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Salvatore Greco
Matthias Ehrgott
José Rui Figueira *Editors*

Multiple Criteria Decision Analysis

State of the Art Surveys

Second Edition



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José Rui Figueira
Editors

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Volume 1 and 2

Second Edition



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Introduction

José Rui Figueira, Salvatore Greco, and Matthias Ehrgott

1 Ten Years of Success of Multiple Criteria Decision Analysis and Reasons for This New Edition

After 10 years we present an updated revision of the collection of state-of-the-art surveys on Multiple Criteria Decision Analysis (MCDA). This is a good occasion to briefly comment on the latest advances in the domain. We believe that in the last 10 years we have seen great progress of MCDA, from both a theoretical point of view and a real-life application point of view. We have seen the consolidation of the main “traditional” methodologies such as multiple attribute utility theory, outranking methods, interactive multiobjective optimization, as well as the growing success of new approaches such as Evolutionary Multiobjective Optimization (EMO). The spectrum of applications has been constantly expanding with particular emphasis on very complex problems such as industrial design or grid optimization. Taking into account this evolution of the domain, we partly modified the structure and the content of the book giving space to new methodologies (e.g., EMO or multi-criteria portfolio decision analysis for project selection) or splitting chapters into several new ones (e.g., the chapter on multiobjective programming of the previous edition that has now been substituted by three chapters, one on vector and set optimization, one on continuous multiobjective, and one on multiobjective combinatorial optimization). Moreover, all authors, sometimes with the help of

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a new colleague, have updated the contents of their contributions incorporating the novelties of the last 10 years. Of course, many sophisticated technical details that appear in the new edition of the book will sooner or later be destined to be superseded by the incessant evolution of research and applications. We think, however, that the basic principles as stated by the experts who prepared the different chapters in the book will remain reference points for the years to come. Moreover, we believe that the spirit with which experts on MCDA are working today, in these so rich and fruitful years, will remain forever in this book. This spirit is strongly related with the spirit with which, in the late 1960s and early 1970s of the last century, the “pioneers” (many of who are among the many authors of the chapters in this book) outlined the basic principles of MCDA with the genuine aim to give a satisfactory answer to concrete real world problems for which the classical methods of operations research were not able to find adequate answers. Therefore the basic principles of the presented methodologies and their relationships with the MCDA spirit are things that we recommend the reader to look for in each chapter. After these words about the intuition that guided the revision of this book, let us enter “in medias res”, coming back to the introduction of the first edition that was of course also updated.

2 Human Reflection About Decision

Decision-making has inspired reflections of many thinkers since ancient times. The great philosophers Aristotle, Plato, and Thomas Aquinas, to mention only a few, discussed the capacity of humans to decide and in some manners claimed that this possibility is what distinguishes humans from animals. To illustrate some important aspects of decision-making, let us briefly quote two important thinkers, Ignatius of Loyola (1491–1556) and Benjamin Franklin (1706–1790).

To consider, reckoning up, how many advantages and utilities follow for me from holding the proposed office or benefice [...], and, to consider likewise, on the contrary, the disadvantages and dangers which there are in having it. Doing the same in the second part, that is, looking at the advantages and utilities there are in not having it, and likewise, on the contrary, the disadvantages and dangers in not having the same. [...] After I have thus discussed and reckoned up on all sides about the thing proposed, to look where reason more inclines: and so, according to the greater inclination of reason, [...], deliberation should be made on the thing proposed.

This fragment from the “Spiritual Exercises” of St. [14] has been taken from a paper by Fortemps and Slowinski [12].

London, Sept 19, 1772

Dear Sir,

In the affair of so much importance to you, wherein you ask my advice, I cannot, for want of sufficient premises, advise you what to determine, but if you please I will tell you how. [...], my way is to divide half a sheet of paper by a line into two columns; writing over the one Pro, and over the other Con. [...] When I have thus got them all together in one view, I endeavor to estimate their respective weights; and where I find two, one on each side, that

seem equal, I strike them both out. If I find a reason pro equal to some two reasons con, I strike out the three. If I judge some two reasons con, equal to three reasons pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly. [...] I have found great advantage from this kind of equation, and what might be called moral or prudential algebra. Wishing sincerely that you may determine for the best, I am ever, my dear friend, yours most affectionately.
B. Franklin

This letter from Benjamin Franklin to Joseph Prestly has been taken from a paper by MacCrimmon [17].

What is interesting in the above two quotations is the fact that decision is strongly related to the comparison of different points of view, some in favor and some against a certain decision. This means that decision is intrinsically related to a plurality of points of view, which can roughly be defined as criteria. Contrary to this very natural observation, for many years the only way to state a decision problem was considered to be the definition of a single criterion, which amalgamates the multidimensional aspects of the decision situation into a single scale of measure. For example, even today textbooks of operations research suggest to deal with a decision problem as follows: To first define an objective function, i.e., a single point of view like a comprehensive profit index (or a comprehensive cost index) representing the preferability (or dis-preferability) of the considered actions and then to maximize (minimize) this objective. This is a very reductive, and in some sense also unnatural, way to look at a decision problem. Thus, for at least 40 years, a new way to look at decision problems has more and more gained the attention of researchers and practitioners. This is the approach considered by Loyola and Franklin, i.e., the approach of explicitly taking into account the pros and the cons of a plurality of points of view, in other words the domain of multiple criteria decision analysis. Therefore, MCDA intuition is closely related to the way humans have always been making decisions. Consequently, despite the diversity of MCDA approaches, methods and techniques, the basic ingredients of MCDA are very simple: A finite or infinite set of actions (alternatives, solutions, courses of action, ...), at least two criteria, and, obviously, at least one decision-maker (DM). Given these basic elements, MCDA is an activity which helps making decisions mainly in terms of choosing, ranking, or sorting the actions.

3 Technical Reflection About Decision: MCDA Researchers Before MCDA

Of course, not only philosophers reasoned about decision. Many important technical aspects of MCDA are linked to classic works in economics, in particular, welfare economics, utility theory, and voting-oriented social choice theory (see [27]). Aggregating the opinion or the preferences of voters or individuals of a community into collective or social preferences is quite similar a problem to

devising comprehensive preferences of a decision-maker from a set of conflicting criteria in MCDA [7].

Despite the importance of Ramon Llull's (1232–1316) and Nicolaus Cusanus' (1401–1464) concerns about and interests in this very topic, the origins of voting systems are often attributed to Le Chevalier Jean-Charles de Borda (1733–1799) and Marie Jean Antoine Nicolas de Caritat (1743–1794), Le Marquis de Condorcet. However, Ramon Llull introduced the pairwise comparison concept before Condorcet [13], while Nicolaus Cusanus introduced the scoring method about three and a half centuries before Borda [26]. Furthermore, it should be noted that a letter from Pliny the Younger ($\approx$ AD 105) to Titus Aristo shows that he introduced the ternary approval voting strategy and was interested in voting systems a long time before Ramon Llull and Nicolaus Cusanus [18, Chapter 2]. Anyway, Borda's scoring method [4] has some similarities with current utility and value theories as has Condorcet's method [10] with the outranking approach of MCDA. In the same line of concerns, i.e., the aggregation of individual preferences into collective ones, Jeremy Bentham (1748–1832) introduced the utilitarian calculus to derive the total utility for the society from the aggregation of the personal interests of the individuals of a community [3]. Inspired by Bentham's works, Francis Ysidro Edgeworth (1845–1926), a utilitarian economist, was mainly concerned with the maximization of the utility of the different competing agents in an economy. Edgeworth tried to find the competitive equilibrium points for the different agents. He proposed to draw indifference curves (lines of equal utility) for each agent and then derive the contract curve, a curve that corresponds to the notion of the Pareto or efficient set [20]. Not long afterward, Vilfredo Federico Damaso Pareto (1848–1923) gave the following definition of *ophelimity* [utility] for the whole community [21].

We will say that the members of a collectivity enjoy maximum *ophelimity* in a certain position when it is impossible to find a way of moving from that position very slightly in such a manner that the *ophelimity* enjoyed by each of the individuals of that collectivity increases or decreases. That is to say, any small displacement in departing from that position necessarily has the effect of increasing the *ophelimity* which certain individuals enjoy, of being agreeable to some, and disagreeable to others.

From this definition it is easy to derive the concept of dominance, which today is one of the fundamental concepts in MCDA.

MCDA also benefits from the birth and development of game theory. Félix Edouard Justin Emile Borel (1871–1956) and John von Neumann (1903–1957) are considered the founders of game theory [5, 6, 19, 29]. Many concepts from this discipline had a strong impact on the development of MCDA.

The concept of efficient point was first introduced in 1951 by Tjalling Koopmans (1910–1985) in his paper “Analysis of production as an efficient combination of activities” [15].

A possible point in the commodity space is called efficient whenever an increase in one of its coordinates (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of a good).

In the same year (1951) Harold William Kuhn (born 1925) and Albert William Tucker (1905–1995) introduced the concept of vector maximum problem [16]. In the 1960s, basic MCDA concepts were explicitly considered for the first time. As two examples we mention Charnes' and Cooper's works on goal programming [8] and the proposition of ELECTRE methods by Roy [22]. The 1970s saw what is conventionally considered the "official" starting point of MCDA, the conference on "Multiple Criteria Decision Making" organized in 1972 by Cochrane and Zeleny at Columbia University in South Carolina [9]. Since then MCDA has seen a tremendous growth which continues today.

4 Reasons for This Collection of State-of-the-Art Surveys

The idea of MCDA is so natural and attractive that thousands of articles and dozens of books have been devoted to the subject, with many scientific journals regularly publishing articles about MCDA. To propose a new collection of state-of-the-art surveys of MCDA in so rich a context may seem a rash enterprise. Indeed, some objections come to mind. There are many and good handbooks and reviews on the subject (to give an idea consider [1, 11, 24, 25, 28]). The main ideas are well established for some years and one may question the contributions this volume can provide. Moreover, the field is so large and comprises developments so heterogeneous that it is almost hopeless to think that an exhaustive vision of the research and practice of MCDA can be given.

We must confess that at the end of the work of editing this volume we agree with the above remarks. However, we believe that a new and comprehensive collection of state-of-the-art surveys on MCDA can be very useful. The main reasons which, despite our original resistance, brought us to propose this book are the following:

1. Many of the existing handbooks and reviews are not too recent. Since MCDA is a field which is developing very quickly this is an important reason.
2. Even though the field of research and application of MCDA is so large, there are some main central themes around which MCDA research and applications have been developed. Therefore our approach was to try to present the—at least in our opinion—most important of these ideas.

With reference to the first point, we can say that we observed many theoretical developments which changed MCDA over the last 20 years. We tried to consider these changes as much as possible and in this perspective strong points of the book are the following:

1. It presents the most up-to-date discussions on well-established methodologies and theories such as outranking-based methods and MAUT.
2. The book also contains surveys of new, recently emerged fields such as conjoint measurement, fuzzy preferences, fuzzy integrals, rough sets, and others.

Following these points we drafted a list of topics and asked well-known researchers to present them. We encouraged the authors to cooperate with the aim to present different perspectives if topics had some overlap. We asked the authors to present a comprehensive presentation of the most important aspects of the field covered by their chapters, a simple yet concise style of exposition, and considerable space devoted to bibliography and survey of relevant literature. We also requested a sufficiently didactic presentation and a text that is useful for researchers in MCDA as well as for people interested in real-life applications.

The importance of these requirements is also related to the specific way the MCDA community looks at its research field. It can be summarized in the observation that there is a very strong and vital link between theoretical and methodological developments on the one hand and real applications on the other hand. Thus, the validity of theoretical and methodological developments can only be measured in terms of the progress given to real-world practice. Moreover, interest of MCDA to deal with concrete problems is related to the consideration of a sound theoretical basis which ensures the correct application of the methodologies taken into account.

In fact, not only the chapters of our book but rather all MCDA contributions should satisfy the requirements stated out above because they should be not too “esoteric” and therefore understandable for students, theoretically well founded, and applicable to some advantage in reality.

5 A Guided Tour of the Book

Of course, this book can be read from the first to the last page. However, we think that this is not the only possibility and it may not even be the most interesting possibility. In the following we propose a guided tour of the book suggesting some reference points that are hopefully useful for the reader.

5.1 Part I: The History and Current State of MCDA

This part is important because MCDA is not just a collection of theories, methodologies, and techniques, but a specific perspective to deal with decision problems. Losing this perspective, even the most rigorous theoretical developments and applications of the most refined methodologies are at risk of being meaningless because they miss an adequate consideration of the aims and of the role of MCDA. We share this conviction with most MCDA researchers.

From this perspective it is important to have a clear vision of the origin of the main basic concepts of the domain. For this reason, Murat Köksalan, Jyrki Wallenius, and Stanley Zionts present the early history of MCDA and related areas showing how many developments in the field were made by major contributors to operations research, management science, economics, and other areas.

Then Bernard Roy discusses “pre-theoretical” assumptions of MCDA and gives an overview of the field. Bernard Roy, besides making many important theoretical contributions, engaged himself in thorough reflections on the meaning and the value of MCDA, proposing some basic key concepts that are accepted throughout the MCDA community.

5.2 Part II: Foundations of MCDA

This part of the book is related to a fundamental problem of MCDA, the representation of preferences. Classically, for example in economics, it is supposed that preference can be represented by a utility function assigning a numerical value to each action such that the more preferable an action, the larger its numerical value. Moreover, it is very often assumed that the comprehensive evaluation of an action can be seen as the sum of its numerical values for the considered criteria. Let us call this the classical model. It is very simple but not too realistic. Indeed, there is a lot of research studying under which conditions the classical model holds. These conditions are very often quite strict and it is not reasonable to assume that they are satisfied in all real-world situations. Thus, other models relaxing the conditions underlying the classical model have been proposed. This is a very rich field of research, which is first of all important for those interested in the theoretical aspects of MCDA. However, it is also of interest to readers engaged in applications of MCDA. In fact, when we adopt a formal model it is necessary to know what conditions are supposed to be satisfied by the preferences of the DM. In the two chapters of this part, problems related to the representations of preferences are discussed.

Stefano Moretti, Meltem Öztürk, and Alexis Tsoukiàs present a very exhaustive review of preference modeling, starting from classical results but arriving at the frontier of some challenging issues of scientific activity related to fuzzy logic and non-classical logic.

Denis Bouyssou and Marc Pirlot discuss the axiomatic basis of the different models to aggregate multiple criteria preferences. We believe that this chapter is very important for the future of MCDA. Initially, the emphasis of MCDA research was on proposal of new methods. But gradually the necessity to understand the basic conditions underlying each method and its specific axiomatization became more and more apparent. This is the first book on MCDA with so much space dedicated to the subject of foundations of MCDA.

5.3 Part III: Outranking Methods

In this part of the book the class of outranking-based multiple criteria decision methods is presented. Given what is known about the decision-maker’s preferences

and given the quality of the performances of the actions and the nature of the problem, an outranking relation is a binary relation S defined on the set of potential actions A such that aSb if there are enough arguments to decide that a is at least as good as b , whereas there is no essential argument to refute that statement [23]. Methods which strictly apply this definition of outranking relation are the ELECTRE methods. They are very important in many respects, not least historically, since ELECTRE I was the first outranking method [2].

However, within the class of outranking methods we generally consider all methods which are based on pairwise comparison of actions. Thus, another class of very well-known multiple criteria methods, PROMETHEE methods, is considered in this part of the book. Besides ELECTRE and PROMETHEE methods, many other interesting MCDA methods are based on the pairwise comparison of actions. José Figueira, Vincent Mousseau, and Bernard Roy present the ELECTRE methods; Jean-Pierre Brans and Yves De Smet present the PROMETHEE methods; and Jean-Marc Martel and Benedetto Matarazzo review the rich literature of other outranking methods.

5.4 Part IV: Multi-attribute Utility and Value Theories

In this part of the book we consider multiple attribute utility theory (MAUT). This MCDA approach tries to assign a utility value to each action. This utility is a real number representing the preferability of the considered action. Very often the utility is the sum of the marginal utilities that each criterion assigns to the considered action. Thus, this approach very often coincides with what we called the classical approach before. As we noted in commenting Part I, this approach is very simple at first glance. It is often applied in real life, e.g., every time we aggregate some indices by means of a weighted sum, we are applying this approach. Despite its simplicity, the approach presents some technical problems. The first is related to the axiomatic basis and the construction of marginal utility functions (i.e., the utility functions relative to each single criterion), both in case of decision under certainty and uncertainty. These problems are considered by James Dyer in a comprehensive chapter about the fundamentals of this approach.

Yannis Siskos, Vangelis Grigoroudis, and Nikolaos Matsatsinis present the very well-known UTA methods, which on the basis of the philosophy of the aggregation–disaggregation approach and using linear programming build a MAUT model that is as consistent as possible with the DM’s preferences expressed in actual previous decisions or on a “training sample”. The philosophy of aggregation–disaggregation can be summarized as follows: How is it possible to assess the decision-maker’s preference model leading to exactly the same decision as the actual one or at least the most “similar” decision?

Thomas Saaty presents a very well-known methodology to build utility functions, the AHP (Analytic Hierarchy Process), and its more recent extension, the ANP (Analytic Network Process). AHP is a theory of measurement that uses pairwise

comparisons along with expert judgments to deal with the measurement of qualitative or intangible criteria. The ANP is a general theory of relative measurement used to derive composite priority ratio scales from individual ratio scales that represent relative measurements of the influence of elements that interact with respect to control criteria. The ANP captures the outcome of dependence and feedback within and between clusters of elements. Therefore AHP with its dependence assumptions on clusters and elements is a special case of the ANP.

Carlos Bana e Costa, Jean-Claude Vansnick, and Jean-Marie De Corte present another MCDA methodology based on the additive utility model. This methodology is MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique). It is an MCDA approach that requires only qualitative judgments about differences of values of attractiveness of one action over another action to help an individual or a group to quantify the relative preferability of different actions. In simple words, the MACBETH approach tries to answer the following questions: How can we build an interval scale of preferences on a set of actions without forcing evaluators to produce direct numerical representations of their preferences? How can we coherently aggregate these qualitative evaluations using an additive utility model?

5.5 Part V: Non-classical MCDA Approaches

Many approaches have been proposed in MCDA besides outranking methods and multi-attribute utility theory. In this part of the book we try to collect information about some of the most interesting proposals. First, the question of uncertainty in MCDA is considered. Theo Stewart and Ian Durbach discuss risk and uncertainty in MCDA. It is necessary to distinguish between internal uncertainties (related to decision-maker values and judgments) and external uncertainties (related to imperfect knowledge concerning consequences of actions). The latter, corresponding to the most accepted interpretation of uncertainty in the specialized literature, has been considered in the chapter. Four broad approaches for dealing with external uncertainties are discussed. These are multi-attribute utility theory and some extensions; stochastic dominance concepts, primarily in the context of pairwise comparisons of alternatives; the use of surrogate risk measures such as additional decision criteria; and the integration of MCDA and scenario planning.

Salvatore Greco, Benedetto Matarazzo, and Roman Słowiński present the decision rule approach to MCDA. This approach represents the preferences in terms of “if . . . , then . . .” decision rules such as, for example, “if the maximum speed of car x is at least 175 km/h and its price is at most \$12000, then car x is comprehensively at least medium”. This approach is related to rough set theory and to artificial intelligence. Its main advantages are the following. The DM gives information in the form of examples of decisions, which requires relatively low cognitive effort and which is quite natural. The decision model is also expressed in a very natural way by decision rules. This permits an absolute transparency of the methodology for the

DM. Another interesting feature of the decision rule approach is its flexibility, since any decision model can be expressed in terms of decision rules and, even better, the decision rule model can be much more general than all other existing decision models used in MCDA.

Michel Grabisch and Christophe Labreuche present the fuzzy integral approach that is known in MCDA for the last two decades. In very simple words this methodology permits a flexible modeling of the importance of criteria. Indeed, fuzzy integrals are based on a capacity which assigns an importance to each subset of criteria and not only to each single criterion. Thus, the importance of a given set of criteria is not necessarily equal to the sum of the importance of the criteria from the considered subset. Consequently, if the importance of the whole subset of criteria is smaller than the sum of the importances of its individual criteria, then we observe a redundancy between criteria, which in some way represents overlapping points of view. On the other hand, if the importance of the whole subset of criteria is larger than the sum of the importances of its members, then we observe a synergy between criteria, the evaluations of which reinforce one another. On the basis of the importance of criteria measured by means of a capacity, the criteria are aggregated by means of specific fuzzy integrals, the most important of which are the Choquet integral (for cardinal evaluations) and the Sugeno integral (for ordinal evaluations).

Helen Moshkovich, Alexander Mechitov, and David Olson present the verbal decision methods MCDA. This is a class of methods originated from the work of one of the MCDA pioneers, the late Oleg Larichev. The idea of verbal decision analysis is to build a decision model using mostly qualitative information expressed in terms of a language that is natural for the DM. Moreover, measurement of criteria and preference elicitation should be psychologically valid. The methods, besides being mathematically sound, should check the DM's consistency and provide transparent recommendations.

Most real-world decision problems take place in a complex environment where conflicting systems of logic, uncertain, and imprecise knowledge, and possibly vague preferences have to be considered. To face such complexity, preference modeling requires the use of specific tools, techniques, and concepts which allow the available information to be represented with the appropriate granularity. In this perspective, fuzzy set theory has received a lot of attention in MCDA for a long time. Didier Dubois and Patrice Perny try to provide a tentative assessment of the role of fuzzy sets in decision analysis, taking a critical standpoint on the state-of-the-art, in order to highlight the actual achievements and trying to better assess what is often considered debatable by decision scientists observing the fuzzy decision analysis literature.

5.6 Part VI: Multiobjective Optimization

The classical formulation of an operations research model is based on the maximization or minimization of an objective function subject to some constraints. A very

rich and powerful arsenal of methodologies and techniques has been developed and continues to be developed within operations research. However, it is very difficult to summarize all the points of view related to the desired results of the decision at hand in only one objective function. Thus, it seems natural to consider a very general formulation of decision problems where a set of objective functions representing different criteria have to be “optimized”. To deal with these types of problems requires not only to generalize the methodologies developed for classical single-objective optimization problems, but also to introduce new methodologies and techniques permitting to compare different objectives according to the preferences of the DM. In this part of the book we tried to give adequate space to these two sides of multiobjective programming problems.

Gabriele Eichfelder and Johannes Jahn discuss recent developments of vector and set optimization. Based on the concept of a pre-order, optimal elements are defined. In vector optimization, properties of optimal elements and existence results are gained. Further, an introduction to vector optimization with a variable ordering structure is given. In set optimization basic concepts are summed up.

Margaret Wiecek, Matthias Ehrgott, and Alexander Engau present their view of the state-of-the-art in continuous multiobjective programming. After an introduction they formulate the multiobjective program (MOP) and define the most important solution concepts. They summarize properties of efficient and nondominated sets and review optimality conditions and solution techniques for MOPs and approximation of efficient and nondominated sets. They discuss also specially structured problems including linear, nonlinear, parametric, and bi-level MOPs, and finally they present a perspective on future research directions.

Within the general field of multiobjective programming, research on combinatorial optimization problems with multiple objectives has been particularly active. Matthias Ehrgott, Xavier Gandibleux, and Anthony Przybylski review exact methods for multiobjective combinatorial optimization problems, covering extensions of single objective algorithms to the multiobjective case, scalarization approaches, the two-phase method and branch and bound algorithms.

Masahiro Inuiguchi, Kosuke Kato, and Hideki Katagiri review fuzzy multicriteria optimization focusing on possibilistic treatments of objective functions with fuzzy coefficients and on interactive fuzzy stochastic multiple objective programming approaches.

Dylan Jones and Mehrdad Tamiz present a review of the field of goal programming describing the current range of goal programming variants and the range of techniques that goal programming has been combined or integrated with is discussed. A range of modern applications of goal programming are also given.

Kaisa Miettinen, Jussi Hakanen, and Dmitry Podkopaev give an overview of interactive methods for solving multi-objective optimization problems. In interactive methods, the decision-maker progressively provides preference information so that her or his most satisfactory Pareto optimal solution can be found. The basic features of several methods are introduced and some theoretical results are provided. In addition, references to modifications and applications as well as to other methods are indicated. As the role of the decision-maker is very important