




Vibration Theory and Applications with Finite Elements and Active Vibration Control

Alan Palazzolo

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Preface

The material in this text is drawn from the author's 35 years of teaching, research, and industrial experiences in the areas of vibrations, finite elements, dynamics, and feedback control. The teaching experiences include both undergraduate and graduate course instruction in vibrations, graduate courses in finite elements and boundary elements, and undergraduate courses in controls and dynamics. The research experiences include performing sponsored research for NASA Glenn and Marshall, ONR, ARL, DOE, and a host of industrial companies. The industrial experience is drawn from employment at Southwest Research Institute, Bently Nevada, and Allis-Chalmers Corp.

The pedagogical motivation for this book resulted from a desire to fulfill the following perceived needs of college and university students and practicing engineers and scientists for learning vibrations and finite elements:

1. Provide a convincing and motivational first chapter about "why" the material presented in the book is important. As was so eloquently expressed in the writings of John Henry Newman, learning for the sake of exercising the intellectual dimension of the person is an important activity which enriches our life experience and strengthens our reasoning faculties with endless benefits. This is very true; however, the application of this knowledge to better understand nature and direct it to better the human condition through engineering practice is also an important motivational benefit for the reader. For this reason, Chapter 1 provides an overview of everyday vibration experiences, fundamental concepts, deleterious effects of vibrations, and industrial standards. A brief introduction to the concept of finite elements is also included, which is the most common tool for vibration analysis in industry and research. Finally, Chapter 1 concludes with a discussion of the concept of active vibration control, which is one area of "smart" technologies that seem ubiquitous in engineering and popular news.
2. Provide a chapter that contains subjects that need to be initially grasped in order to more quickly comprehend and utilize the core material that appears in the later chapters. The preparation subjects include computer coding, mathematical theory, modeling, and kinematic constraints. Quite often, the above subjects are folded into the major areas such as free and forced vibration theory or are relegated to appendices that in the author's experience are rarely referenced. Chapter 2 presents these preparatory subjects in an isolated, front, and center manner with the goal of providing a solid background for the reader before he or she embarks on using them in sometimes subtle steps, nested in more complicated vibration theory and problems. Divide and conquer is a very effective strategy in vibrations, as in all educational pursuits!

   This chapter also introduces the student to the use of the symbolic math codes MAPLE and MATLAB symbolic. These tools are utilized in many of the chapters to facilitate complicated and tedious algebraic and differential and integral calculus calculations in an elegant and minimal error manner. Working knowledge of these tools will aid the reader in many areas of engineering practice which fulfills a holistic learning goal of engineering education.

   The common thread for implementation of structural modeling methods is kinematic constraints (deformation assumptions). As discussed in Chapter 2, rigid body, assumed modes (Rayleigh–Ritz), finite elements, and boundary element models all impose kinematic constraints (deformation assumptions) that reduce the dimensionality of a structural model in order to provide a practical solution path to its governing differential equations. Understanding this common thread removes some of the apprehension for learning or instructing assumed modes or finite elements, when they are viewed as extensions of more elementary and familiar kinematic constraints. These may include coupled rigid body systems or examples from strength of materials, for example, plane sections remain plane in the beam deformation theory.

3. Provide initial chapters that elucidate the understanding and application of Newton's laws (Chapter 3) and the energy-based (Chapter 4) approaches (conservation and Lagrange equations) for deriving governing differential equations. Instructional experience has so often exposed the inability of students to derive accurate governing differential equations, prior to utilizing them to obtain vibration-related response characteristics such as natural frequencies, transient response, etc. The student's solution is maimed by an erroneous governing equation from the start, and the ensuing results are misleading and often nonsensical. This occurs in spite of the possible mastery of other modeling, simulation, and presentation skills. Frankly speaking, garbage in leads to garbage out (no matter how polished and visual is its presentation format).

Chapter 4 demonstrates that the most widely used engineering vibration simulation tool—finite elements—follows naturally from Lagrange equations with the removal of the kinematic (deformation) constraints (assumptions) of the simpler models. This requires a somewhat rigorous demonstration that Lagrange's equations are valid for flexible members and their assemblages in structural systems. Most texts leap over this demonstration by implicitly invoking a variant of "it can be shown." Thus, it is presupposed that the demonstration of the generalized force of a coil spring being obtained from the derivative of its potential energy is sufficient for justifying the application of the same approach for modeling the elastic properties of a 10 000-degree-of-freedom solar panel array on a satellite. This approach, although ultimately valid, is deficient for leaving an important gap in the sound understanding of the approach by the reader. The chapter also provides detailed derivations of the Lagrange equations for rigid body, assumed modes, and finite element-type models with a wide variety of stiffness and damping interconnections.

The assumed modes section of Chapter 4 is included for its intrinsic modeling value and as an introduction to the finite element approach. The chapter also utilizes bar/truss elements for the initial presentation of deriving finite element stiffness and mass matrices and force vectors and for the matrix assembly procedure. The assembly procedure is presented with significant detail for fully automating in a computer code, for both free and constrained structures. The method presented is nearly universal and is applied without significant modification for beams in Chapter 9, 2D and axisymmetric solids and membranes in Chapter 10, and 3D solids in Chapter 11.

Symbolic math examples are provided in both **Chapters** 3 and 4 to demonstrate their usage for automating steps in deriving equations of motion, such as substitutions, combination, sorting, integrations, and differentiations, which typically are steps prone to error when worked by hand.

4. Provide a more pedagogically effective approach for instructing free, transient, and harmonic vibrations as compared with the traditional approach. The major simulation application areas of vibrations—free vibrations (F), transient forced vibrations (T), and steady-state forced harmonic vibrations (SSH)—are treated in **Chapters** 5, 6, and 7, respectively. A pedagogical goal for this arrangement of the text was to provide uninterrupted treatments of these three major areas of vibrations. The format of many vibrations textbooks is frequently to present F, then T, and then SSH for single-degree-of-freedom models; next to present F, then T, and then SSH for 2-degree-of-freedom models; then to present F, then T, and then SSH for multiple-degree-of-freedom models; and finally to present F, then T, and then SSH for continuous member models. The author's pedagogical experience is that a more effective approach is to cover F for 1, 2, and multiple degree of freedom and continuous members and then present similar learning sequences for T and finally for SSH. Instructing free vibrations from single degree of freedom through continuous members without multiple circulations through transient and SSH vibrations seems far less confusing and more effective and logical. A similar conclusion holds for treatments of transient vibrations and steady-state harmonic vibrations. In the author's opinion, the format of prior texts, as outlined above, has lost a significant justification with the advent of modern math tools, which have greatly lessened the solution difficulties encountered in transitioning from single-degree-of-freedom to 2-degree-of-freedom to *N*-degree-of-freedom models.

Chapter 5 expands the conventional content covered in free vibrations by including treatment of rotating systems with gyroscopic moments, the destabilizing effect of circulatory forces, flexible unconstrained structures, orthogonal damping matrices, and unstable systems. Likewise, Chapter 6 expands the conventional content covered in transient vibrations by including response spectrums, modal condensation for general $\underline{M}, \underline{K}$, and $\underline{C}$ systems, flexible unconstrained structures, base excitation, participation factor and modal effective mass, and numerical integration methods. Finally, Chapter 7 expands the conventional content covered in steady-state harmonic response by including peak amplitude and frequency for the simple single-degree-of-freedom oscillator (SDOFO), parameter identification methods for the SDOFO, high spot–heavy spot and influence coefficient balancing for a simple Jeffcott rotor, demonstration that resonance may occur in any general $\underline{M}, \underline{K}$, and $\underline{C}$ linearized vibrating system, use of receptances for the synthesis of substructures and mode shape identification, and use of the modal assurance criterion (MAC) for mode shape correlation.

5. Provide a treatment of techniques for improving computational efficiency for larger-order models by utilizing approximate methods. Large-scale finite element models are utilized throughout industry and in research and economic solutions are typically a necessity. Long run times inhibit use of optimization approaches such as genetic algorithm guided design which requires a large multitude of simulations with parameter variations. Modal condensation for accelerating system transient solutions is covered very thoroughly in Chapter 6, including use of the modal acceleration method. Chapter 7 also introduces a receptance approach for economically determining the response of coupled substructures through receptance synthesis. Chapter 8 covers other areas for economic, large-order system model solutions including Guyan reduction-static condensation, substructures–superelements, modal synthesis, eigenvalue–eigenvector perturbations with reanalysis, and the Rayleigh quotient approach.

6. Provide an in-depth presentation of finite elements that far surpasses the conventional content of only 2D Euler–Bernoulli beams and present an implementation algorithm that is universally applicable among the various types of elements and treats both fixed and time-varying boundary conditions. This goal reflects the author's experiences with finite elements in industry and research, namely, various types of elements are utilized and in most cases 3D models are inevitably required. Chapter 9 presents theory and examples for 2D Euler–Bernoulli and 2D and 3D Timoshenko beams with shear deformation effects. The Timoshenko beam development includes a truly "consistent mass matrix" utilizing the Timoshenko shape functions, derived from the solution of the beam's static governing equations, in the kinetic energy expression for deriving the element mass matrix. Most developments employ a lumped mass formulation or an "inconsistent" mass matrix formulation utilizing Timoshenko beam shape functions for displacements and Euler–Bernoulli (Hermite cubic polynomials) shape functions for velocities in the beam's kinetic energy expression. A general approach and accompanying 2D beam-frame example are provided for the case of imposed motion excitation at boundary points in a finite element model. The Timoshenko beam theory presented is for general 3D frames including I beams, box beams, etc. The matrix assembly algorithm presented in Chapter 4 is again utilized for all beam-frame models in Chapter 9. The standard format of only including 2D Euler–Bernoulli beams in vibration texts is clearly surpassed in Chapter 10 which includes treatment of 2D solid elements for plane stress and plane strain and axisymmetric and 2D vibrating membranes. Detailed algorithms are provided for determining stresses at interior and surface points for use in high-cycle fatigue studies. Both bilinear (2 node) and quadratic (9 node) isoparametric element formulations are presented. The extra (incompatible) shape function approach is utilized in order to accelerate convergence especially in 2D bending-type problems. Most commercial finite element codes utilize automated mesh generators with lower-order finite element models. The formulation for a constant strain triangle is presented for this purpose. The MATLAB code MESH2D is utilized for creating an automated triangular element mesh, which is then solved for natural frequencies and mode shapes. Large-order problems create large systems of linear algebraic equations that must be solved for the unknown nodal vibrations. The corresponding matrices may be highly sparse as described by a small bandwidth to order ratio. This fact may be exploited to economize on the required computation time for solving the equations. A banded solver assembly procedure and coding are provided and demonstrated with a steady-state harmonic vibration response example.

Chapter 11 provides theory, assembly procedures, and an example for a general 8-node, 3D solid (brick) hexahedral isoparametric element including extra (incompatible) shape functions for improved bending deformation modeling. The example reveals modes and natural frequencies that are absent from the corresponding 2D solid and Timoshenko beam models. A detailed discussion is provided for determining interior and surface point stresses for usage in high-cycle fatigue studies.

7. Provide an intermediate-level treatment of active vibration control (AVC) which is often categorized as an area of smart structures and materials. The need for lightweight, high-performance structures, vehicles, machines, and devices that may be required to function in extreme environments and adapt to various operating conditions has spawned a vast amount of research and development efforts in AVC. Chapter 12 provides in-depth treatments of both electromagnetic and piezoelectric actuator types, ideal (infinite) and finite bandwidth modeling and effects, and closed-loop stability and steady-state response determination. Closed-loop feedback control models that assume infinite bandwidths for all feedback components (sensors, controllers, power amplifiers, and actuators) are prone to miss unstable poles that appear in the as-built system and preclude the use of predetermined design feedback gains. This point is elucidated by both theory and example in Chapter 12. Examples are provided for systems with electromagnetic actuators or with piezoelectric stack or patch (layer) actuators.

8. Provide an appendix which contains a summary of the basic equations of elasticity (equilibrium, constitutive law, strain displacement, compatibility, strain energy) for easy reference when deriving the assumed modes and finite element stiffness matrices.

All chapters have a generous number of EXERCISES. Limitations on the size and cost of the textbook precluded including the EXERCISES within the textbook. The exercises are accessible from a dedicated website (www.wiley.com/go/palazzolo), which is maintained by Wiley. The website contains a wide variety of intermediate to challenging exercises. A typed solution manual for the exercises is available from Wiley for instructors.

MATLAB and MAPLE codes are utilized in the examples throughout the text and in the exercise solutions. Many of the code listings are contained in the chapters or in Appendices B–F. The remaining codes are provided in a dedicated website (www.wiley.com/go/palazzolo) maintained by Wiley for instructors.

Limitations on the size and cost of the textbook precluded including sections on test instrumentation and sensors, nonlinear vibrations, and random vibrations. These are all very important subjects although much can be obtained on instrumentation and sensors by web search. Other texts that are readily accessible to students have introductory sections on nonlinear and random vibrations. The author has taught nonlinear vibrations at Texas A&M for the past 12 years and is planning a specialized book in this area.

Acknowledgments and Dedication

The author acknowledges the dedicated and excellent effort provided by his wife Chang-chun "Esther" Palazzolo in preparing the figures, typing, formatting, and detailed submission of this book. The author dedicates this book to his parents, Jerome and Beverly Palazzolo; his loving wife, Esther; his children, Stephanie, Elizabeth, and Justin; his siblings, Thomas, Steven, Jerome, and Marianne; and his friends, Msgr. John McCaffrey and Marty and Karen Smith. The author also dedicates the book to all those whose enthusiasm sparked and sustained his interest in vibrations, dynamics, and finite elements instruction and research:

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Many past and present graduate students, all very special sources of enthusiasm, imagination and hope

About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/palazzolo

This website includes:

- Appendices B through F which contain listings of MATLAB and MAPLE Codes for major examples in the text
- Exercises
- Matlab and Maple Codes

Exercises will be updated to reflect reader comments and the database of exercises will be expanded. This technological innovation will make the text a 'living' document, whilst having an expanding and polished Exercises section on a website reduces the size and cost of the book.

List of Acronyms

AF	amplification factor
AFM	atomic force microscope
AM	amplitude modulation
ANS	American National Standard
API	American Petroleum Institute
AVC	active vibration control
BC	boundary conditions
CCW	counterclockwise
COAM	conservation of angular momentum
COLM	conservation of linear momentum
CS	commercial software
CV	complex variables
DCTM	direction cosine transformation matrices
DE	differential equation
DFCA	“degree of freedom” connectivity array
DOF(s)/dof(s)	degree(s) of freedom
DSP	digital signal processor
EA	electrorestrictive actuator
EM	electromagnetic
ENI	Euler numerical integration
EOM(s)	equation of motion(s)
ES	equilibrium state
FBD(s)	free body diagram(s)
FE	finite element
FEM(s)	finite element method(s)
FRF(s)	frequency response function(s)
FS	Fourier series
GP	Gauss points
GQ	Gauss quadrature
GRSC	Guyan reduction and static condensation
HAVS	hand-arm vibration syndrome
HCF	high-cycle fatigue
HP	high pressure
HST	Hubble space telescope
HVAC	heating, ventilation and air-conditioning
IC(s)	initial condition(s)
IP	Intermediate pressure
ISO	International Standard Organization
ISS	international space station
JRM	Jeffcott rotor model
LE	Lagrange’s equation
LHS	left-hand side
LP	low pressure
LPF	low-pass filters

LT	Laplace transform
LTM	Laplace transform method
MA	mode acceleration
MAC	modal assurance criterion
MB	magnetic bearings
ME	microgravity experiment
MIMO	multiple input multiple output
MSF	mode scale factor
NASA	National Aeronautics and Space Administration
NB	Newmark beta
NCA	"nodal" connectivity array
NI	numerical integration
NIOSH	National institute of occupational safety and health
ODE(s)	ordinary differential equation(s)
OP	operating point
OSR	operating speed range
PD	proportional-derivative
PDE	partial differential equation
PE	potential energy
PID	proportional-integral-derivative
PLA	piezoelectric layer (patch) actuator
RB	rigid body
RBM	rigid body model
RBMPI	receptance-based modal parameter identification
REOM	rotational equation of motion
RHS	right hand side
RK	Runge-Kutta
RK4NI	fourth-order Runge-Kutta numerical integration
rms	root mean square
SDOF	single-degree-of-freedom
SDOFO	single-degree-of-freedom oscillator
SEP	static equilibrium position/static equilibrium reference
SISO	single input single output
SM	separation margin
SPA(s)	servo power amplifier(s)
SSB	simply supported beam
SSHR	steady-state harmonic response
SSME	space shuttle main engine
TB	Timoshenko beam
TEOM	translational equation of motion
VDV	vibration dose value
VM	Von Mises
WBV	whole body vibration

Chapter 1

Background, Motivation, and Overview

1.1 INTRODUCTION

The word "university" is derived from the word "universal" (Newman, 1927) in that the university is the foremost setting for teaching universal knowledge. Philosophy, chemistry, agriculture, mechanics, theology, biology, and so on are all topics of learning, teaching, and exploring at the true university. The study of vibrations is a microcosm of the ideal university, encompassing aspects of dynamics, fluid mechanics, structural deformation and fatigue, electromagnetism, feedback control, sound, and other phenomena. Confronting this, the eager investigator feels great satisfaction in drawing ideas from each area and then forging solutions to vibration problems. As an athlete develops calves and biceps, shoulders, and forearms and then enjoys using these in harmony and mutual support in competition, so the vibration engineer delights in recognizing and using many disciplines to tame vibrations.

With its arsenal of anomalies—fastener looseness, structural member fatigue and failure, noise, internal rubs in machinery, human fatigue and distractions, optical instrumentation and machining errors, and so on—vibration continues to present formidable engineering challenges and to limit energy efficiency and cost reduction in machinery and structures in the twenty-first century. New machinery that pushes the envelopes of efficiency and power density; new structures that stretch the imagination in size, materials, light weights, and locations; and new vehicles that propel us through land, air, sea, and space with ever increasing speed and comfort level all hold great promise for an efficient and convenient future. These advances will come at a price though and vibration will be there to collect its due. *The author extends his best wishes for success to those who meet the vibration challenges that continue to arise in mankind's quest to subdue nature and use its awesome forces for peace, human dignity, and prosperity.*

1.2 BACKGROUND

The following sections provide discussions of many important aspects of vibration. The intent of this section is to provide some basic background material to facilitate understanding of the following sections. Vibration is the study of dynamic motions of mechanical, structural, or anatomical components or systems about their static equilibrium configurations. The motion may be sinusoidal periodic, complex periodic, quasiperiodic, transient, chaotic, or random. Monotone (single-frequency) sinusoidal vibration is characterized by an equilibrium position x_{eq} and the dynamic displacement *amplitude* (A_x), *phase angle* (ϕ_x), *frequency* (f), and *period* (T) as shown in Figure 1.2.1.

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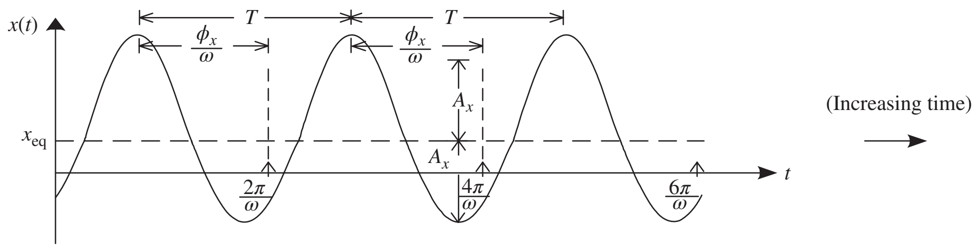

Figure 1.2.1 Pure tone sinusoidal vibration

The period and frequency are related by

$$f = \frac{1}{T} \text{ cycles/s or Hz}, \quad \omega = 2\pi f \text{ circular frequency in rad/s} \tag{1.2.1}$$

Period markers are seen to occur at $0, 2\pi/\omega, 4\pi/\omega, \ldots$. This may represent a once per revolution event on a rotating shaft or just some arbitrarily referenced pulse that indicates the beginning of a new forcing period. The motion is described using the expression

$$x(t) = A_x \cos(\omega t + \phi_x) \tag{1.2.2}$$

The positive peaks occur when the argument of the cosine function is a multiple of 2π, that is,

$$\omega t_{pn} + \phi_x = 2\pi n \quad n = 1, 2, \ldots \tag{1.2.3}$$

which implies

$$t_{pn} \in \left\{ \frac{2\pi}{\omega} - \frac{\phi_x}{\omega}, \frac{4\pi}{\omega} - \frac{\phi_x}{\omega}, \frac{6\pi}{\omega} - \frac{\phi_x}{\omega}, \ldots \right\} \tag{1.2.4}$$

Thus, it is seen by comparison of (1.2.4) and Figure 1.2.1 that the phase angle ϕ_x has a physical interpretation, namely, it provides a measure of the time between $x(t)$ experiencing a positive peak and the occurrence of a period marker. This time lag is

$$\Delta t_p = \frac{\phi_x}{\omega} \tag{1.2.5}$$

The velocity and acceleration expressions are obtained by differentiating (1.2.2)

$$v(t) = \dot{x}(t) = A_v \cos(\omega t + \phi_v) \tag{1.2.6}$$

$$a(t) = \dot{v}(t) = \ddot{x}(t) = A_a \cos(\omega t + \phi_a) \tag{1.2.7}$$

where

$$\begin{aligned} A_v &= \omega A_x, & \phi_v &= \phi_x + \frac{\pi}{2} \\ A_a &= \omega A_v = \omega^2 A_x, & \phi_a &= \phi_v + \frac{\pi}{2} = \phi_x + \pi \end{aligned} \tag{1.2.8}$$

The motion depicted in Figure 1.2.1 could result from displacing or striking the component and allowing it to freely vibrate as in the case of a swing, traffic light, car antenna,