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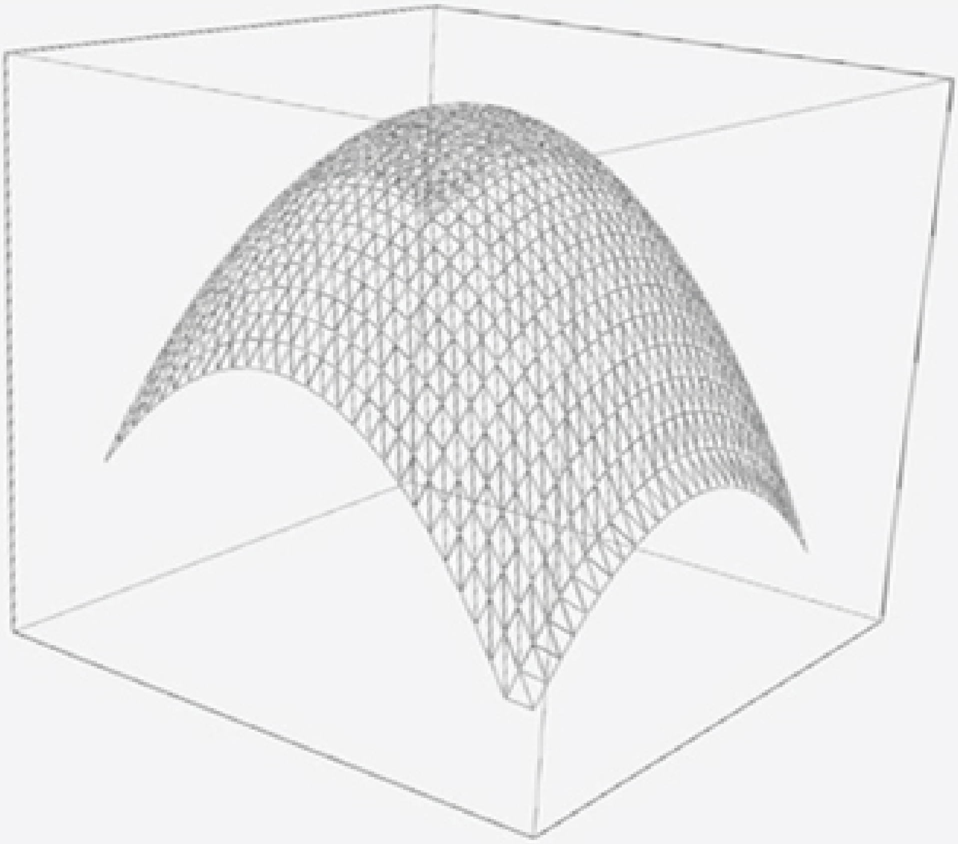
RESPONSE SURFACE METHODOLOGY

PROCESS AND PRODUCT OPTIMIZATION USING DESIGNED EXPERIMENTS

FOURTH EDITION



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RESPONSE SURFACE METHODOLOGY

Process and Product Optimization Using Designed Experiments

Fourth Edition

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