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Stefan Nanz

Toroidal Multipole Moments in Classical Electrodynamics

An Analysis of their Emergence and Physical Significance

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Preface

> *If I have seen further it is by standing on the shoulders of giants.*
> Isaac Newton

This book is based on my master thesis, on which I worked from January 2014 until February 2015 at the Institute for Theoretical Solid State Physics at Karlsruhe Institute of Technology (KIT). The topic regarding the toroidal multipole moments was originally suggested by my supervisor, Professor Carsten Rockstuhl, who had just started to establish a new working group at KIT at the end of 2013. Pointing out some inconsistencies in various derivations and descriptions of the electromagnetic multipole expansion, he motivated me to dig into the depth of the theory of electrodynamics and to find out what's really going on in the multipole expansion.

Soon after I started working, I had the fortune that the working group was joined by an analytically well-experienced postdoctoral researcher, Dr. Ivan Fernandez-Corbaton, who took the role as my advisor and helped me to tackle the sometimes dry theoretical analysis. In the course of time, it became apparent to us that the derivations which argue in favor of the physical significance of toroidal moments are suffering from some assumptions, which turn out to be not justified in general. Condensing these insights then lead to a master thesis which was approved by the supervisors, Prof. Carsten Rockstuhl and Prof. Martin Wegener, and later on also by Springer Spektrum, resulting in their decision to publish it as a book.

At this point, I want to take the chance to express my cordial thanks to several people without them my master thesis and therefore this book would not have been possible. First of all, I would like to thank Prof. Dr. Carsten Rockstuhl for giving me the possibility to work on such an interesting and rewarding topic and for being a great mentor regarding all aspects of research. I would also like to thank Dr. Ivan Fernandez-Corbaton, who did a great job as advisor with his analytical skills and extensive knowledge of literature.

Furthermore, I would like to thank: Dr. Christoph Menzel for fruitful discussions and proofreading the thesis; Iris Schwenk for providing me a nice template for the layout of the thesis; Dr. Andreas Poenicke for his support regarding computer problems; Clemens Baretzky for creating the images; my half-brother Philipp for proofreading the thesis; and my roommates Dr. Giuseppe Toscano and Alexander Kwiatkowski for sharing the office with me and helping me with all kinds of problems. I am grateful to Springer Spektrum for publishing my master thesis, and I want to thank my contact persons there, Nicole Schweitzer and Marta Schmidt. Last but not least I would like to thank my parents for their support during my education.

Heidelberg, November 2015

Stefan Nanz

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List of Symbols

This list includes the most important symbols which are at least used once in the thesis.

- $\vec{\nabla}$ nabla operator
- Δ Laplace operator
- $\mathcal{P}$ parity operator
- $\mathcal{T}$ time inversion operator
- $\mathcal{L}$ angular momentum operator
- $\mathcal{D}$ detracing operator
- Λ detracing functional
- Tr trace of a tensor
- $\vec{k}$ wave vector
- $\dot{x}$ time derivative of x
- a^* complex conjugation of a
- $\vec{r}$ position vector of evaluation point
- $\vec{r}\,'$ position vector of the source
- $\vec{e}_i$ unit vector in direction i
- μ_0 vacuum permeability
- ε_0 vacuum permittivity
- c speed of light in vacuum
- ω angular frequency
- $\vec{A}$ vector potential
- φ electric scalar potential
- $\vec{j}$ electric current density
- ρ electric charge density
- $\vec{B}$, $\vec{H}$ magnetic field
- $\vec{E}$ electric field
- $\hat{P}^{(n)}$ Cartesian electric multipole moment of order n
- $\hat{M}^{(n)}$ Cartesian magnetic multipole moment of order n
- $\hat{T}^{(n)}$ Cartesian toroidal multipole moment of order n
- $\vec{p}$ electric dipole moment
- $\vec{m}$ magnetic dipole moment
- $\vec{t}$ toroidal dipole moment
- $\hat{Q}^{(e)}$ Cartesian traceless electric quadrupole moment
- $\hat{Q}^{(m)}$ Cartesian traceless symmetric magnetic quadrupole moment
- $\hat{Q}^{(t)}$ Cartesian toroidal quadrupole moment
- Q_{lm} spherical electric multipole moment
- M_{lm} spherical magnetic multipole moment
- T_{lm} spherical toroidal multipole moment
- a_{lm} spherical electric parity multipole moment
- b_{lm} spherical magnetic parity multipole moment
- j_n spherical Bessel function
- $h_n^{(1)}$ spherical Hankel function of first kind

1 Introduction and Overview

The analysis of electromagnetic radiation is an important resource to investigate the properties of materials. Often, unknown materials are irradiated by a suitable light source, e.g. a laser, and the scattered electric and magnetic fields are used to obtain information about the materials. This requires to link the properties of the incident to the scattered radiation. This asks to solve Maxwell's equations with spatially distributed materials whose properties are introduced on phenomenological grounds. A prototypical example for such approach is ellipsometry. When designing e.g. antennas, the opposite is of relevance: An incident and scattered electromagnetic field is given and the structure, which produces this field, shall be constructed.

The latter problem also applies in the context of metamaterials. Metamaterials are made from small scattering objects. If their optical response is dominated not just by an electric dipole moment but also by higher order electromagnetic multipole moments, material properties not available in nature can be reached. This is possible, since on phenomenological grounds natural materials at optical frequencies usually are considered as an ensemble of polarizable entities with a scattering response that corresponds to an electric dipole. This only allows to observe a dispersion in the optical permittivity

Now, by composing metamaterials from strongly scattering unit-cells, also called meta-atoms [1], other material properties are allowed to be dispersive. The key is to make meta-atoms sufficiently small and to arrange them sufficiently densely in space, such that light will experience a homogeneous medium with properties derived from the scattering response of the individual meta-atom. For example, if their scattering response is dominated by a magnetic dipole moment, a dispersive permeability can be observed. If their scattering response is dominated by an electromagnetic coupling, a strong optical activity can be observed. Studying the light propagation in metamaterials and the scattering response from individual meta-atoms is a major challenge for contemporary theoretical optics.

From the many interesting properties meta-atoms may exhibit, we discuss in this thesis the toroidal multipole moments. Toroidal moments are a third multipole family besides the electric and magnetic moments. They have several properties which attract the interest of research. It has been shown [2] that current distributions, which cause toroidal moments, violate Newton's third law. With toroidal moments it is possible to generate non-zero vector potentials without electromagnetic fields [3], hereby realizing non-radiating charge-current distributions [4, 5]. Also, a negative index of refraction can be caused by toroidal moments [6]. Though the experimental possibility to realize materials with such properties is quite new, there has been a theoretical interest for such characteristics for long time. Although being considered in the context of electrodynamics usually as exotic, the properties of toroidal moments came again in

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