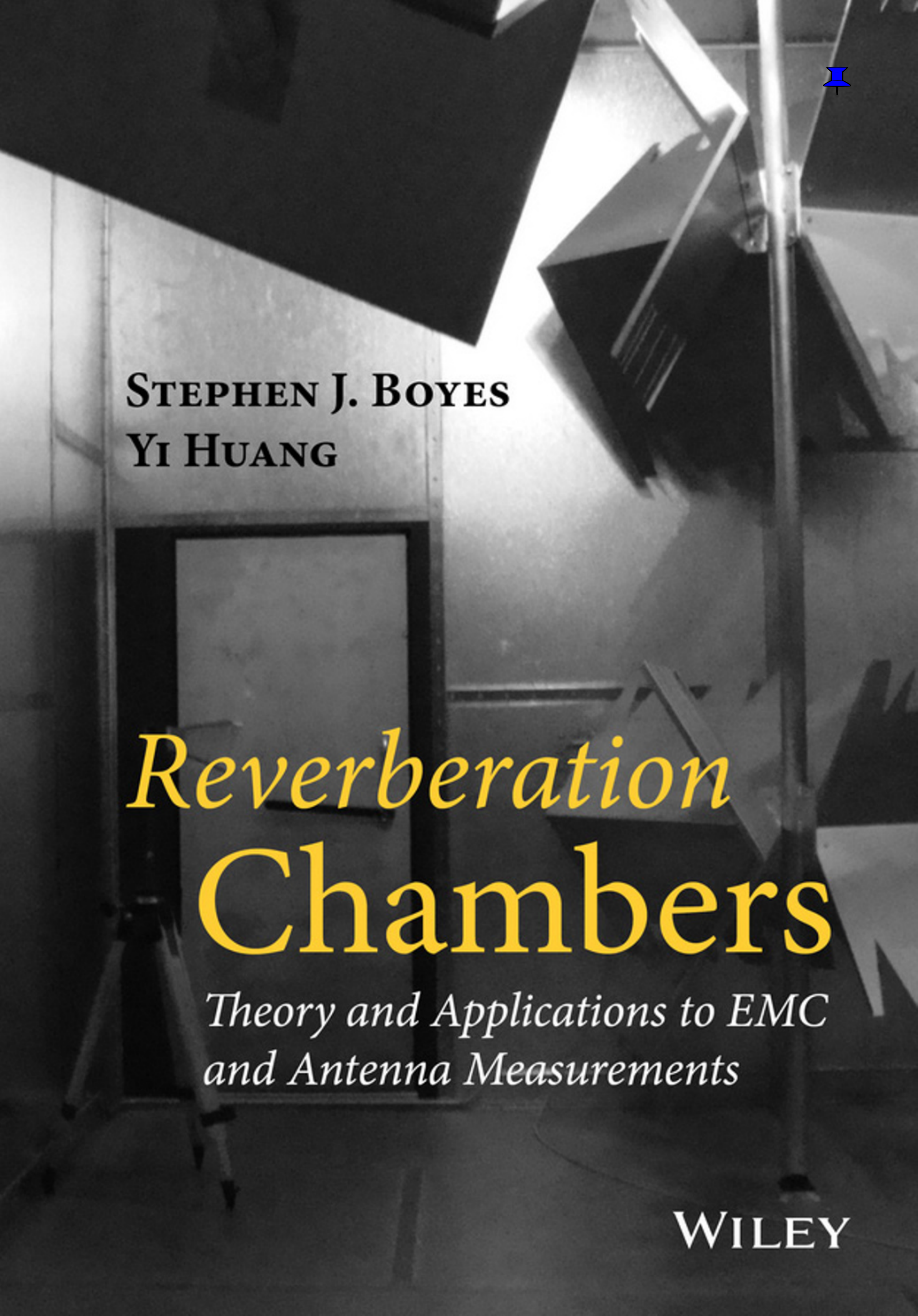

Stephen J. Boyes
Yi Huang

Reverberation Chambers

Theory and Applications to EMC and Antenna Measurements

Wiley

REVERBERATION CHAMBERS

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THEORY AND APPLICATIONS TO EMC AND ANTENNA MEASUREMENTS

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Dr. Stephen J. Boyes received his BEng (Hons) in electronics and communications engineering, MSc (Eng) in microelectronic systems and telecommunications and PhD in antennas/electromagnetics, all from the University of Liverpool, United Kingdom.

His academic research activities and PhD at the University of Liverpool was centred on reverberation chambers, with an emphasis on a wide variety of antennas and antenna measurements for communication applications. His academic research work covered novel textile antennas, multiple input multiple output (MIMO) antennas and novel antenna arrays.

In addition to work in academia, Dr. Boyes has also held various posts throughout 10 years of his working in the industry. In 2013, Dr. Boyes joined the Defence Science & Technology Laboratory where he is now a senior RF Research Scientist leading all antennas and electromagnetics research activities within his group. His present research interests and activities are associated with novel antennas for military applications which include body-worn and vehicle-borne types. He also leads research into novel frequency-selective surfaces and holds numerous patents in both these areas. Dr. Boyes has published many papers in leading international journals and conferences and is also a reviewer for leading academic journals. He also participates and sits on major international committees on behalf of the United Kingdom.

Prof. Yi Huang received his BSc in Physics (Wuhan, China), MSc (Eng) in Microwave Engineering (Nanjing, China) and DPhil in Communications from the University of Oxford, United Kingdom, in 1994. He has been conducting research in the areas of radio communications, applied electromagnetics, radar and antennas since 1986. He has been interested in reverberation chamber theory and applications since 1991. His experience includes 3 years spent with NRIET (China) as a Radar Engineer and various periods with the Universities of Birmingham, Oxford and Essex as a member of research staff. He worked as a Research Fellow at British Telecom Labs in 1994 and then joined the Department of Electrical Engineering & Electronics, University of Liverpool, as a Lecturer in 1995, where he is now the Chair in Wireless Engineering, the Head of High Frequency Engineering Group and the Deputy Head of the Department.

Dr. Huang has published over 250 refereed research papers in leading international journals and conference proceedings and is the principal author of the popular book *Antennas: From Theory to Practice* (John Wiley & Sons, 1st edition, 2008; 2nd edition, 2016). He has received many research grants from research councils, government agencies, charity, EU and industry; acted as a consultant to various companies; served on a number of national and international technical committees (such as the UK Location & Timing KTN, IET, EPSRC and European ACE); and has been an Editor, Associate Editor or Guest Editor of four international journals. He has been a keynote/invited speaker and organiser of many conferences and workshops (e.g. IEEE iWAT, WiCom and Oxford International Engineering Programmes). He is at present the Editor-in-Chief of *Wireless Engineering and Technology* (ISSN 2152-2294/2152-2308), Leader of Focus Area D of European COST-IC0603 (Antennas and Sensors), Executive Committee Member of the IET Electromagnetics PN, UK national representative to the Management Committee of European COST Action IC1102 (VISTA), a Senior Member of IEEE and a Fellow of IET.

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1

Introduction

1.1 Background

The concept of the Reverberation Chamber (RC) was first proposed by H. A. Mendes in 1968 as a novel means for electromagnetic field strength measurements [1]. The RC can be characterised as an electrically large shielded metallic enclosure with a metallic stirrer to change the field inside the chamber that is designed to work in an 'over-mode' condition (i.e. many modes). It has taken some time for the facility to gain universal acceptance, but by the 1990s, their use for performing Electromagnetic Compatibility (EMC) and Electromagnetic Interference (EMI) measurements was well established and various aspects were studied [2–11]. An international standard on using the RC for conducting EMC testing and measurements was published in 2003 [12]. The RC is now used for radiated emission measurements and radiated immunity tests, as well as for shielding effectiveness measurements. It was in this role that the facility was known for a long time and in part still continues to be [13, 14]. More recently the RC has been employed for antenna measurements due to the rapid development of wireless communications.

It is clear that the role and function of wireless technology in everyday life have reached unprecedented levels as compared to 20–30 years ago. For this change to take place, it has meant that antenna designs and their characterisation have had to evolve also. A question exists as to how the RC has risen to prominence to be proposed

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and also to be used for antenna measurements, which represents a brand new capability for the chamber that diverges from its initial intended use. To answer this question, we must partly examine the nature of antenna designs and their operational use.

Traditionally, antennas have always been orientated, and their communication channels configured in a Line of Sight (LoS) manner. For example, we have terrestrial antennas mounted on roof tops, and other directive types of antenna that are employed in satellite communications. The characterisation of these types of antenna for use in LoS communications are widely defined by the application of an equivalent free space reflection-free environment, which is typified by the Anechoic Chamber (AC). In a real application environment, reflection, scattering and diffraction effects may still exist to a certain extent, which brings about the creation of additional wave paths within the communication channel. However, the AC is still the preferred environment to characterise these types of antennas as their radiation patterns (and other subsequent parameters of interest) are of prime importance to the LoS scenario.

When we consider the modern mobile terminals (such as the mobile/cell phone), they do not operate under the premise of an LoS scenario. The antennas inside mobile phones might seldom 'see' the base station and they are expected to work perfectly in Non-Line of Sight (NLoS) environments. This type of environment will readily give rise to signals that will be exposed to reflections caused by large smooth objects, diffraction effects caused by the edges of sharp objects and scattering effects caused by small or irregular objects. When these effects occur, they will cause the creation of additional wave paths which will eventually add at the receiving side. These wave contributions have independent complex amplitudes (i.e. magnitude and phase information), such that at recombination, they may add constructively or destructively or anything in between these extremes. The wave paths and their complex amplitudes are also subject to rapid changes with time, with the terminal moving or parts of the environment (communication channel) changing. This brings about variations in the signal at the receiver and is commonly referred to as fading. The largest variations occur when there is a complete block on the LoS, which is more accurately referred to as small- scale fading, as opposed to large-scale fading, which is usually applied to variations only in the distance from the transmitter or due to part shadowing [15].

As it is important to characterise any antenna in a manner befitting its operational scenario or intended use to accurately reflect the performance merits, another measurement facility is required that can emulate this type of fading environment – this is where the RC comes in. The antenna measurement inside an RC can be closer to a real-world scenario than inside an AC. Furthermore, some measurements, such as the antenna radiation efficiency measurement, may be more efficient and accurate if preformed in an RC than in an AC as we will see later in the book.

When the regulations of the American Federal Communications Commission (FCC) released the unlicensed use of the Ultra Wide-Band (UWB) frequency domain between 3.1 and 10.6 GHz in 2002 [16], a vast amount of interest and industrial/academic research followed. The RC offers little restriction concerning large operational frequencies while also allowing for a vast range of device types and sizes.

Financial implications have also played a part in the subsequent rise to prominence of the RC. With the construction and operation of an AC (here we will only specifically compare with far field ACs), large amounts of anechoic absorber must be purchased to line the walls in order to suppress reflections, and this can be expensive. The RC requires no such absorber, leaving the walls purely metallic to actively encourage reflections – a trait which allows for a cost saving. Furthermore, if one was to compare the relative size of each facility against the lowest frequency of operation, it is possible to conclude that the RC can be constructed smaller in size than its far field anechoic counterpart – again offering a potential cost saving.

The RC is a unique, stand-alone facility, in the sense that it will allow a user full control over the time frame and uncertainty inherent in a given measurement; this distinguishes itself from any other facility. The operational principles for the RC allows for the measurement resolution to be clearly defined which in turn controls the overall measurement time. Mathematical procedures can be defined which link the expected uncertainty to the resolution and thus the time frame. Therefore, before a measurement commences all the parameters can be defined accordingly.

Perhaps one of the more important factors that led to the increasing popularity of the RC concerns the ease of measurement. Due to its unique operation of multiple reflected waves, the angle of arrival of these waves reaching any receiving antenna or device is uniformly distributed over three-dimensional space [17]. What this effectively means is that the angle of arrival and wave polarisation is equally probable which can simplify the characterisation of any device, as in such an environment, their performance is then insensitive to their orientation – this aspect is particularly acute for EMC and antenna measurements.

1.2 This Book

The RC is a very powerful tool for EMC and antenna measurements and has many advantages to offer. There have been many journal articles published over the years on the subject which signifies its increasing popularity; however, few published reference books exist. The most relevant ones are probably references [18] and [19]. They have provided an excellent and comprehensive coverage on the electromagnetic theory on cavities and RCs, and a very good introduction to EMC tests and measurements using an RC. But very limited information is on the RC design and its application to antenna measurements. There are still some important issues on how to use the RC for EMC and antenna measurements in practice.

This book is different from other works on RCs. It is designed to encompass both EMC and antenna measurements together which is important as there are subtleties between how the RC facility is used with respect to these different domains – it is crucial therefore to understand what these are and how to apply the operational principles of the chamber for the benefit of the intended measurement.

The book is also designed to take a reader from the very basic theory of the chamber to a more complicated stirrer design and measurements. It is written to include both detailed theory and practical measurements so a reader can appreciate and understand not only what the theory means, but also how to apply and configure it in a practical manner to complete a desired measurement. With all this information in one place, this book aims to ensure that the reader has a comprehensive yet compact reference source so that the RC can be studied and understood without needing to access a number of different sources which may not be well correlated.

The material covered in this book is underpinned by accepted theory on the RC that is published worldwide along with our own detailed research which covers many of the very latest and cutting-edge trends. The information in the book is also used as part of the Antennas (antenna measurements section) and EMC modules for students at the University of Liverpool. This subsequently is also where all of the measurement work for the book has been wholly conducted. A major feature of this book is to apply the theory to practice.

The book is organised as follows:

Chapter 2: Reverberation Chamber Cavity Theory. This chapter details all the important theoretical concepts that uphold and support the use of the RC as a measurement facility. In this chapter, all the theories are explained, all equations detailed and these are supplemented with practically measured quantities from the RC at the University of Liverpool, to illustrate the magnitude of the quantities derived.

Chapter 3: Mechanical Stirrer Designs and Chamber Performance Evaluation. This chapter presents a general method that can be employed to go through the process of designing mechanical stirring paddles for use in the chamber. New mechanical stirring paddles are designed and presented in this chapter. Also detailed are the complete equations and practical procedures of how the performance of any given chamber may be assessed in an accurate and robust manner.

Chapter 4: EMC Measurements Inside Reverberation Chambers. This chapter focuses specifically on EMC tests in RCs. The relevant standards for EMC tests in RCs are discussed, after which immunity and emission tests are introduced. Practical procedures of how to conduct EMC tests are explained which is followed by practical tests. A comparison between RCs and ACs for EMC radiated emissions is also presented to benchmark both facilities.

Chapter 5: Single Port Antenna Measurements. This chapter is dedicated solely to single port antenna measurements in RCs. The measurements are based around some of the very latest trends in the antenna field with the use of textile antenna which is selected as an example to demonstrate measurement procedures. This chapter not only shows how to measure single port antenna quantities in free space conditions, but also shows how body worn antennas can be measured that include the use of live human beings in the chamber. The radiation efficiency is the major concern of the measurement. This chapter also includes some of the subtle measurement issues that should be avoided when conducting such measurement work in addition to a comprehensive uncertainty assessment, including both procedures and equations.

Chapter 6: Multiport and Array Antennas. This chapter discusses how multiport and array antennas can be measured using the RC. The multiport section includes all measurement procedures and equations for quantities such as diversity gain, correlation and channel capacity and details the performance merits of a new multiport (diversity) antenna for Multiple Input Multiple Output (MIMO) applications. The array section shows how the efficiency of large-scale arrays can be measured using the chamber and develops a new equation to allow this characterisation to take place.

Chapter 7: Further Applications and Developments. This chapter presents and discusses some of the very latest research in RCs that includes how to measure antenna performance parameters without reference antennas. Also included is the use of the RCs for emulating different 'channel' characteristics which is important for many over- the-air measurements.

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2

Reverberation Chamber Cavity Theory

2.1 Introduction

A grounding in the theoretical principles concerning Reverberation Chambers (RCs) is important and essential for any practical undertaking to be completed accurately. The overall field of electromagnetics that contains the foundations of RC cavity theory is a well studied and classical area, and we owe much to authors such as Harrington [1], Balanis [2], Jackson [3], Kraus [4] and more recently Hill [5] for their excellent work in this area. We should also not forget the contribution made by James Clark Maxwell who provided the unified theory and equations that made work in the entire field possible, and Henrich Hertz who practically validated Maxwell's theorems by demonstrating the existence of electromagnetic radiation.

The purpose of this chapter is to present a review and discuss the important theoretical concepts that uphold and support the RCs' use as a measurement facility. We will firstly present a review of the concept of resonant modes and show how these can be used to calculate the electromagnetic fields in an RC. Next, we will show how RCs uniquely deal with the nature of the fields inside the facility and discuss the main principles of operation. The concepts of angle of arrival of plane waves, mode bandwidths and chamber quality (Q) factors will then be introduced and explained, which will be backed up with practically measured data.

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A discussion on the statistical forms will be also included in this chapter and a practical way of deducing the statistical forms will be illustrated. At all stages, where appropriate, figures depicting any measurement set-ups and measurement details will be provided to clearly show how all quantities are practically acquired. By the end of this chapter, the reader will have a firm grasp of the important theoretical concepts and be able to appreciate how they impact upon practical measurements.

2.2 Cavity Modes and Electromagnetic Fields

Since most RCs are rectangular in shape, this discussion will be centred solely on rectangular-shaped cavities. It is well known that a metallic rectangular-shaped cavity is resonant when its dimensions satisfy the condition

$$k_{mnp}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \tag{2.1}$$

where k_{mnp} is an eigenvalue to be determined; m, n and p are integer numbers and a, b and d are the chamber width, height and length in metres, respectively.

For convenience (2.1) can also be expressed as:

$$k_{mnp}^2 = k_x^2 + k_y^2 + k_z^2 \tag{2.2}$$

where

$$k_x = \left(\frac{m\pi}{a}\right), k_y = \left(\frac{n\pi}{b}\right), k_z = \left(\frac{p\pi}{d}\right) \tag{2.3}$$

It is stated that the simplest method to construct the resonant modes for a rectangular cavity is to derive modes that are Transverse Electric (TE) or Transverse Magnetic (TM) to one of the three axes [5]. The standard convention in this sense is normally chosen to be the z axis with respect to Figure 2.1.

The TE modes are referred to as magnetic modes as the E_Z field component is zero. Similarly, the TM modes are referred to as electric modes because the H_Z field component is also zero [5]. The corresponding fields in a rectangular cavity can be given as follows, beginning first with the TM components. Please note that the equations presented at this stage do not take any current source into account.

$$E_{zmnp}^{\mathrm{TM}} = E_O \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{2.4}$$

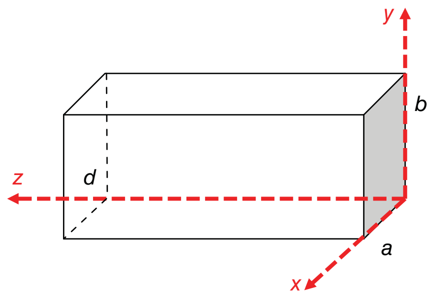

Figure 2.1 Rectangular cavity principle axis.

where E_O is a constant with units of V/m [5]. The transverse components can be issued as follows

$$E_{xmnp}^{\mathrm{TM}} = \frac{k_x k_z E_O}{k_{mnp}^2 - k_z^2} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.5}$$

$$E_{ymnp}^{\mathrm{TM}} = \frac{k_y k_z E_O}{k_{mnp}^2 - k_z^2} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.6}$$

The z component of the magnetic field is zero (owing to the definition of the TM mode), and the transverse components of the magnetic field can be given as [5]:

$$H_{xmnp}^{\mathrm{TM}} = -\frac{i\omega_{mnp}\varepsilon k_y E_O}{k_{mnp}^2 - k_z^2} \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{2.7}$$

$$H_{ymnp}^{\mathrm{TM}} = \frac{i\omega_{mnp}\varepsilon k_x E_O}{k_{mnp}^2 - k_z^2} \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{2.8}$$

where ε is the permittivity of the media inside the cavity and ω is the angular frequency. By virtue that E_{zmnp}^{TM} be non-zero (Eq. 2.4), the allowable values of the mode coefficients are $m = 1,2,3,\ldots;\ n = 1,2,3,\ldots;\ p = 0,1,2,3,\ldots$.

The *TE* (magnetic modes) can be derived in an equivalent manner. The z component of the magnetic field satisfies the scalar Helmholtz equation and the boundary conditions are such that the following is prevalent:

$$H_{zmnp}^{\mathrm{TE}} = H_O \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.9}$$

where H_O is a constant of units A/m. The eigenvalues and axial wave numbers are the same as given in Equations (2.1)–(2.3). The transverse components can now be issued as [5]:

$$H_{xmnp}^{\mathrm{TE}} = -\frac{H_O k_x k_y}{k_{mnp}^2 - k_z^2}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{2.10}$$

$$H_{ymnp}^{\mathrm{TE}} = \frac{H_O k_y k_z}{k_{mnp}^2 - k_z^2}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.11}$$

The z component of the electric field is zero by definition of a TE mode and the transverse components of the electric field can be stated as [5]:

$$E_{xmnp}^{\mathrm{TE}} = -\frac{i\omega_{mnp}\mu k_y H_O}{k_{mnp}^2 - k_z^2}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.12}$$

$$E_{ymnp}^{\mathrm{TE}} = \frac{i\omega_{mnp}\mu k_x H_O}{k_{mnp}^2 - k_z^2}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{2.13}$$

The allowable mode coefficients in this regard are $m=0,1,2,3,\ldots$; $n=0,1,2,3,\ldots$ and $p=1,2,3,\ldots$; with the only exception that $m=n=0$ is not allowed.

The resonant frequencies f with respect to each individual set of mode coefficients can be deduced by (2.14), adopting the earlier defined notation.

$$f_{mnp} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \tag{2.14}$$

where μ, ε are the permeability and the permittivity of the medium inside the cavity, respectively.

When assessing the number of modes that are present in a given cavity, three common methods exist. The first is based on what is termed 'mode counting', which can be performed by the repeated solution of (2.1) for both TE and TM modes, giving the total number of modes present with eigenvalues less than or equal to k (a practical limit for the propagation of modes). The second method is by an approximation referred to as 'Weyl's formula' [5], which is valid for cavities of general shape and is given in (2.15).

$$N = \frac{8\pi}{3}\left(a\times b\times d\right)\frac{f^3}{c^3} \tag{2.15}$$

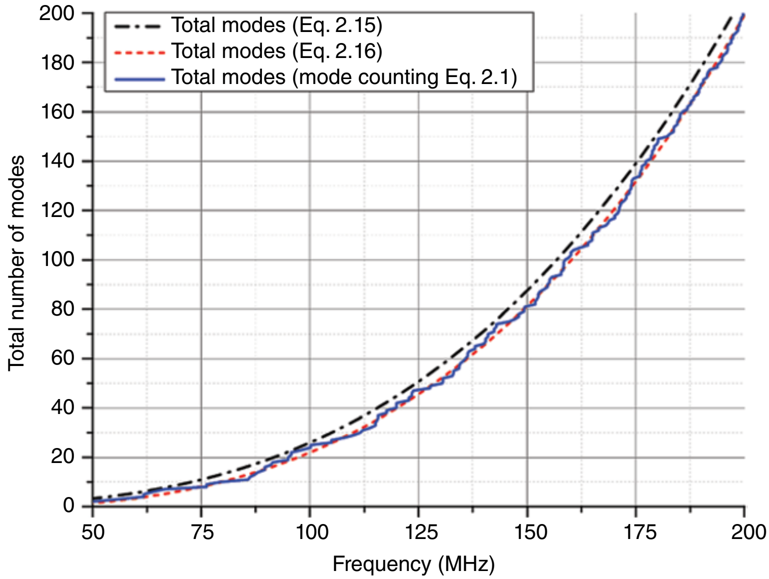

Figure 2.2 Mode numbers vs frequency for the University of Liverpool RC.

where N is the number of modes, f is the frequency in Hertz and c is the wave speed in the cavity medium in meter per second.

The third method is an extension to Weyl's formula specific to rectangular cavities [5] and is stated in (2.16).

$$N = \frac{8\pi}{3}(a \times b \times d)\frac{f^3}{c^3} - (a + b + d) + \frac{1}{2} \tag{2.16}$$

A comparison of the modal numbers present in the University of Liverpool RC (dimensions of $a = 3.6\,\text{m}$, $b = 4\,\text{m}$ and $d = 5.8\,\text{m}$) by all three methods is shown in Figure 2.2.

From Figure 2.2 we see that the extra terms in (2.16) improve the agreement obtained with the numerical mode counting as opposed to using the original Weyl's formula in (2.15). For increasing frequency, it is seen that the number of modes increases with respect to the cavity volume and the third power of frequency.

The mode density is another important parameter to be able to assess. This quantity charts the amount of modes available in a small bandwidth about a given frequency [5]. This quantity can be found by differentiating (2.16) and is issued in (2.17) after consultation with [5].

$$D_S(f) = 8\pi(a \times b \times d)\frac{f^2}{c^3} - \frac{a + b + d}{c} \tag{2.17}$$

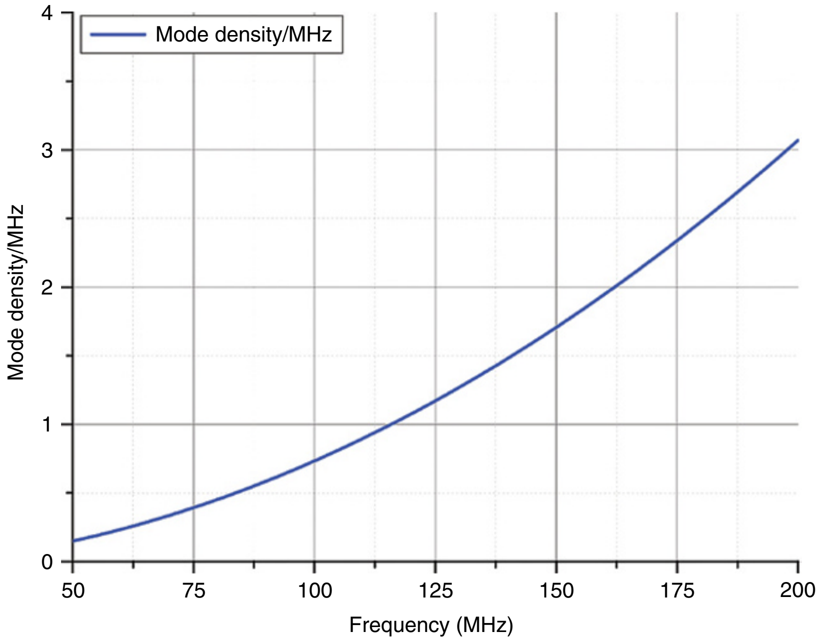

Figure 2.3 Mode density/MHz vs frequency in the University of Liverpool RC.

A plot of the mode density per megahertz vs frequency can be viewed in Figure 2.3. From Figure 2.3 we see that the chamber has a mode density of at least one mode per megahertz from 116 MHz upwards.

A low mode density in any given chamber means that chamber would not have adequate performance, as the mode density is too small to obtain spatial field uniformity [5].

Knowledge of the modal condition inside any given RC is important. As can be seen from Equations (2.4) to (2.13), the fields inside a given cavity can be depicted in terms of resonant modes and their subsequent integer coefficients (m, n and p) satisfying the boundary conditions at the walls of the chamber. In operational practice, different modes are required to be excited in order to promote a sufficient change in the field distribution (to achieve spatial uniformity). The first five allowable modes in the University of Liverpool RC can be seen in Table 2.1.

It is advantageous to be able to calculate and visualise the fields created inside a given chamber by the excitation of resonant modes. However, different from the prior field equations in (2.4)–(2.13), it is advantageous to visualise these fields in a 'non-empty' chamber; that is, with a realistic excitation involved. This provides confidence to be able to interpret the conditions inside the chamber and link them to operational

Table 2.1 The first five resonant modes in University of Liverpool RC.

Distinct mode	m	n	p	Resonant frequency (MHz)
1	0	1	1	45.55
2	1	0	1	49.04
3	1	1	0	56.06
4	1	1	1	61.73
5	0	1	2	63.89

conditions that are witnessed in practice. With respect to the modes and the subsequent fields, it is known from [6] that:

1. The Electromagnetic (EM) fields inside a rectangular chamber outside of the source area are purely the superposition of all TE and TM modes generated within it. In the source area, an extra term needs to be added from the contribution of other hybrid modes.
2. For a current with any polarisation inside the chamber, an electric field with three components may be generated. This essentially means that a signal transmitted by an antenna with one polarisation can be received by an antenna with any polarisation, which is advantageous for measurement purposes.
3. Modes inside the chamber can be controlled by the choice of the polarisation and location of the excitation source which is important as we will see later in this chapter.

In Ref. [6] a computationally efficient series of equations based on cavity Green's functions was derived in order to study the resultant electric fields inside shielded enclosures. The Green's function is essentially a compact means of describing an electric or magnetic field distribution due to a current source, and is desirable here because a realistic source type can be incorporated. The main equations can be stated as (2.18), (2.19), (2.20) and (2.21).

$$E = \frac{1}{j\omega\varepsilon} \int_{\text{source}} \underline{G} \cdot J\left(x', y', z'\right) dv' \tag{2.18}$$

where $J(x', y', z')$ = the excitation current, ω is the angular frequency in radians and $\underline{G}$ = dyadic Green's function.

$$E = \frac{1}{j\omega\varepsilon} \int_{\text{source}} \left[G_{xy}\hat{x} + G_{yy}\hat{y} + G_{zy}\hat{z}\right] J\left(x', y', z'\right) dv' \tag{2.19}$$

$$
\begin{aligned}
\underline{G} = {} & \sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\frac{2\varepsilon_{0m}}{da\alpha\sin\alpha b}\sin k_z z'\cos k_x x' \\
& \left\{\begin{array}{l}
\left\{k_z k_x \cos k_z z \sin k_x x \begin{array}{l}\sin\alpha y\sin\alpha\left(b-y'\right) \\ \sin\alpha y'\sin\alpha\left(b-y\right)\end{array}\right\} \begin{array}{l} y<y' \\ y>y' \end{array} \hat{z}\hat{x} \\
\left\{+\left(k_x^{\,2}-k^2\right)\sin k_z z\cos k_x x \begin{array}{l}\sin\alpha y\sin\alpha\left(b-y'\right) \\ \sin\alpha y'\sin\alpha\left(b-y\right)\end{array}\right\} \begin{array}{l} y<y' \\ y>y' \end{array} \hat{x}\hat{x} \\
\left\{+k_x\alpha\sin k_z z\sin k_x x \begin{array}{l}\cos\alpha y\sin\alpha\left(b-y'\right) \\ -\sin\alpha y'\cos\alpha\left(b-y\right)\end{array}\right\} \begin{array}{l} y<y' \\ y>y' \end{array} \hat{y}\hat{x}
\end{array}\right\} \\
& +\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{2\varepsilon_{0n}}{ab\beta\sin\beta d}\sin k_x x'\cos k_y y' \\
& \left\{\begin{array}{l}
\left\{k_x k_y \cos k_x x \sin k_y y \begin{array}{l}\sin\beta z\sin\beta\left(d-z'\right) \\ \sin\beta z'\sin\beta\left(d-z\right)\end{array}\right\} \begin{array}{l} z<z' \\ z>z' \end{array} \hat{x}\hat{y} \\
\left\{+\left(k_y^{\,2}-k^2\right)\sin k_x x\cos k_y y \begin{array}{l}\sin\beta z\sin\beta\left(d-z'\right) \\ \sin\beta z'\sin\beta\left(d-z\right)\end{array}\right\} \begin{array}{l} z<z' \\ z>z' \end{array} \hat{y}\hat{y} \\
\left\{+k_y\beta\sin k_x x\sin k_y y \begin{array}{l}\cos\beta z\sin\beta\left(d-z'\right) \\ -\sin\beta z'\cos\beta\left(d-z\right)\end{array}\right\} \begin{array}{l} z<z' \\ z>z' \end{array} \hat{z}\hat{y}
\end{array}\right\} \\
& +\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\frac{2\varepsilon_{0p}}{bd\gamma\sin\gamma a}\sin k_y y'\cos k_z z' \\
& \left\{\begin{array}{l}
\left\{k_y k_z \cos k_y y \sin k_z z \begin{array}{l}\sin\gamma x\sin\gamma\left(a-x'\right) \\ \sin\gamma x'\sin\gamma\left(a-x\right)\end{array}\right\} \begin{array}{l} x<x' \\ x>x' \end{array} \hat{y}\hat{z} \\
\left\{+\left(k_z^{\,2}-k^2\right)\sin k_y y\cos k_z z \begin{array}{l}\sin\gamma x\sin\gamma\left(a-x'\right) \\ \sin\gamma x'\sin\gamma\left(a-x\right)\end{array}\right\} \begin{array}{l} x<x' \\ x>x' \end{array} \hat{z}\hat{z} \\
\left\{+k_z\gamma\sin k_y y\sin k_z z \begin{array}{l}\cos\gamma x\sin\left(a-x'\right) \\ -\sin\gamma x'\cos\gamma\left(a-x\right)\end{array}\right\} \begin{array}{l} x<x' \\ x>x' \end{array} \hat{x}\hat{z}
\end{array}\right\}
\end{aligned}
\tag{2.20}
$$

where $\varepsilon_{0n} = \begin{array}{l} 1 \text{ when } n=0, \\ 2 \text{ when } n\neq 0, \end{array}$ $\alpha=\sqrt{k^2-k_z^2-k_x^2}$, $\beta=\sqrt{k^2-k_x^2-k_y^2}$ and $\gamma=\sqrt{k^2-k_y^2-k_z^2}$

Now, assuming a y polarised unit current (with respect to the principle axis defined in Figure 2.1, meaning that the current source is vertically linearly polarised), and with reference to (2.18), (2.19) and (2.20), the resultant E_y electric field on an xz plane can be obtained using (2.21).