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Edited by

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PREFACE

Vibration problems dealing with advanced Mathematical and Numerical Techniques have extensive application in a wide class of problems in aeronautics, aerodynamics, space science and technology, off-shore engineering and in the design of different structural components of high speed space crafts and nuclear reactors. Different classes of vibration problems dealing with complex geometries and non-linear behaviour require careful attention of scientists and engineers in pursuit of their research activities. Almost all fields of Engineering, Science and Technology, ranging from small domestic building subjected to earthquake and cyclone to the space craft venturing towards different planets, from giant ship to human skeleton, encounter problems of vibration and dynamic loading.

This being truly an interdisciplinary field, where the mathematicians, physicists and engineers could interface their innovative ideas and creative thoughts to arrive at an appropriate solution, Bengal Engineering and Science University, Shibpur, India, a premier institution for education and research in engineering, science and technology felt it appropriate to organize 8th International Conference on "Vibration Problems (ICOVP-2007)" as a part of its sesquicentenary celebration. The conference created a platform and all aspects of vibration phenomenon with the focus on the state-of-the art in theoretical, experimental and applied research areas were addressed and the scientific interaction, participated by a large gathering including eminent personalities and young research workers, generated many research areas and innovative ideas.

This proceedings being published by Spinger containing good number of scientific research articles and state-of-the art lecture by the distinguished scientists and engineers across the globe will definitely be a good reference for the scientists who would be working in the relevant field in theoretical, experimental and applied research areas.

We express our sincere thanks and gratitude to all the guests, participants, speakers, sponsors and supporters for their kind patronage and valued cooperation to make the conference a grand success. Special thanks are due to Ministry of Human Resources, Department of Science and Technology (DST), Council for Scientific and Industrial Research (CSIR), Indian Space Research Organization (ISRO), Govt. of India, Centre for Applied Mathematics and Computational

Science, Saha Institute of Nuclear Physics (SINP), Kolkata, India for their kind cooperation and support for the conference and making the publication of the proceeding a reality.

Prof. D. Sengupta   Prof. N. R. Bandyopadhyay   Prof. B. Mukhopadhyay
Chairperson   Co-Ordinator   Secretary
Organizing Committee   Organizing Committee   Organizing Committee
ICOVP-2007   *ICOVP-2007*   *ICOVP-2007*

"International Conference on Vibration Problems"

ICOVP-2007 and a short history

The 8th International Conference on Vibration Problems (ICOVP-2007) took place in Shibpur, West Bengal, India, between 1-3 February, 2007.

First ICOVP was held during October 27-30, 1990 at A.C. College, Jalpaiguri, under the co-Chairmanship of two scientists, namely, Professor M. M. Banerjee from the host Institution and Professor P. Biswas from the sister organization, A.C. College of Commerce, in the name of "International Conference on Vibration Problems of Mathematics and Physics". The title of the Conference was changed to the present one during the third conference.

The Conferences of these series are:

1. ICOVP-1990, 20-23 October 1990, A.C. College, Jalpaiguri-India,
2. ICOVP-1993, 4-7 November 1993, A.C. College, Jalpaiguri-India,
3. ICOVP-1996, 27-29 November 1996, University of North Bengal, India,
4. ICOVP-1999, 27-30 November 1999, Jadavpur University, West Bengal, India,
5. ICOVP-2001, 8-10 October 2001, (IMASH), Moscow, Russia,
6. ICOVP-2003, 8-12 September 2003, Tech. Univ. of Liberec, Czech Republic,
7. ICOVP-2005, 5-9 September 2005, Işik University, Şile, İstanbul, Turkey,
8. ICOVP-2007, 1-3 February 2007, Bengal Engineering and Science University, Shibpur, West Bengal, India.

There was also a pre-conference tutorial for the faculty members of Colleges, Universities Scientists from Research Laboratories, Research Scholars and for Post-Graduate students. Foreign and Indian scientists delivered lectures in pre-conference tutorial which was held on January 30-31, 2007.

Fifteen very well-known scientists were very kind in accepting our invitation to give General Lecture in the conference. These lectures were followed by 44 Research Communications. 15 General Lectures and 24 Research Communications are included to this Proceedings. There was 13 Session in the conference and 5 Technical Sessions in the Pre-Conference Colloquium.

As with the earlier Conferences of the ICOVP series, the purpose of ICOVP-2005 was to bring together scientists with different backgrounds, actively working on vibration problems of engineering both in theoretical and applied fields. The main objective did not lie, however, in reporting specific results as such, but rather in joining different languages, questions and methods developed

in the respective disciplines and to stimulate thus a broad interdisciplinary research. Judging from the lively discussions, the friendly, unofficial and warm atmosphere created both in side and outside Conference rooms, this goal was achieved.

The following broad fields have been chosen by the International Scientific Committee to be special importance for the ICOVP-2007.

- Mathematical modeling and computational techniques in sound and vibration analysis,
- Instrumentation and experimental techniques in sound and vibration engineering,
- Structural dynamics: seismic effect, fluid structure interaction, soil structure interaction, sensitivity problems,
- Vibration problems in structural dynamics, structural vibration in non-linear range and damage mechanics, fracture mechanics, composite and granular materials,
- Analysis of deterministic and stochastic vibration phenomena,
- Uncertainties in structural dynamics and acoustics,
- Nonlinear dynamics, stability, bifurcation and chaos and its application,
- Vibration of transport system,
- Vibration problems related to Bio-mechanics and Bio-engineering,
- Vibration in micro-systems,
- Vibrational Technology in industrial devices and processes, dynamic materials of second kind,
- Nanotechnology in vibration-phononic band gap structures and materials,
- Signal processing and analysis,
- Other topics related to vibration problems.

Other topics concerned with vibration problems, in general, were also open as well, but the bulk of presentations were within the above fields. All of the lecturers were carefully reviewed by the International Scientific Committee, so as to illustrate the newest trends, ideas and the results.

As the Editor-in-Chief, I would like to express my deep gratitude to the Faculty of Bengal Engineering and Science University (BESU) and the Scientific Committee and functional committees of the conference for their immeasurable efforts to organize this conference. They did a wonderful job for the realization of this traditional conference.

In the meantime, I would like to express my great sympathy, regards and thanks to my very best friend Professor M. M. Banerjee who is the "father" of these conference series. His efforts are immeasurable to keep this conference series as going on successfully.

I would also like to thank to Dr. Ahmet KIRIŞ for his great help on the editing of the book.

Finally, on behalf of the Editorial Board, I would like to send our cordial thanks to all lecturers for their excellent presentations and careful preparation of the manuscripts. We are looking forward to come together at 9th ICOVP conference, which will tentatively take place in India in 2009.

Esin İNAN,
Editor-in-Chief
Şile, İstanbul, December 2007

CONTENTS

FREE VIBRATIONS OF DELAMINATED COMPOSITE CYLINDRICAL SHELL ROOFS

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Abstract. Recently laminated composites are widely used in civil engineering, which may suffer from delamination damage resulting from improper fabrication and overloading at service. A review of literature that exists on composite shells reveals that the research reports on delaminated shells are very few in number. Hence the present endeavor is to work on delaminated simply supported cylindrical shell with different extents of delaminations. An eight noded isoparametric element with five degrees of freedom per node is used together with Sander's strain displacement relationships and multipoint constraint equations to satisfy the compatibility of displacements and rotations along the cracked edges. The study reveals that there is a consistent decrease in the fundamental frequency value as the area of the delamination damage increases. Further the fundamental frequency of angle ply shells undergo relatively more prominent decrease compared to that of cross ply shells. It seems that delamination damage brings about greater reduction in frequency values as the number of layers increases for angle ply shells, especially for symmetric ones.

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1. Introduction

Gim[1] worked on delaminated plates including the effect of transverse shear deformation. He employed the finite element technique and to ensure the compatibility of deformation and equilibrium of forces and moments at the delamination crack tip, a multiple constraint algorithm was developed and incorporated in the code. He considered delaminations at tips of cantilever plates and reported the strain energy release rate. Dynamic analysis of composite plates and shells with mutiple delamination was reported by Parhi and his colleagues[2] using an 8-noded finite element to study the free vibration and transient response with different parametric variations. Effects of delamination on free vibration characteristics of graphite-epoxy composite pretwisted shallow cylindrical shells were reported by Karmakar et al[3]. Although among civil engineering shells cylindrical configuration is commonest but delamination problems of commonly used composite cylindrical shells received very limited attentions only. Hence in this paper a free vibration analysis of simply supported delaminated composite cylindrical shells with different delamination areas and stacking sequences are presented.

2. Mathematical formulation

An eight noded curved quadratic isoparametric finite element with five degrees of freedom u, v, w, α, β at each node is employed for solution, where u, v, w are displacements at any general point at a distance z from the midplane within the shell thickness and α and β are rotations about Y- and X- axes respectively. The corresponding midplane displacement components are u^0, v^0, w^0. R is the radius of the shell surface.

For shallow cylindrical shells the linear inplane strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ are expressed as

$$\varepsilon_x = u_{,x} = u^0_{,x} - z\alpha_{,x}$$

$$\varepsilon_y = v_{,y} - \frac{w}{R} = v^0_{,y} - z\beta_{,y} - \frac{w}{R} \tag{1}$$

$$\gamma_{xy} = u_{,y} + v_{,x} = u^0_{,y} + v^0_{,x} - z(\alpha_{,y} + \beta_{,x})$$

Multipoint constraint equations, used here in the finite element formulation, are to satisfy the compatibility of displacements and rotations. These are suggested by Gim[1].

3.   Numerical examples

The numerical examples solved in this paper include a benchmark problem and number of other problems, which are authors' own. In each problem, E_{11}, E_{22} (elastic moduli), G_{12}, G_{13}, G_{23} (shear moduli), v_{12} (Poisson's ratio) and ρ (density) represent material properties. Length and width of the cylindrical shell are represented by a and b along x (beam) and y (arch) directions respectively. Similarly length and width of central delamination area of the shell are represented by c and d parallel to x and y directions respectively. Shell thickness is h. Natural frequency is ω and non-dimensional natural frequency is $\overline{\omega} = \left[\omega a^2 \left(\rho / E_{22} h^2 \right)^{1/2} \right]$. Fundamental frequencies (Hz) of cylindrical shells for different centrally located mid-plane delaminations, solved by Parhi[2] and obtained by the present method are compared in Table 1 as benchmark problem. Authors' own problems are about simply supported cylindrical shells. The area of delamination zone is varied from zero to 56.25% of the total plan area of the shell. Twelve different types of laminations are considered including orthotropic, anisotropic, cross and angle plies, both antisymmetric and symmetric, furnished in Table 2.

TABLE 1. Fundamental frequencies (Hz) of cylindrical shells for different centrally located (area = $c \times d$) midplane delaminations for simply supported boundary conditions.

c/a	Parhi[2]	Present approach
0	175.55	175.26
0.25	167.62	167.37
0.5	158.67	158.40
0.75	127.39	127.03

Material: Carbon-epoxy composite
E_{11}=172.5GPa, E_{22}=6.9GPa, G_{12}=G_{13}=3.45GPa, G_{23}=1.38GPa,
v_{12}=0.25, ρ=1600 kg/m³, a=b=0.5m, R=1.5m, h=5mmz.
Lamination: (0º/90º)$_2$

4.   Results and discussions

The results of the fundamental frequencies as obtained by Parhi[2] and the present method show excellent agreement as evident from Table 1. The other problems are studied from different angle and discussed hereafter (Table 2).

TABLE 2. Non-dimensional fundamental frequencies of cylindrical shells for different centrally located (area = $c \times d$) midplane delaminations for simply supported boundary conditions.

Stacking sequence	c/a			
	0	0.25	0.5	0.75
0°	24.252	23.079	21.556	20.765
90°	22.957	22.509	19.118	16.301
+45°	37.161	36.377	30.618	25.836
–45°	37.161	36.377	30.618	25.836
0°/90°	24.520	24.403	24.182	23.994
0°/90°/0°	27.050	26.001	24.641	23.959
+45°/–45°	32.150	32.013	30.860	29.036
+45°/–45°/+45°	40.195	38.694	29.901	12.245
(0°/90°)$_2$	26.513	25.321	23.963	19.218
(0°/90°)$_S$	27.135	26.081	24.696	24.055
(+45°/–45°)$_2$	44.891	43.056	33.348	26.634
(+45°/–45°)$_S$	43.383	41.245	10.519	24.241

E_{11}=25, E_{22}, G_{12}=G_{13}=0.5E_{22}, G_{23}=0.2 E_{22}, v_{12}=0.25,
a=b, h=0.01b, R=3b.

4.1. EFFECT OF EXTENT OF DELAMINATION ON NATURAL FREQUENCY VALUES

Firstly introduction of delamination reduces the bending stiffness in the damaged zone without any change in mass and hence a decrease in the value of the fundamental frequency is quite expected. But for (+45°/–45°)$_S$ laminate, the fundamental frequency value for c/a = 0.5 is even less than that when c/a = 0.75. This observation makes the author more curious to study the higher mode frequencies for this laminate, which are shown in Table 3. It is found that the second and third natural frequencies show a consistent decay as the area of damage increases. These observations leaves the room for further research to exactly determine the cause of such behaviour. Apparently it seems that the fundamental vibration mode in this case has its highest point of bending curvature within the delamination zone and hence the reduction of bending stiffness in the area of damage has affected the fundamental frequency to a very great extent. Secondly orthotropic and cross ply shells are less susceptible to frequency loss with introduction of delamination. In fact the fundamental frequency of 0°/90° is hardly affected by delamination and three and four layered symmetric cross

TABLE 3. Non-dimensional natural frequencies of (+45°/–45°)s laminate (area of damage = $c \times d$, centrally located at the midplane) for simply supported boundary conditions.

Natural frequency	c/a			
	0	0.25	0.5	0.75
First mode	43.383	41.245	10.519	24.241
Second mode	62.335	60.969	37.648	31.895
Third mode	69.196	66.744	56.463	37.542

$E_{11}=25$ E_{22}, $G_{12}=G_{13}=0.5E_{22}$, $G_{23}=0.2$ E_{22}, $v_{12}=0.25$,
$a=b$, $h=0.01b$, $R=3b$.

ply laminates exhibit a marginal loss of about 10% only even when 56.25% of the total shell area is damaged. Among angle ply laminates only +45°/–45° shell has the frequency value not much affected with the introduction of damage. But one observation is true in general that introduction of small area of damage ($c/a = 0.25$) hardly affects the fundamental frequency value and in no case, considered here, the loss is more than 5%.

4.2. EFFECT OF STACKING SEQUENCE ON FUNDAMENTAL FREQUENCY VALUES

Firstly the single layered anisotropic laminates perform considerably better than orthotropic ones. In the same way, angle ply laminates for undelaminated ($c/a = 0$) and slightly delaminated ($c/a = 0.25$) turn out to be better than cross ply ones. So relative performances of orthotropic and anisotropic laminates can form the basis of predicting that of multi-layered laminates for undelaminated and slightly delaminated shells. Secondly among two and four layered antisymmetric laminates four layered ones show higher frequency values than two layered ones for undelaminated and slightly delaminated shells. Among three and four layered symmetric laminates four layered ones show increased frequency values for $c/a = 0$ and $c/a = 0.25$. For higher delamination areas some deviation is noted. Thirdly symmetric cross ply and antisymmetric angle ply laminates are preferable than the other types in case of four layered laminates. Comparing three layered symmetric and four layered antisymmetric laminates, symmetric cross ply and antisymmetric angle ply laminates show higher values, as before.

5. Conclusion

Firstly the formulation, presented here, can successfully analyze problems of cylindrical laminates as established through solution of benchmark problems. Secondly for simply supported shells, the fundamental frequency values of

delaminated shells are always less than the corresponding undelaminated shells for all types of stacking sequences. Thirdly the relative performances of single layered anisotrpic laminates are better than the orthotropic ones. Fourthly undamaged ($c/a = 0$) and slightly damaged shells ($c/a = 0.25$) show that variation of fundamental frequencies is not more than 5%. Also for the same case it is seen that angle ply shells perform better than cross ply shells. So performances of multi-layered shells can be predicted from single layered shells. Moreover in this case increase in number of layers has positive effect on frequency values and symmetric cross ply and antisymmetric angle ply laminates show better performances that the other types. Fifthly moderately damaged ($c/a = 0.5$) and highly damaged ($c/a = 0.75$) shells show cross ply shells are less susceptible to frequency loss than angle ply ones. Among the angle ply laminates frequency loss is more for symmetric ones.

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2. P. K. Parhi, 2000. *Finite element dynamic analysis of multiple delaminated composite plates and shells*, PhD Thesis.
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COMPUTATIONAL DYNAMIC ANALYSIS OF SINGLE-MASS FREELY SHAKING CONVEYERS WITH CENTRIFUGAL VIBRATION EXCITER

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Abstract. Single-mass freely shaking (vibrational) conveyers with a centrifugal vibration exciter transmit their load based on the jumping method. The trough is oscillated by a common unbalanced-mass driver. This vibration causes the load to move forward and upward. The moving loads jump periodically and move forward with relatively small vibration. The movement is strictly related to the vibrational parameters. This is applicable in laboratory conditions in the industry that accommodate a few grams of loads, up to those that accommodate tons of loading capacity. In this study, the variation of motional parameters has been represented graphically using the software *Mathematica* with respect to i) vibrational parameters, ii) parameter of the unbalanced mass, iii) operation mode, iv) friction coefficient, and v) angle of attack. The proper parameters for the transport of the numerous loads and the transportation velocity can be chosen with the help of the graphics. The results obtained in this study have been compared with the experimental results in the pertaining literature, and have been found to be well matched.

Keywords: shaking (vibrational) conveyers, operation mode, unbalanced mass

1. Introduction

In the shaking conveyers, the load moves relative to the trough and the force of the load pressure on the trough's bottom is variable. The load periodically rises

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('*jumps*') above the trough and moves in small jerks. For the load to move in the desired direction, the drive must be mounted so that the line of action of the excitation force is at an angle of β to the longitudinal axis of the conveyer.

A suspended shaking conveyer with a freely oscillating single-mass system (Fig. 1a) consists of a load-carrying element (pipe or trough) (1) which is freely suspended on the elastic tie-dampers (3) from a stationary structure, and is oscillated in a directed manner by a centrifugal driver (4). The conveyer is equipped with a safety belt to hold the trough in the event of occasional breakage of the elastic ties. The loading and unloading connections of a vibrating pipe of a shaking conveyer are connected to stationary structures, such as bunkers or funnels, by means of flexible corrugated pipe connections (2) made of strong fabric, rubber, or plastics. The drive is in the form of a doubled centrifugal (inertia-type unbalanced) vibration exciter (Fig. 1b). The drive may be arranged above or below the load-carrying element.

(a)   (b)
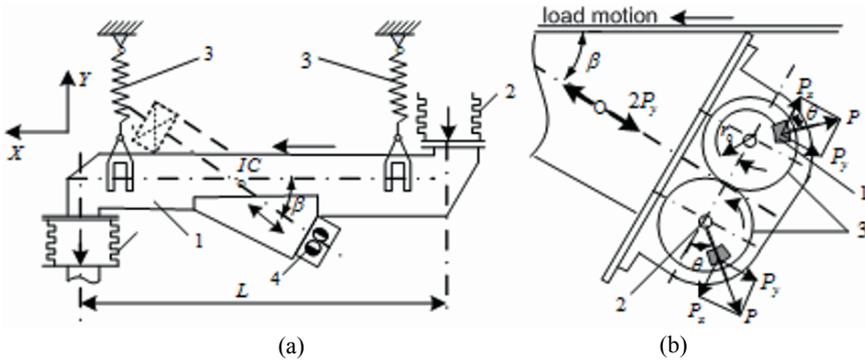
Fig. 1 (a) Single-mass freely shaking conveyers with centrifugal vibration exciter (IC − inertia centre) (b) Two equal unbalanced centrifugal vibration exciters.

2. Dynamics of the motion

The following derivation can be made for the maximum velocity attained by the load on the trough. When the equation of motion is written according to the Newton's second law along the horizontal axis (X − axis) for the load on the trough, the following equation is obtained:

$$\sum F_X = m a_x \rightarrow f = m a_x \tag{1}$$

were f represents the frictional force affecting the load, m is the mass of the load, and a_X is the horizontal component of the trough's acceleration.

If ma_X is greater than the maximum value of the frictional force (f_{max}), the load starts to slide on the trough. Therefore, the maximum friction force can be expressed as follows:

$$f_{max} = \mu_s N \tag{2}$$

where μ_s represents the static friction and N is the normal force.

The equation of motion along the vertical axis $(Y-\text{axis})$ for the load on trough can be written as

$$N = m g + m a_Y \tag{3}$$

If equation (3) is substituted into equation (2) at the beginning of the slide of the load, f_{max} is obtained as follows:

$$f_{max} = \mu_s m (g + a_Y). \tag{4}$$

Recalling that the load starts to slide on the trough when $ma_x > f_{max}$ and using equation (4), the condition of sliding of the load on the trough may be expressed with the following inequality:

$$a_X > \mu_s (g + a_Y). \tag{5}$$

By arranging inequality (5), the horizontal component of the acceleration of the load is obtained by using the kinetic friction constant μ_k as follows:

$$a_X = \mu_k (g + a_Y). \tag{6}$$

If equation (6) is integrated as shown below, the horizontal component of the velocity of the load can be expressed as[1,2]

$$V_X = \int_0^t \mu_k (g + a_Y)\, dt. \quad \rightarrow \quad V_X = \mu_k\, g\, t + \int_0^t \mu_k\, a_Y\, dt. \tag{7}$$

The vertical motion of the load is the same as the motion of the trough until the normal force vanishes or the acceleration of trough reaches the value of $-g$. Thereafter, the load is separated from the trough.

3. Forced vibration of the motion

In a doubled centrifugal driver (Fig. 1b), the two equal unbalanced masses (1) are mounted on two gear wheels (3) (or on two shafts (2)) which are engaged

with each other. As the wheels rotate, the centrifugal force P appears. The longitudinal components of the centrifugal forces of the two unbalanced masses P_y are added together and the transverse components of the centrifugal forces of the two unbalanced masses P_x counter balance each other. The equation of motion according to the Newton's second law along the y-axis for the shaking conveyer[3] can be obtained as:

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = \frac{m_0 r_0 \omega^2}{m}\sin\omega t, \qquad (8)$$

where k is the stiffness of the elastic ties, c is the internal resistance, m_0 is the total unbalanced mass of a centrifugal exciter, r_0 is the eccentricity of the unbalanced masses, and m is the total mass of vibrating elements of the conveyer including the attached mass of the load. If the natural frequency, viscous damping factor, forced amplitude, and the angular frequency ratio in this equation are defined as

$$\omega_n^2 = k/m, \quad \zeta = c/(2m\omega_n), \quad A = (m_0 r_0)/m, \quad \eta = \omega/\omega_n, \qquad (9)$$

respectively, then the differential equation (8) can be expressed as:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 \left(\eta\right)^2 A\sin\omega t. \qquad (10)$$

The solution of the differential equation (10) is obtained as[4]:

$$y = \Phi(\omega)\sin(\omega t - \phi). \qquad (11)$$

The amplitude of vibrations of the load-carrying element of a freely shaking conveyer can be determined by solving the differential equation of the forced oscillations of the inertia centre of the system (Fig. 1). The amplitude function, and the phase difference can be expressed as

$$\Phi(\omega) = AG(\omega), \quad \phi = \arctan\left(\frac{2\zeta\eta}{1-\eta^2}\right) \qquad (12)$$

respectively, where $G(\omega)$ is the amplitude magnification factor

$$G(\omega) = \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + (2\zeta\eta)^2}}. \qquad (13)$$

The components of y along the trough and perpendicular to the trough with β (the angle of attack) can be obtained as:

$$y_X = \Phi(\omega)\sin(\omega t - \phi)\cos\beta, \; y_Y = \Phi(\omega)\sin(\omega \mp \phi)\sin\beta. \qquad (14)$$

The components of the velocity and acceleration can be found by taking the first and second derivatives of the equation (14).

4. The graphics of the motion

The graphical representations of the dynamic analysis obtained by Mathematica software are shown in Figs. 2-5 below.

For the values close to 1 of the angular frequency ratio $\eta = \omega/\omega_n$ (resonance region), it is observed that the amplitude at small viscous damping factor increases (Fig. 2a). Even though the load transmission velocity is higher in this region, the lifetime of the system is shorter as a result of the additional dynamic loads.

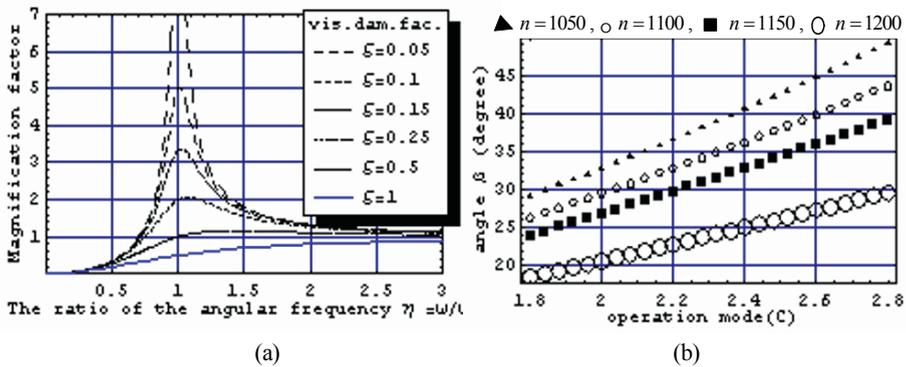Fig. 2 (a) The change in the amplitude magnification factor depending on the angular frequency ratio (b) Diverse velocity of rotating unbalanced mass according to attack angle-operation mode.
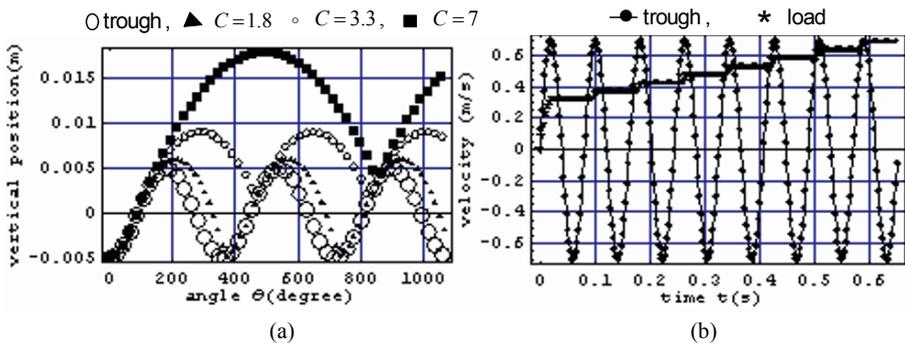

Fig. 3 (a) The vertical position of the trough and the load at diverse operation modes (b) The velocity-time graph of the trough and the load in horizontal direction from the first movement until the steady state.

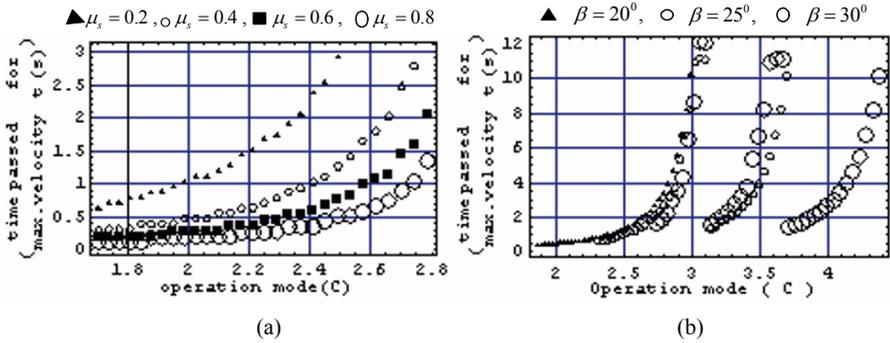(a)   (b)

Fig. 4 (a) The time passed for the load to reach maximum velocity at diverse friction coefficients according to the C operation mode (b) The time passed for the load to reach maximum velocity at diverse angles of attack depending on the C operation mode.

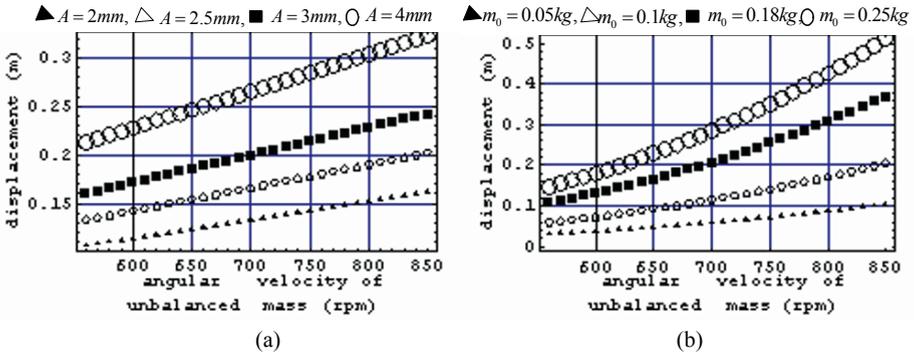(a)   (b)

Fig. 5 (a) Displacement of the load in one second at diverse amplitude values depending on velocity of the rotating unbalanced mass (b) at diverse unbalanced mass values.

C is the coefficient defining the dynamic working conditions (dynamic forces acting on the driver and other conveyer elements) of an oscillating conveyer and the motion style of the load particles. If $C < 1$, the load is always in contact with the trough of the conveyer and the load is not disengaged from the trough. If $C > 1$, the load is disengaged from its conveyer trough at some instants and moves in micro-jumps. For the values $1 < C < 3.3$, the load particle will be launched upward as soon as it reaches the trough ground (Fig. 3a). The most efficient movement of the load particles on a shaking conveyer $(C > 1)$ depends on the correct chosen time, when a particle falls on the trough. During the forward motion of the plane, the particle must be caught by the plane, and it must move with the plane until it is disengaged from the plane in the shortest possible time period. The most suitable displacement of the load and the most suitable style of transportation are achieved through the equivalence of the disengagement time interval from the trough to one complete oscillation period (Fig. 3a).

The velocity of the load increases until it equals to the velocity of the trough and the motion continues at this velocity (Fig. 3b).

The following values are used for drawing of the graphics [3, 4] (Fig. 3, 4): the spring constant $k = 62550 N/m$, the friction coefficient $\mu_s = 0.5$, the operation mode $C = [(a_Y)_{max} sin(\beta)]/g$, the eccentricity of the unbalanced masses $r_0 = 0.2m$, and the angular frequency ratio $\eta = \omega/\omega_n = 1$.

5. Conclusions

In this study, the graph of the displacement by the load in one second depending on the velocity of the rotating unbalanced mass (Fig. 5a, b) is drawn for numerous amplitude values and the unbalanced mass values. Fig. 2, 3, 4 illustrate the changes necessary for the load to reach the maximum velocity, depending on the static friction coefficient and the angle of attack (Fig. 4a, b). The numerical calculations are performed with the help of the software *Mathematica*[4-5] The graphs drawn in this study are compared with the experimental results in the literature, and it has been observed that they are well-matched. Using these graphs, the most proper parameters can be chosen. In this system at the load transmission; especially the coefficient of friction, the coefficient of spring, the coefficient of amortization of the elastic ties, the angle of attack, the values of the unbalanced mass, the radius of rotation, and the velocity of rotation must be well-arranged.

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ON THE APPLICATION OF CONSTANT DEFLECTION CONTOUR METHOD TO NON-LINEAR VIBRATION ANALYSIS OF ELASTIC PLATES AND SHELLS

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Abstract. The present paper aims at establishing the validity of the constant deflection contour (CDC) method to the nonlinear analysis of plates of arbitrary shapes vibrating at large amplitude. It begins with a review of the basic ideas developed earlier by the present author. The deduction of the governing differential equations have been established. The author has made an attempt here to develop the concept of constant deflection contour method and specifically to make its introduction into the nonlinear analysis of plates. A combination of the constant deflection contour method and the Galerkin procedure has been employed for solution. The numerical results obtained for the illustrative problems are in excellent agreement with those of available studies. Application of the present analysis to structures with complicated geometry has also been attempted in this paper. It has been demonstrated that this method provides a powerful tool to tackle problems involving structures with uncommon boundaries. The comparison of the present results with others strongly supports this. The analysis carried out in this paper may readily be applied to other geometrical structures and as a byproduct the static deflection is also obtainable.

Keywords: constant deflection contours, iso-deflection curves, contour, integration and Green's function

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1.   Introduction

The paucity of literature concerning nonlinear (large amplitude) vibration ana-
lysis is, probably, due to the fact that the two basic Von Kármán field equations
extended to the dynamic case, involved the deflection and the stress functions in a
coupled form. Moreover, these equations are of fourth order, posing analytical
problems and necessitating a numerical approach. Large amplitude vibration
analysis has been treated well by many authors mostly for plates and shells
having regular shapes[1-3] and almost all problems involved considerable amount
of mathematical computation. Simplified approaches had been attempted[4,5] but
later on some reservations were made and the accuracy of the modifications
were questioned.[6,7] Recently a new idea has been put forward by Banerjee[8] to
study the dynamic response of structures of arbitrary shapes based on "constant
Deflection Contour" method. Mazumdar and others[9-13] have previously developed
the method. However, the application of this method has been restricted to linear
cases only.

2.   Some preliminary remarks on the Constant Deflection Contour
Method

The fundamental concept of the constant deflection contour method may be best
explained by considering transverse vibrations of a plate, referred to a system of
orthogonal coordinates *Oxyz* for which *Oz* is the transverse direction (positively
downward). The horizontal plane *Oxy* coincides with the middle plane of the
plate. Consider such a plate statically deflected, vibrating freely or forced to
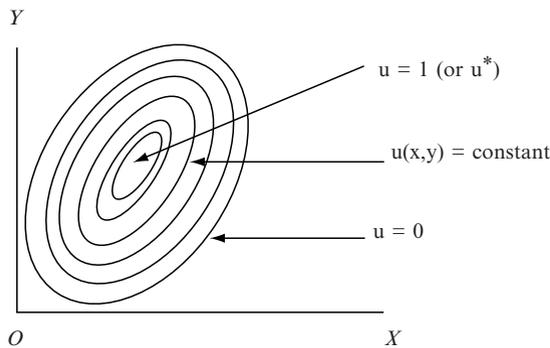vibrate, all due to normal static or dynamic loads.

Fig. 1 Iso-deflection Curves.

When the plate vibrates in a normal mode then at any instant t_θ, the intersections between the deflected surface and the parallels $z = constant$ will yield contours which after projection onto $z = 0$ surface are a set of level curves, $u(x,y) = constant$, called the "Lines of Equal Deflections"[10], which are iso-amplitude contours.

The boundary of the plate, irrespective of any combination of support, is also a simple curve C_u belonging to the family of lines of equal deflections.

As defined by Mazumdar[9] the family of nonintersecting curves may be denoted by C_u, for $C_u, 0 \le u \le u^*$, so that $C_u (u = 0)$ is the boundary and C_u coincides with the point(s) at which the maximum $u = u^*$ is obtained.

3. Deduction of basic equations (a different approach)

The usual procedure is to consider Kármán type field equations extended to the dynamic case

$$D\nabla^4 w = hS(\phi, w) + q - \rho h w_{tt} \tag{1}$$

$$\nabla^4 \phi = -(E/2)S(w, w), \tag{2}$$

in which the flexural rigidity D and the two-dimensional Laplacian operator ∇^2 are defined by

$$D = \frac{Eh^3}{12(1 - v^2)}, \qquad \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \equiv \nabla^2 \tag{3}$$

with h the thickness of the plate, E the Young's modulus, q the uniform normal load, ρ the material density, w is the deflection function and ϕ the Airy stress function. In addition a suffix is taken as an indication of partial differentiation with respect to the implied variable and the operator S is defined by

$$S(I, J) = I_{xx}J_{yy} - 2I_{xy}J_{xy} + I_{yy}J_{xx} \tag{4}$$

It should be noted here that the use of the stress function is equivalent to disregard of inertia terms in the equations of in-plane motions of the particles of the plate. This assumption is legitimate when the oscillations primarily take place in the transverse direction, perpendicular to the middle plane of the plate. We choose the deflection function and the stress function in the separable form[3]

$$w(x, y, t) = h \ W(x, y) \ F(t), \ \phi(x, y, t) = h \ \Phi(x, y) \ F^2(t) \tag{5}$$

where $F(t)$ is an unknown function of time to be determined.