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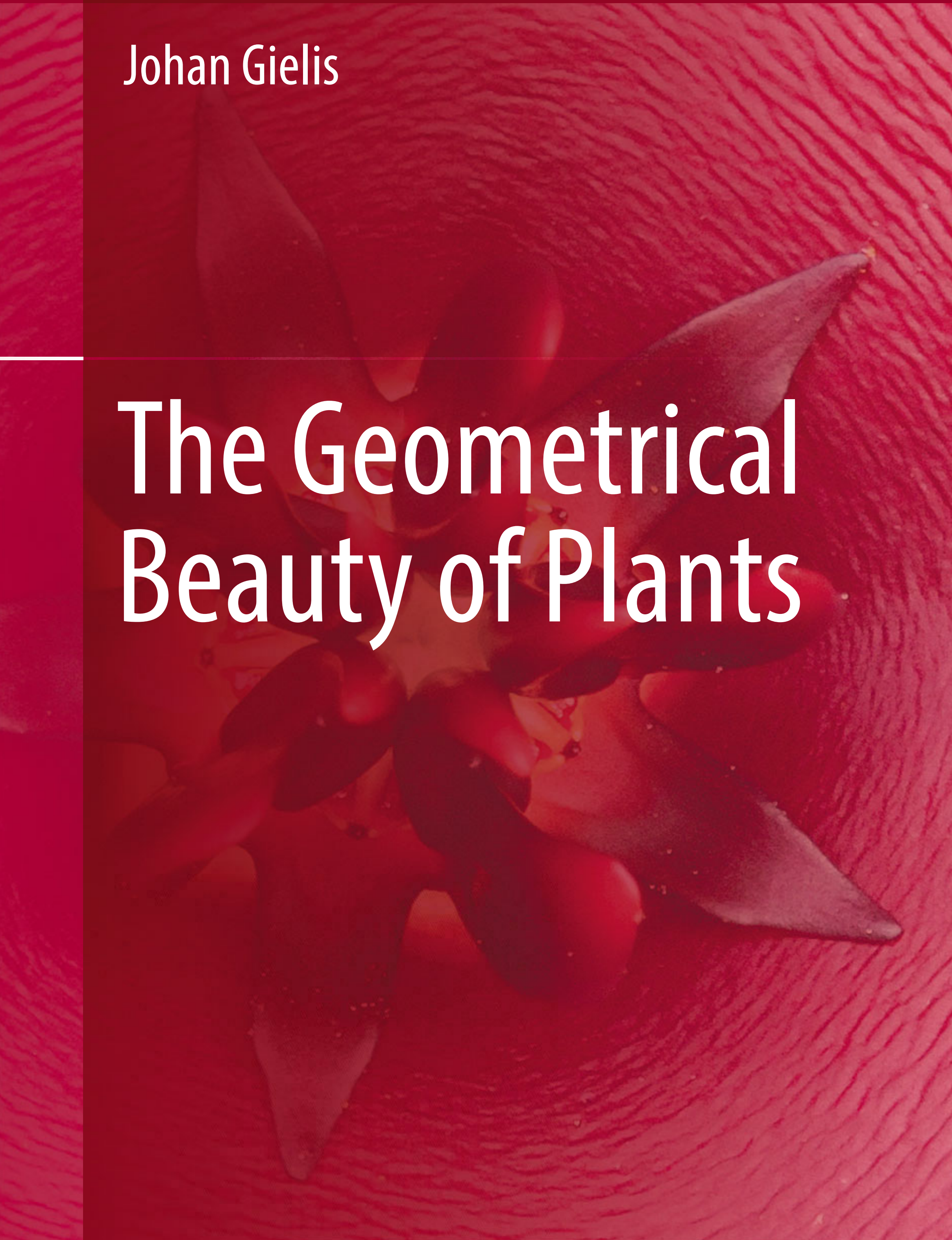

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An equation has no meaning for me if it does not express a thought of God.

Ramanujan

"One must study not what is interesting and curious, but what is important and essential".

Pafnuty Lvovich Chebyshev's advice to his students

A special dedication to Walter Liese and Tom Gerats

Preface

In my study of natural shapes, more specifically of bamboo, I started using the superellipses and supercircles of Gabriel Lamé around 1994 to study the shape of certain square bamboos. The first publication was in the Belgian Bamboo Society Newsletter in 1996 followed by a presentation by Prof. Freddy Van Oystaeyen in the same year at a meeting at the University of Louvain organized by the Belgian Plant and Tissue Culture Group and published in the journal *Botanica Scripta Belgica*. Three years later, in 1997, I was able to generalize these curves into what I originally called superformula, as a generalization of supercircles, following joint work with Bert Beirinckx on superellipses. In 1999, I founded a company with the explicit aim to disseminate these ideas in science, technology, and education, with better than expected results.

My first presentation on the more general use of Lamé curves in botany was in 1997 at the Symposium Morphology, Anatomy and Systematics in Leuven, in honor of the great German plant scientist Wilhelm Troll. The symposium was co-organized by the Deutsche Botanische Gesellschaft and by the Botany Department of the University of Louvain. The talk went quite well, and in the closing speech, Erik Smets remarked that it was hoped that I could bring fresh ideas to mathematical botany; the untimely death of the late Aristid Lindenmayer had left a deep gap in that field. On advice of Focko Weberling, one of Troll's students, I was contacted by Springer Verlag that same week to publish a book on my work. In a sense, this book is 20 years overdue, but *pauca sed matura* was Gauss' motto.

In 2003, the first major scientific paper was published in the *American Journal of Botany*, on invitation by the editor in chief, Karl J. Niklas. The title "*A generic geometric transformation which unifies a wide range of natural and abstract shapes*" expresses the gist of the matter. This publication attracted a lot of attention, and I still think it was a good timing and (hoped for but unexpected) strategy for dissemination. In the same year, the English version of my book *Inventing the Circle* was published, two years after the Dutch version in 2001. My journey took me from horticulture and plant biotechnology to geometry. The article "*Universal Natural Shapes*" with Stefan Haesen and Leopold Verstraelen in 2005 introduced the equation into the field of geometers, and also substituted the name superformula by the names *Gielis curves, surfaces, (sub-)manifolds, and transformations*. In geometry, a whole new world unveiled before me. This, along with several publications by others ensured the adoption and absorption of the formula in mathematics, science, and education. Two of my main goals, formulated in 1999, have been or are being realized, along various paths.

Technology was my third, long-term goal, and many papers in science and technology have been published using the formula. In many cases (antennas and nanotechnology, for example), the formula allowed to go beyond the classical and canonical shapes, opening many doors. My own passion about technology is this: No matter what field we consider, I (with many others) think our current technology, no matter how advanced, is essentially a bag-of-tricks, aimed at deception (which is perhaps a major feature of our times and culture). We are working toward new applications following my dream of unifying and simplifying, at the same time appreciating complexity.

In this respect, it is important to note that my background is horticulture and plant biotechnology. I have been involved as researcher and research director in plant research for more than 25 years, and methods were developed in our team for mass propagation of plants, in particular temperate and tropical bamboos, the former for ornamental purposes, the latter for reforestation in the tropics. Over the past years, we have produced over 20 million bamboos that have been distributed and planted worldwide. Bamboo is indeed a multipurpose plant, a beautiful plant for our gardens but providing building materials, food, and much more for the poorest one billion humans on this planet.

Key to this was focused in-depth research using molecular markers and high-throughput determination of plant hormones, but never losing sight of the end goal: plant production. The same procedure of combining science with technology, we use now in the development of antennas, where optimization is an ongoing activity. We are now able to produce very powerful antennas, at costs which could be up to ten times less than existing ones, optimizing margin, while delivering the highest possible quality and efficiency. In all my (scientific and engineering) activities, this combination of wide interests, a generalist (rather than myopic) view, and stamina has always led to remarkable results.

Always keep focused on what you want to achieve. The current book is a combination of such focus, combining wide interests (nature and science) and a generalist (rather than specialist) attitude, inspired by the vision of natural scientists and philosophers from a long gone era. Along the way, I learned many other things, which one cannot learn but by a constant drive and strive to understand. My scientific education continues daily.

I wish to convey my sincere gratitude to my parents and all teachers, botanists, mathematicians, and engineers who played an enormously important role in my personal scientific development and the various developments described in this book: These include my teachers in high school Fred Verstappen (Greek) and Gerard Bodifée (sciences); in my professional horticultural and plant biotechnological life: Pierre Debergh (University of Ghent), Walter Liese (University of Hamburg, Germany), Tom Gerats (Radboud University, The Netherlands), and

Paul Goetghebeur (University of Ghent, Belgium); in mathematics and geometry: Freddy Van Oystaeyen (University of Antwerp), Leopold Verstraelen (University of Louvain, Belgium), Paolo Emilio Ricci (La Sapienza University, Rome), and Ilia Tavkhelidze (Tbilisi State University, Georgia). During the past two decades, fantastic collaborations developed with Bert Beirinckx, with Diego Caratelli (Antenna Company, University of Tomsk, Russia), Yohan Fougerolle (University of Dijon, France), Dishant Pandya (India), and with Shi Peijian, Yulong Ding, and the team at Nanjing Forestry University. I must also thank all collaborators in Genicap Beheer BV and Antenna Company.

Special thanks for their important contributions and advice in various chapters of this book are due to Yohan Fougerolle, Diego Caratelli, Dishant Pandya, Paolo Emilio Ricci, and Ilia Tavkhelidze. The results of the collaborations with all four are fundamental to this book. Many thanks also to Albert Kiefer for graphics to Violet for help on references, corrections and pictures, and to Arjen Sevenster and the Atlantis and Springer teams.

Many thanks to my whole family, past–present–future, for their continuous support. With a most special dedication and many, many thanks to my wife and soul mate Christel, and our children, Violet and Fabian, for making my life complete.

Antwerp, Belgium Johan Gielis
December 2016

About the Book

This book focuses on the origin of the Gielis curves, surfaces, and transformations in the plant sciences. It is shown how these transformations, as a generalization of the Pythagorean theorem, play an essential role in plant morphology and development. New insights show how plants can be understood as developing mathematical equations, which opens the possibility of directly solving analytically any boundary value problem (stress, diffusion, vibration…). This book illustrates how form, development, and evolution of plants unveil as a musical symphony. The reader will gain insight in how the methods are applicable in many diverse scientific and technological fields.

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Chapter 1
Universal Natural Shapes

> *The geometrical description of curves and surfaces and the shapes that are derived via Gielis-transformations, describe and determine in a uniform and universal way an enormous diversity of natural shapes.*
>
> Leopold Verstraelen

Conic Sections at the Core, Once More

In the book *Inventing the Circle* [1] it was shown how one generic geometric description, a generalization of Pythagoras and Lamé, allows for the description of many natural shapes, illustrated with many pictures and illustrations. The same ideas were published in the *American Journal of Botany* in April 2003 [2]. In 2004 this generic geometric transformation was named Gielis Transformations, giving rise to the class of Gielis curves and surfaces, by the geometer Leopold Verstraelen, who understood that this transformation allows for a broadening of some crucial concepts in geometry. Its actions on classic curves could describe a wide range of shapes in nature [3]: "*The basic shapes of the highly diverse creatures, objects and phenomena, as they are observed by humans, either visually or with the aid of sophisticated apparatus, can essentially, either singular or in combination, be considered as derived from a limited number of special types of geometric figures. From Greek science up to the present this is probably the most important subject of natural philosophy… The geometrical description of curves and surfaces and the shapes that are derived* via *Gielis-transformations, describe and determine in a uniform and universal way an enormous diversity of natural shapes*".

Because of this wide applicability—the Gielis Formula describes shapes at nano, micro, macro and gigascale—the idea of Universal Natural Shapes was born, providing for a uniform description of natural shapes (Fig. 1.1). T. Philips of the Courant Institute wrote [4]: "*A botanical Kepler awaiting his Newton*". Obviously, this waiting can take a while. In the meantime we have deepened our understanding of its applicability to study natural shapes and broadening of concepts in geometry and mathematics. This book describes some of these developments, and shows how

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Fig. 1.1 Universal Natural Shapes. Copyright Johan Gielis/Martin Heigan

it connects various ideas and fields, starting from the principles developed by Ancient Greek mathematicians.

Like *Inventing the Circle* this book is one of ideas, rather than technical. It can be considered as the second part of a trilogy. *Inventing the Circle* introduced the new transformations, and provided many examples of such shapes in nature. In this book we will focus on some of the underlying geometrical aspects based on the subtitle of the first book: *The geometrical beauty of nature*.

We will follow the universal scheme of building arguments as developed by the ancient Greeks [5], the Demonstratio artis geometriae.

Πρότᾰσις or Propositio exposes what needs to be shown, namely that beyond the mere analogy, we can develop a rigorous geometrical and mathematical approach to study natural shapes based on Lamé-Gielis curves and surfaces.

Εκθεσις or Expositio describes what we have to start with. From elementary notions in mathematics that are at work in every field of mathematics and its applications in the natural sciences, we derive Lamé-Gielis curves and surfaces.

Διορισμός: in this Determinatio step we investigate "*whether what is sought is possible or impossible, and how far it is practicable and in how many ways*". It will be shown how the shapes and their combinations can be easily combined with many existing concepts in geometry and mathematics. This wide applicability applies to a wide variety of natural shapes, Universal Natural Shapes.

Κατασκευή or Constructio: Observations from botany open the door for understanding many key concepts and their connections and allow for analytical solutions of boundary value problems, using classical 19th century methods, thereby broadening various concepts (Gielis is an acronym) and analytical methods. What we can learn from flowers and plants can be applied in virtually all fields of science and technology.

Απόδειξις or Demonstratio: in the final chapters it is demonstrated how Lamé and Gielis curves provide better models for studying various natural shapes, and how they can help understanding evolutionary and developmental aspects in biology, based on geometrical considerations.

Συμπέρασμα: This then leads to the conclusion that many of the suggestions made in the original book and articles in the period 1999–2005 have been validated. Description always precedes understanding the connections between mathematics and nature. Moreover, Lamé-Gielis curves and Gielis transformations have opened the door for many new developments, among others for a geometrical theory of shape and morphogenesis.