Digital Games and Mathematics Learning

Potential, promises and pitfalls
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Digital Games and Mathematics Learning

Potential, Promises and Pitfalls
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Digital Games and Learning: What’s New Is Already Old?

Tom Lowrie and Robyn Jorgensen (Zevenbergen)

Keywords Technology · Mathematics education · Digital · Literacy · Games environment · Digital environment · Authentic problem solving · Mathematics · Dynamic visual imagery · Spatial reasoning · Dynamic imagery

Context

The genesis of this manuscript was inspired by a series of presentations (in 2011) undertaken via a Discussion Group at the 35th conference of the International Group for the Psychology of Mathematics Education held in Ankara, Turkey. In fact, several of the participants in the Discussion Group are chapter authors. Collectively, the authors of this manuscript were given the challenge to consider the affordances (or not) of digital games for mathematics learning. Their international perspectives are drawn from a diverse range of cognitive, psychological and sociocultural viewpoints, from foundations within and outside mathematics education. It was not our intent to have a book that was driven solely by data, but rather to make a contribution to the field by drawing on a wide range of authors whose methodologies and approaches would create a discussion forum for considering the worth (or not) of games in bringing about better ways of teaching and learning mathematics. At the same time, we were also interested in seeing the affordances that this new genre may create for new forms of learning and mathematics.

The manuscript addresses the potential, promises and pitfalls of digital games for mathematics learning by measuring, monitoring and analysing the development of students’ sense making as they engage in games technologies, both in- and out-of-school. Technology is clearly a catalyst for significant educational and social change—and although technology has become intrinsic to most of our daily practices, education systems rely much less on technology than is the case in society.
more generally. As citizens, we have been forced to be adaptors of digital technology—from paying bills to how we decode a map. To date, education systems have been protected somewhat, and mathematics education in particular. Indeed, there is some sense that there may be some artificiality in terms of the potential for digital tools to radically reform education. It is in this context that we have actively sought to bring a broad collection of authors and perspectives to create a forum for debate.

In the last chapter of the book, a secondary data analysis of digital game impact over the past 5 years, Logan and Woodland (Chap. 14) highlight the influence digital games are having beyond the entertainment industry. They speculate that the current generation of children is experiencing a parallel education, with out-of-school learning highly influenced by gaming. They suggest that these children will “grow and compound the use of digital games in learning as they themselves become our future educators and policy-makers”. Potentially, we are at the advent of a digital era that could impact dramatically on education and school classrooms. In the past, such expectations and predictions have had much less effect than initially conceived [remember Pappert’s (1980) Mindstorms].

We trust that this book will provide readers with a relatively global perspective of the influence of digital games in education, and particularly the nature and role of gaming in mathematics education. We are mindful of the fact that digital technologies ‘change’ at a much greater rate than education curricula systems, and that today’s new hardware or peripherals are likely to be redundant in a few years. Nevertheless, gaming may well be the next major influence on learning and education, and it is certainly the case that mathematics has a role in new developments and initiatives.

**Positing Digital Games Within Literacy Contexts**

In the field of literacy education, there is a strong recognition of the possibility of the digital games environment creating new opportunities for literacy and literacy learning. Gee’s (2003) seminal work with digital games has highlighted two salient features that may have application in the field of mathematics education. First are the opportunities for new forms of literacy that are made possible through the digitized literacy format of the games platforms. Second, the digital games environment itself creates and fosters new learning opportunities that appear to engage learners for long, sustained periods of time. Gee contends that much can be learnt from the principles that underpin the games technologies that need to be adopted into modern learning environments.

Gee (2003) has examined the digital games environments to explore the principles used by the gaming industry to engage players in games. As a highly competitive industry where millions of dollars can be spent on developing games, the industry has designed games that engage players for extended periods of time. Gee’s principles have been used to justify reforms in education that will engage the students as they enter schools. Gee and his advocates argued that the current practices
in school are failing to cater for today’s learners (often termed ‘digital natives’). He proposed that 36 principles used in games designs could radically offer new learning environments that cater for learning and learners in the digital era.

Drawing on three discourses (situated cognition, new literacy studies and connectionism), Gee provides a comprehensive account of the possibilities of games to create exciting and engaging learning opportunities. Primarily, Gee focuses on literacy learning and how the games environment allows for new forms of literacy and engagement in literacy texts. The literacy demands of these digital worlds are substantially different from the linear text models of the printed media that has dominated literacy since the industrial revolution. It is beyond the scope of this chapter to outline each of these principles in detail but we provide the full list here without description. Fundamental to Gee’s principles are the notions that gamers identify with the game and develop an identity (and affinity) with the game that aids in the engagement with the game. Once in the game, the player then is further engaged through the underlying structures of the game where there is a progression through the game from simple activities that progressively increase in difficulty. As the player engages with these increasing complexities, he/she is strongly scaffolded through a range of design principles including low-failure and where failure is not public so that there is encouragement to engage with game. The game is also structured so that skills learnt in one level will be used and extended in subsequent levels. The principles are compelling and clearly work in the games industry. Given many of the principles mirror practices most educationalists value and indeed strive for, one could easily suggest a ‘magic bullet’ has been identified, at least in terms of literacy education.

Nevertheless, Gros (2007) argued that overall the field of research into digital games is fragmented, disjointed and has no real sense of boundaries. This is due, in part, to the burgeoning pace at which technology develops. Although there have been strong advocates supporting the notion that digital games may revolutionise education, others (Warschauer 2007) caution such support and take a more measured view in terms of the types and speed of reform that has emerged. One of the fundamental tenants for this book was to challenge the authors to consider the limitations in conjunction with the affordances of digital games for learning. It is here that this book contributes significantly to the series in which it is located.

Digital Games and Mathematics Thinking

Since the explicit positioning of literacy education within the digital games environment, researchers have begun to explore the possibilities for digital games to enhance learning of mathematical concepts and processes. Relatedly, problem solving in a digital games environment requires varying levels of goal-orientated decision making. In a mathematics context, Schoenfeld (2010) argued that such goal-orientated processing included three components, namely: (1) resources (general knowledge); (2) goals; and (3) orientations (including beliefs and dispositions).
He argued that most “in the moment” decision making had links to these three mathematics components. Fregola (Chap. 10) maintains that games environments promoted the process of mathematical abstraction, which included decision making about the characters and language of the environment. In a similar vein to that of Schoenfeld, he points out that problem solving consisted of a set of skills that included a self-regulatory that was mathematical in nature.

Collectively, much of the research in this area has been the catalyst for imagining the possibilities of the digital games environment to enable new ways of thinking and working mathematically. For example, Dalla Vecchia, Maltempi and Borba (Chap. 4) understand that the mathematical modeling that takes place in the process of electronic games construction may contribute to the mathematisation process, since the process considers the students’ choices and interests, and adopts learning frameworks which are essentially constructivist in nature. In particular, they raise issues about the potential role of mathematical modelling in creating new virtual environments within games contexts. It would seem to be the case that the more open-ended and multidirectional games become, the greater the need to model the environments mathematically.

Technology advances provide scope for digital games to become more complex and certainly more challenging. As a consequence, user engagement can be multidimensional and storylines can have realistic implications and outcomes. In fact, serious games tend to be more effective in terms of both learning and retention when compared to conventional instructional practices (Wouters et al. 2013). Bossomaier (Chap. 11) maintains that the potential and perhaps real impact of this burgeoning area of serious games “is the complex environment surrounding the game, the meta-game and affinity spaces. This rich, creativity extension of the gaming world offers in-depth, contextualised understanding”. It also offers huge potential for mathematical thinking, not only associated with problem solving but also the development of engagement in spatially and visually rich environments. However, as Bossomaier points out, “One of the huge gains, and possibly, one of the challenges, is integrating these powerful frameworks into conventional courses and educational program…”.

As Van Eck (Chap. 9) asserts, it is unwise to rely on the game as the source for learning development. Rather, a sound understanding of what embedded theories promote quality instructional design is required. As with many authors in this volume, he argues that sound psychological, cognitive and sociocultural principles must surround the games environment. This chapter outlines a model (one that encourages situated and authentic problem solving) that can be used with digital games to promote transfer and improve attitudes toward mathematics. In concert with the fundamental intent of the book, both Bossomaier and Van Eck acknowledge the games themselves cannot enhance learning opportunities—no matter how good the learning designs may be. Gros (Chap. 3) indicates this can only occur if user experiences are carefully linked to context and learning. Indeed, Gros maintains that this integrated understanding of the artifact (the game) and the process is critical since general perceptions of the usefulness of digital games to enhance learning are likely to grow in the immediate future. This rationale is based on the
fact that the generation experiencing learning through games in the classroom today will expect such engagement when they reach tertiary education. Moreover, he predicts that teachers will receive tools and learning materials developed specifically for game-based learning that will cater for groups of learners with different skills, levels and competencies. This notion of inevitability is certainly apparent within the Logan and Woodland chapter.

Mathematics and Digital Games in Schools

There are a number of approaches in mathematics education where the possibilities of digital games are explored. The types of games used as the basis of this research vary considerable, making it challenging to find effective definitions of what constitutes a ‘digital game’. As Rothschild and Williams (Chap. 8) point out, the availability of products and applications to enhance basic mathematics and literacy skills is overwhelming, even at the early childhood and preschool levels. They argue that software developers “would be well advised to move beyond enumeration activities and look into supporting the transition from enumeration to number application” since seemingly simple cognitive progression contains numerous leaps toward higher-order number sense. In a similar vein, Beavis (Chap. 7) argues that digital games are enabling high-level understandings to be gained. Beavis’s chapter describes how digital tools expose students to sophisticated disciplinary and process knowledge, via tools that encourage engagement and fun—while exposing students to new forms of text and literacy.

Somewhat disturbingly, at least to us, some of these best design features of games are not being used to promote higher-order thinking and deep learning, but rather visually appealing drill-and-practice games. Although the reinforcement of facts and skills form a critical part of mathematics understanding, it is noteworthy that these are the type of game genre that are most likely to be introduced to classrooms. For example, in their work on the Mathletics software (3P Learning 2012), the designers adopted gaming principles and applied them to the learning of mathematics. The authors argue that the “material and relational organization of Mathletics play emerges over time through the entanglement of object design and ownership, the context and governance of use, and collaboration in play” (Nansen et al. 2012, p. 2) where the players can engage with either “maths-related activities and courses or play Live Mathletics” (p. 3). Such games are penetrating school classrooms and are increasingly used as revision and homework tools.

As new hardware and platforms become commonplace, software used on tablets and other mobile devices are likely to penetrate classroom learning environments. Two chapters of the manuscript are devoted to the use of apps in classrooms. As Larkin (Chap. 13) points out, the vast number of apps available to time-poor teachers is overwhelming (there are more than 500,000 ‘education’ apps in the Apple iTunes store). He recognizes that this is problematic for teachers to be able to make informed decisions about suitability and relevance, unless they can spend
considerable time actually engaging with the respective apps. In a detailed analysis of apps that report to promote mathematics learning, he identifies a large discrepancy in the quality of apps, with many of limited to no use at all in terms of mathematical learning. Nevertheless, he identifies some apps with huge potential for mathematics engagement. In his chapter, Calder (Chap. 12) maintains that the most useable learning apps allow individuals to pace their learning and self-select apps with more challenging concepts or processes. However, he reports that the nature and design of most apps lead to rapid familiarity and, consequently, disengagement. In many ways, most apps are at the opposite end of the spectrum to that of serious games—with the design sophisticated and potential for open-ended engagement similar to computer software of 30 years ago. Some popular entertainment apps have less functionality than some of the very first computer games (such as Space Invaders and Pacman). However, the relative low cost of most apps, and the fact that they can be used on increasingly popular tablet devices, ensure impact in and out of classrooms. Calder reports that the best function of apps is within an integrated program. The challenge in terms of eventual familiarity leading to relative disengagement is to keep the apps as part of a varied program, to ensure that they are relevant and appropriate for the students, and for the development of apps to be ongoing and responsive to critical review. He concludes that mathematics educators and students need to be influential in the development of apps, to especially ensure that mathematical thinking is given primacy. Such reasoning is constant throughout the manuscript, yet challenging given resources for entertainment games far exceeds that of games with an educational focus.

Mathematics and Digital Games in Other Learning Contexts

The mobile nature of digital games ensures that the lines between in-school and out-of-school gameplay is blurred. Thus, it is important to explore the possibilities of these games to create new spaces for learning and engaging with mathematics. From a social learning perspective, research has been concerned with the ways in which the games industry has been influencing ‘interactive’ learning via computers (Scanlon et al. 2005); creating spaces for students to create their own digital games in order to teach concepts to peers (Li 2010); or the ways in which the games are arranged to motivate learners to engage with the games (Habgood and Ainsworth 2011) and engage with higher-order problem solving abilities (Sun et al. 2011). These and many other studies seem to support the possibilities of digital games to promote learning.

The potential of the games environment to create dynamic visual imagery (Gros 2007) is a vast leap from the static pencil-and-paper tools of the classroom. Not only are spatial images important in terms of new forms of spatial reasoning, but the capacity to read such images is critical to success. Lowrie’s (2002) work with Pokémon attests to this substantial leap in learning possibilities within mathematics.
engagement and learning. The games environment creates many new possibilities for imagery that is beyond the scene as well as dynamic imagery—a far cry from the limited opportunities available in traditional teaching of mathematics. While there is some debate as to the value that games have in terms of the education environment, there is some sense that the inability for games to prosper and be valued in education is not because of the games per se but due to the conservative view of educationalists (Moreno-Ger et al. 2009). As Lowrie (Chap. 5) proposes, digital games appear to accommodate the visuospatial-reasoning skills required to interpret and manage information systems than traditional classroom practices and pedagogies. Digital games also allow gamers with different preferences and skills (or game profiles) to access and navigate the spatial demands of information.

Some studies have been more open-ended and have attempted to document the ways in which learners navigate through games and the strategies they used (Augustin et al. 2011; Bottino and Ott 2006). However, to explore the potential of games without an understanding of learner context and engagement is problematic. Squire (2006) has called for a much richer understanding of how identities are shaped through the games contexts and the impact of this engagement to wider social contexts. Indeed, there are dangers in taking a game that successfully engages learners in an out-of-school context and assuming it would be effective in classrooms. Avraamidou, Monaghan and Walker (Chap. 2) maintain it is necessary to:

[…] view mathematics as a cultural practice and doing mathematics as an artefact, person and sign mediated, object-oriented activity…. Taking non-school games, which are designed to be played for leisure, and trying to integrate them into a classroom setting, following a curriculum that expects school mathematics teaching and real-world rules, is a transition that needs further exploration and preparation on behalf of the students, teachers, curriculum developers and other education stakeholders.

Moreover, Jorgensen (Zevenbergen) (Chap. 6) highlights the fact that the social fabric of gameplay provides different levels of equity, access and preference. Since her work found that low socio-economic status students were reporting greater use of the digital games environment, and the potential for learning that can arise from these environments, she maintained that digital games could create new opportunities for constructing mathematical habitus for this group of learners. This is particularly important, as these students are most at risk of performing poorly in measures of mathematical learning.

Coda

Collectively, these 14 chapters explore the possibilities of the games environment to create new opportunities for learning for mathematics. The manuscript sought to examine a range of implications of the use of games to enhance and/or develop new mathematical understandings and dispositions. We have deliberately and intentionally sought authors whose work would disrupt current thinking of the potential for games to enhance (or not) mathematical learning. It was the intent to seek
authors whose work could be theoretical or empirical but are always seeking to push boundaries in educational thought. Whether this disruption was around pedagogy, the technology per se, the potential for learning mathematics or issues associated with access and success, it was our intent to bring some of the leading thinkers and thoughts to what is potentially a new era in mathematics learning. The relative cost and pervasiveness of digital games in the modern world means that it is accessible to many—students, educators, policy makers and families. This makes it a potentially viable medium for learning and for the masses. But within this context, caution and limits need to be established as well. It is the case that the authors in this collection bring some of these debates and affordances into a forum for discussion. If this book achieves this, then we have attained our goal.

References

Robyn Jorgensen (Zevenbergen) is a Professor of Education: Equity and Pedagogy at the University of Canberra. Her work has been focused on issues of equity and access in relation to mathematics education. This work has sought to understand the ways in which mathematics practices are implicated in the success (or not) of students who have been traditionally marginalised in the study of school mathematics. Her work usually draws on the theoretical frameworks offered by French sociologist, Pierre Bourdieu, to better understand the ways in which practices within the field of mathematics education are implicated in the (re)production of equity and inequities.

Tom Lowrie is a Centenary Professor at the University of Canberra. Tom has an established international research profile in the discipline area of mathematics education and he has attracted considerable nationally competitive funding from the Australian Research Council. A substantial body of Tom’s research is associated with spatial sense, particularly students’ use of spatial skills and visual imagery to solve mathematics problems. He also investigates the role and nature of graphics in mathematics assessment. Tom has investigated the extent to which digital technologies impact on the education community including teachers, children, and their parents, as well as children’s engagement in out-of-school settings. A particular focus of Tom’s work has been on disadvantaged students (particularly Indigenous students and students living in remote areas). He was selected to publish an entry on rural and remote mathematics education in the Springer Encyclopedia of Mathematics Education (2014) and is co-author of the book, Mathematics for Children: Challenging Children to Think Mathematically (the most widely distributed undergraduate mathematics book in Australia and published in its fourth edition in 2012 by Pearson Australia).
Mathematics and Non-School Gameplay

Antri Avraamidou, John Monaghan and Aisha Walker

Abstract This chapter investigates the mathematics in the gameplay of three popular games (Angry Birds, Plants vs. Zombies and The Sims) that are unlikely to be played in mathematics lessons. The three games are different but each has been observed to provide opportunity for mathematical activity in gameplay. After describing each game, and the mathematics that can arise in gameplay, the chapter explores two questions: What kind of mathematics is afforded in these games? Can these games be used in/for school mathematics? Issues considered under the first question include: the nature of mathematics and the difficulty of isolating the mathematics in non-school gameplay; players’ strategic actions as mathematical actions; and ‘truth’ and its warrants in different mathematical worlds. Issues considered under the second question include: tensions between curricular expectations and the mathematics that arise in gameplay; and possible changes in gameplay when a game is moved from a leisure to an educational setting.

Keywords mathematics/Mathematics · Non-school gameplay · Strategies · Abstraction-in-context · Theory of didactical situation · Three worlds of mathematics

Introduction

Gameplay can be used to present and structure mathematical activities in classrooms: Nim, for example, has been used extensively in French primary mathematics lessons (see Brousseau 1997); in England teachers have used the Shell Centre
Design a Board Game resource box in their lessons; in North America, the National Council of Teachers of Mathematics [NCTM] (2004) claim that mathematical games “can foster mathematical communication...can motivate students and engage them in thinking about and applying concepts and skills” <http://www.nctm.org/fractiontrack/>. Research, however, reminds us that learning mathematics through gameplay is not automatic: “games can be used to teach a variety of content in a variety of instructional settings...there is no guarantee that every game will be effective” (Bright et al. 1985, p. 133); “it appears that assumptions that students will see the usefulness of mathematics games in classrooms are problematic” (Bragg 2006, p. 233). However, these examples focus on mathematical games used in classroom settings which leaves a question about games that are not deemed appropriate for classrooms.

By a non-school game we mean a game that is unlikely to be offered for students to play in a mathematics lesson. It has been argued that non-school games can have beneficial impact on players’ problems solving skills (Chuang and Chen 2009) and spatial ability (Dye et al. 2009); and Gee and Hayes (2010) claim that some games require a considerable knowledge of geometry. The adoption of non-school games in a classroom largely depends on the classroom teacher (Bakar et al. 2006). When a digital game is used in a mathematics lesson, it is likely that the game meets a teacher’s interpretation of a curriculum objective (NCTM 2004). When a student chooses to play a new non-school game, they are highly unlikely to play this for reasons that a teacher might have in introducing the game in a lesson, such as curriculum content. Studies have shown that the content of non-school games is often irrelevant or not aligned with that of school curricula (Egenfeldt-Nielsen 2005). Further to this, students do not necessarily appreciate it when non-school games are used for education rather than fun (Bourgonjon et al. 2010). The issue of mathematics and non-school gameplay is, thus, far from straightforward. We restrict our attention, unless otherwise stated, to digital games, and all references below to game or gameplay may be assumed to concern digital games.

This chapter investigates the question: What mathematics is there in non-school gameplay?1 How one understands and addresses such a question depends, amongst other factors, on one’s theoretical framework. Our framework is sociocultural in as much as we view mathematics as a cultural practice and doing mathematics as an artefact, person and sign mediated, object-oriented activity. From this position, our understanding of the question is that mathematics resides in mathematical activity and the answer to the question depends on the game, the player and the context of the gameplay.

To address the question, we focus on three popular (circa 2013) games: Angry Birds, Plants vs. Zombies and games in The Sims series. The next section presents these games and discusses mathematics that can arise in gameplay. This is followed by a discussion of two further questions arising from our considerations of the three games: What kind of mathematics is afforded in gameplay? Can these games be used in/for school mathematics?

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1 Note that we use the word gameplay and not games in this question. This reflects an ontological assumption that mathematics, if it exists at all, does not reside in the game itself but in the gameplay.
Three Games

We focus closely on three games, rather than surveying a large number, because of a conviction that the detail of gameplay is important in a consideration of mathematics in gameplay. We chose the three games below because: they are clearly non-school games; they have each given rise to observed gameplay which can, in a sense to be discussed in this chapter, be viewed as mathematical activity; there are differences in the nature of the mathematical activity in these three games; and they are popular games. For each game, we first describe the game and then raise issues concerned with mathematics.

Angry Birds

Angry Birds is a casual game developed by Rovio Entertainment which was first issued for the Apple iPhone and is now available for a range of iOS and Android devices, including high-definition versions for tablet devices such as the Apple iPad. An underpinning principle of casual games is that they can be played in very small blocks of time such as a 10-min bus journey (although some players may devote more time to the game). Typically, each level takes a short time to complete. Angry Birds begins with the narrative premise that the pigs stole eggs from the birds. The birds are consequently angry and take revenge on the pigs by firing themselves from catapults to destroy the pigs and their shelters. The task of the player is to aim the catapult to fire the birds at the pigs. As the game progresses, the shelters in which the pigs take refuge become increasingly complex and incorporate a wider variety of materials which present different constraints (for example, stone is more difficult to destroy than wood). In addition the structures often require a chain of actions so that the bird cannot be fired directly at the target but needs to hit, for example, a boulder which will strike a pedestal at the bottom of a structure and knock away support for higher levels. The birds also change as the game progresses with new attributes triggered by swiping the screen during the flight. A small blue bird, for example, splits into three smaller birds each flying at a different height whereas a white bird drops an egg when the screen is swiped. The player cannot choose which bird to deploy but is presented with a fixed number, type and sequence for each level. In order to achieve successful destruction of a pig, the player has to think about the nature of the structure and which part of the structure to target. The player then has to consider the flight path curve that the bird needs to take and so the angle at which the catapult must be pulled back in order to achieve the required trajectory. In addition, the further the catapult is pulled, the further the bird will travel, although speed is constant (whereas in real life, the further the catapult is drawn back, the greater the speed of the projectile/bird). The game draws the flight path for the current bird as it travels and the player can use this as a guide when launching the next bird.

In the following two subsections we recount two instances of individuals playing Angry Birds. The first arose from a chance encounter with a young person playing
it. The second was an attempt to replicate the first encounter with a very different person, a mature mathematician. In both cases the (same) observer simply made notes on the gameplay.

**Emily Plays Angry Birds**

Emily is 4 years old. Her older sister has an iPod Touch so Emily is familiar with touchscreen games, although she is not often allowed (by her sister) to play them. She is familiar with the Angry Birds concept but has not previously played the game. She is excited to be playing games on an iPad. It is briefly explained to Emily that she needs to fire birds from the catapult to hit the pigs but she is given no direction about how best to achieve pig destruction. Emily fires a bird but the flight path is too low so the bird hits the ground before it reaches the pigs’ shelter. When asked what happened, Emily says “I needed to go upper”. The second shot is successful. Emily chooses higher levels to play and these require a strategic approach. Emily plans her attack by tracing the prospective flight path of the first bird. It might be expected that a 4-year-old would aim for the easy birds but Emily does not do this. Instead she aims the bird high, so that it will knock down the coping stones (Fig. 1) which fall behind the structure thus destroying two pigs. The bird falls forward and catches one pig.

It might be assumed that this was simply a lucky shot had Emily not carefully traced the arc before aiming the catapult. It should be noted that to an observer it seemed as though the shot would simply bounce off the structure and be wasted. However, Emily’s reaction made clear that she had achieved the intended result. In order to plan the shot, Emily needed to consider how the blocks were arranged, the shapes of the blocks, the direction in which blocks would fall, the optimum point at which the bird should hit the structure and, finally, the flight path and the angle/distance at which the catapult should be released.

**Fig. 1** Emily aims for the top
We believe that Emily’s strategic thinking is mathematical (and we provide an argument that this is so in the Discussion section below); we also feel that Emily’s strategic thinking is pretty impressive for a 4-year-old. Emily navigates this mathematics effortlessly but without analysis, which may be expected in a classroom. Her intention was to perform the necessary moves to destroy the pigs and she did this easily. However, she was not able to explain what she had done; she could give only simple description. Her lack of explicit knowledge of the mathematics is made clear by her inability to put in words the decisions that she has made.

Rich Plays Angry Birds

Rich is an adult, an academic in the field of mathematics education. Although a confident user of digital tools, Rich is not a player of electronic games and had not previously encountered Angry Birds. Rich takes aim and fires the first bird at the structure but the bird falls short. The same happens with the second bird. The third (of three) overshoots. Rich becomes frustrated with the game and gives up, saying, “As a mathematician and a scientist, this makes no sense to me”. The problem for Rich is that although the game is mathematically accurate in some respects, for example, in terms of angles and curves, it does not completely replicate real-world physics. In real life, the further the catapult is drawn back the greater the speed of the projection of the bird. In Angry Birds, pulling the catapult back further increases the distance that the bird will travel but does not increase either the speed of projection or the force with which the bird strikes the structure. Rich is correct; in this respect the game makes no sense. Unlike Emily, he is able to explain the mathematics (and physics) of the game. However, Emily is able to use the mathematics within the game whereas Rich cannot.

The ‘Magic Circle’ and Mathematics

As with many games, the gameplay of Angry Birds takes place within a closed environment. Moore (2011) calls this the magic circle and relates it to the spaces in which traditional games are played, for example chessboards or card tables. Within the magic circle the rules of everyday life are suspended and replaced by the rules of the game. With traditional games the boundaries of the magic circle are clear and the rules are explicit; all players know how and when behaviours within the magic circle diverge from everyday life. Moore argues that the ubiquitous nature of digital gaming, especially on mobile devices, blurs the distinction between the magic circle and everyday life because the games do not have to be played in special places but are available everywhere. However the boundaries are blurred in other, perhaps more important ways. With traditional games it is obvious that the games operate in specialised contexts. For example, a game board clearly delineates the space in which the game is played and it is obvious to the players that the board is not real life. With Angry Birds there are aspects of the game which are clearly artificial
such as the cartoon characters. There is no attempt to replicate reality with the birds and pigs, indeed, there seems to be a clear attempt to make sure that nobody could confuse them with real creatures as that could be distressing. The birds and pigs are clearly *magic circle* characters. However, the materials used in the structure are designed to look similar to real-world wood and stone and, to a certain extent, share the characteristics of their real-world counterparts. Wood is much easier to break than stone. The parabolas of the birds also appear to be real-world rather than *magic circle*.

The mathematics of Angry Birds is real and is explained clearly by Chartier (2012) and by teaching websites such as InThinking Teach Maths (2013). For example, InThinking Teach Maths provides resources for working with quadratic equations based on Angry Birds. Clearly, Rich is capable of understanding these equations where Emily is not. Yet Emily can play the game whereas Rich is puzzled by the mechanics. Because Emily does not yet have any real-world understanding of the mathematics employed in Angry Birds, she is able to enter the *magic circle* of Angry Birds completely and therefore can make the practical calculations that she needs to play the game successfully. In future years, when she reaches the curriculum stage that addresses the mathematics employed in Angry Birds she may be able to relate the skills she has developed inside the ‘magic circle’ to the abstract concepts of real-world mathematics.

**Plants vs. Zombies**

Plants vs. Zombies (PvZ) is another casual game: a *tower defence* real-time strategy game where you, the player, plant plants in your garden to repel zombies from entering your house (where they promptly eat your brains and you lose). There are a variety of plants and zombies with different defensive and attack attributes. The basic game has five levels: front garden by day/night; back garden by day/night; and roof. Each level has ten *adventures* (zombie attacks). Collecting suns allows plants to be planted. Successful planting strategies vary with the adventure as the zombies vary. In addition to the basic game, there are a variety of *puzzles*. We present the *last stand—roof* puzzle. Last stand puzzles have *onslaughts* (each with several *waves* of zombie attacks) and you successfully complete the puzzle when you have withstood five onslaughts.

Figure 2 shows the screen at the beginning of the puzzle (where plants are inserted into flower pots) of *last stand—roof* with the *plants* available to use (and their individual costs, measured in *suns*) displayed on the left and zombies (who will start their attack after the *set up*) in the inset. Going down from the top: plants 2, 3, 4, 5 and 8 are attacking plants (plants 2 and 4; 4 is an upgrade of plant 3, which also slow zombies down); plants 6 and 7 are defensive plants (‘tall nuts’ and ‘umbrella leaves’); plant 1 is actually a plant pot (only needed on roof levels as there is no soil as there is in garden levels). To the right of the plant pot are the available suns (in *last stand* puzzles, most of the suns available during an adventure are available at the outset). The zombies (not present at this stage in this game) come in *waves,*
mainly from the right hand side of the screen; the exceptions to this are bungee jumping zombies who can land zombies to, or steal plants from, the left hand side of the screen. Once an onslaught has been successfully defended, the player gets an additional 500 suns.

Figure 3 shows a possible configuration of plants and the start of the first wave of zombies. It is not a particularly good configuration but serves initial explanatory purposes at this point in this section. Rows (of 5) of plants 2, 3 and 7 have been planted. The cost of these rows is \(5 \times 100 + 5 \times 300 + 5 \times 125 = 2675\) (suns) and