frac{T - T_{00}}{T_0 - T_{00}}\frac{q_v - \left(q_{qws} + q_{is}\right)}{1 + \left(\frac{A_1 q_c q_{ws} + A_2 q_i q_{is}}{q_c + q_i}\right)\left(\frac{L_v(T - T_{00}) + L_s(T_0 - T)}{c_p(T_0 - T_{00})}\right)}, \qquad (1.16)$$

where q_{is} is the saturation mixing ratio with respect to ice; $A_1 = 237.3B_1/(T\text{-}35.86)^2$; $A_2 = 265.5B_2/(T\text{-}7.66)^2$; $B_1 = 17.2693882$; $B_2 = 21.8745584$.

$$P_{GMLT} = -\frac{2\pi}{\rho L_f}K_a\,(T - T_o)\,N_{0G}\left[\frac{0.78}{{\lambda_G}^2} + 0.31\left(\frac{\overline{a}\rho}{\mu}\right)^{\frac{1}{2}}\left(\frac{\rho_o}{\rho}\right)^{\frac{1}{4}}\frac{\Gamma\left(\frac{\overline{b}+5}{2}\right)}{{\lambda_G}^{\frac{\overline{b}+5}{2}}}\right], \qquad (1.17)$$

$$P_{SMLT} = -\frac{2\pi}{\rho L_f} K_a (T - T_o) N_{0S} \left[\frac{0.65}{{\lambda_S}^2} + 0.44 \left(\frac{a'' \rho}{\mu} \right)^{\frac{1}{2}} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{4}} \frac{\Gamma\left(\frac{b+5}{2}\right)}{{\lambda_S}^{\frac{b+5}{2}}} \right], \quad (1.18)$$

$$P_{RACI} = \frac{\pi}{4} q_i E_{RI} N_{0R} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \left[\frac{a_o \Gamma(3)}{{\lambda_R}^3} + \frac{a_1 \Gamma(4)}{{\lambda_R}^4} + \frac{a_2 \Gamma(5)}{{\lambda_R}^5} + \frac{a_3 \Gamma(6)}{{\lambda_R}^6} \right], \quad (1.19)$$

where E_{RI} (=1) is the rain/cloud ice collection efficiency coefficient; $a_0 = 0.267\ \mathrm{m\,s^{-1}}$, $a_1 = 5.15 \times 10^3\ \mathrm{s^{-1}}$, $a_2 = -1.0225 \times 10^6\ \mathrm{m^{-1}\,s^{-1}}$, $a_3 = 7.55 \times 10^7\ \mathrm{m^{-2}}$ $\mathrm{s^{-1}}$, which are the coefficients in polynomial fall-speed relation for raindrops.

$$P_{RACW} = \frac{\pi}{4} q_c E_{RC} N_{0R} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \left[\frac{a_o \Gamma(3)}{{\lambda_R}^3} + \frac{a_1 \Gamma(4)}{{\lambda_R}^4} + \frac{a_2 \Gamma(5)}{{\lambda_R}^5} + \frac{a_3 \Gamma(6)}{{\lambda_R}^6} \right], \quad (1.20)$$

where E_{RC} (=1) is the rain/cloud water collection efficiency coefficient.

$$P_{RACS} = E_{SR} \pi^2 \frac{\rho_s}{\rho} \left| \overline{V}_R - \overline{V}_S \right| N_{0R} N_{0S} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \left[\frac{5}{{\lambda_S}^6 \lambda_R} + \frac{2}{{\lambda_S}^5 {\lambda_R}^2} + \frac{1}{2{\lambda_S}^4 {\lambda_R}^3} \right], \quad (1.21)$$

where E_{SR} (=1) is the snow/rain collection efficiency coefficient;

$$\overline{V}_R = \left(-0.267 + \frac{206}{\lambda_R} - \frac{2.045 \times 10^3}{{\lambda_R}^2} + \frac{9.06 \times 10^3}{{\lambda_R}^3} \right) \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}}, \quad (1.21a)$$

$$\overline{V}_S = a'' \frac{\Gamma(4+b)}{6{\lambda_S}^b} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}}, \quad (1.21b)$$

where $\overline{V}_R$ and $\overline{V}_S$ are the mass-weighted fall-speed for rain and snow, respectively.

$$P_{RAUT} = \alpha (q_c - q_o), \quad (1.22)$$

where α (=10^{-3} $\mathrm{s^{-1}}$) is the rate coefficient for auto-conversion; q_o(= $1.25 \times 10^{-3}\mathrm{gg^{-1}}$) is the mixing ratio threshold.

$$P_{IDW} = \frac{n_0 e^{\frac{1}{2}|T-T_o|}}{10^3 \rho} b_1 \left(\frac{\rho q_i}{n_0 e^{\frac{1}{2}|T-T_o|}} \right)^{b_2}, \quad (1.23)$$

where $n_0 = 10^{-8}\mathrm{m^{-3}}$; b_1 and b_2 are the positive temperature-dependent coefficients tabulated by Koenig (1971).

$$P_{IACR} = n_{ci} E_{RI} \frac{\pi^2}{24} \frac{\rho_L}{\rho} N_{0R} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \left[\frac{a_o \Gamma(6)}{{\lambda_R}^6} + \frac{a_1 \Gamma(7)}{{\lambda_R}^7} + \frac{a_2 \Gamma(8)}{{\lambda_R}^8} + \frac{a_3 \Gamma(9)}{{\lambda_R}^9} \right], \tag{1.24}$$

where $n_{ci} \left(= \rho q_i / \overline{M}_i\right)$ is the number concentration of cloud ice crystals; $\overline{M}_i \left(= 6 \times 10^{-12} \text{kg}\right)$ is the average mass of a cloud ice particle.

$$P_{IHOM} = \frac{q_c}{\Delta t}, \tag{1.25}$$

$$P_{DEP} = \frac{1}{\Delta t} \frac{T_0 - T}{T_0 - T_{00}} \frac{q_v - \left(q_{qws} + q_{is}\right)}{1 + \left(\frac{A_1 q_c q_{ws} + A_2 q_i q_{is}}{q_c + q_i} \right) \left(\frac{L_v (T - T_{00}) + L_s (T_0 - T)}{c_p (T_0 - T_{00})} \right)}, \tag{1.26}$$

$$P_{SAUT} = \frac{\rho q_i - M_{\max} n_0 e^{0.6(T - T_o)}}{\rho \Delta t}, \tag{1.27}$$

where $M_{\max} \left(= 9.4 \times 10^{-10} \text{kg}\right)$ is the maximum allowed crystal mass.

$$P_{SACI} = \frac{\pi a'' q_i E_{SI} N_{0S}}{4} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \frac{\Gamma\,(b+3)}{{\lambda_S}^{b+3}}, \tag{1.28}$$

where E_{SI} (=0.1) is the snow/cloud ice collection efficiency coefficient.

$$P_{SACW} = \frac{\pi a'' q_c E_{SC} N_{0S}}{4} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \frac{\Gamma\,(b+3)}{{\lambda_S}^{b+3}}, \tag{1.29}$$

where E_{SC} (=1) is the snow/cloud water collection efficiency coefficient.

$$P_{SFW} = \frac{q_i \Delta t}{m_{ir} \Delta t_1} \left(b_1 {m_{ir}}^{b_2} + \pi \rho q_c r^2 U_{ir} \right), \tag{1.30}$$

where $\Delta t_1 \left[= \left({m_{ir}}^{1-b_2} - {m_{iro}}^{1-b_2}\right) / b_1 \left(1 - b_2\right)\right]$ is the timescale needed for a crystal to grow from radius r_o to radius r; $m_{ir} \left(= 3.84 \times 10^{-9} \text{kg}\right)$ and U_{ir} (=1 m s^{-1}) are the mass and terminal velocity of an ice crystal r (=10^2 μm); $m_{ir} \left(= 2.46 \times 10^{-10} \text{kg}\right)$ is the mass of an ice crystal r_o (=40 μm).

$$P_{SFI} = \frac{q_i}{\Delta t_1}, \tag{1.31}$$

$$P_{SACR} = E_{SR} \pi^2 \frac{\rho_L}{\rho} \left| \overline{V}_S - \overline{V}_R \right| N_{0R} N_{0S} \left(\frac{\rho_o}{\rho} \right)^{\frac{1}{2}} \left[\frac{5}{{\lambda_R}^6 \lambda_S} + \frac{2}{{\lambda_R}^5 {\lambda_S}^2} + \frac{1}{2 {\lambda_R}^4 {\lambda_S}^3} \right], \tag{1.32}$$