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**RF SYSTEM DESIGN  
OF TRANSCEIVERS FOR  
WIRELESS  
COMMUNICATIONS**

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# RF SYSTEM DESIGN OF TRANSCEIVERS FOR WIRELESS COMMUNICATIONS

**Qizheng Gu**

*Nokia Mobile Phones, Inc.*



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**To my wife, Lixian,  
and  
to our family's younger generations, Ye and Ethan**

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## Preface

This book is about radio frequency (RF) transceiver system design for wireless communication systems. Most digital communications texts focus on the system design of the digital base-band rather than the RF section. The text is written for RF system design engineers as well as RFIC design engineers involved in the design of radios for digital communication systems. It is also appropriate for senior undergraduates and graduate students in electrical engineering.

The text develops systematic design methods of RF receivers and transmitters along with a corresponding set of comprehensive design formulas. Attention is given equally to the analysis of the RF systems. The book is focused on mobile communication systems implemented in RF application specific integrated circuits (ASICs) but it is applicable to other wireless systems such as, for examples, WLAN, Bluetooth and GPS. It covers a wide range of topics from general principles of communication theory, as it applies to digital radio designs, to specific examples on the implementation of multimode mobile systems. It is assumed that the reader has a good RF background, basic knowledge of signal and communication theory, and fundamentals of analog and mixed signal circuits.

Completion of this book is the result of helps and encouragement from many individuals, to all of whom I express my sincerest thanks and appreciation. To Dr. Bjorn Bjerede, I wish to pay tribute for his constant encouragement, many valuable inputs and technical discussions during the course of writing this book. I am deeply indebted to Professor Peter Asbeck of UCSD, for his time to review the whole manuscript, and his very useful comments and helpful suggestions. Much of my RF knowledge and analysis skill were learned and developed when I was engaged in research at the Center for Electromagnetic Wave Theory and Application headed by Professor Kong, in the Research Laboratory of Electronics at MIT. I would like to express my sincerest gratitude to Professor Jin Au Kong of MIT for his creating and providing an academic environment in which there is much knowledge sharing, learning, and searching. My knowledge of wireless communication systems and RF transceivers was learned and accumulated from practical designs when working for wireless companies, such as, PCSI,

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Qizheng Gu  
April, 2005

# Chapter 1

## Introduction

### 1.1. Wireless Systems

In wireless systems, the connection between equipments, such as a mobile station and a base station, is by means of electromagnetic waves instead of a cable or a wire. In this sense, optical and infrared communications systems are also wireless systems. However, the wireless systems discussed in this book are only those based on the connection medium of *radio frequency (RF)* electromagnetic waves. The present practicable limits of radio frequency are roughly 10 kHz to 100 GHz [1]. Wireless systems at present commonly operate in hundreds MHz or a few GHz frequency. Electromagnetic waves with a frequency in these regions have a propagation distance with an acceptable attenuation and a good penetrating capability through buildings and vehicles and are able to carry wide-band signals.

#### 1.1.1. Mobile Communications Systems

Mobile communications started in 1920s, but the real mobile communication era began in the early 1980s. The cellular mobile system, *advanced mobile phone service (AMPS)*, first operated for commercial telecommunication service in the United States in 1983. All the first-generation mobile communications systems — including AMPS, *TACS (total access communications system)* in the U.K., *NTT (Nippon Telephone and Telegraph)* systems in Japan, and *NMT (Nordic Mobile Telephones)* in Europe — are analog systems.

The second-generation mobile systems — digital mobile systems — were introduced in late 1980s. There are several competing worldwide digital standards, such as the *global system for mobile communications (GSM)*, *IS-95/98 code-division multiple-access system (CDMA)*, *IS-136 time-division multiple-access system* (called as U.S. *TDMA* or *D-AMPS*), and the *personal digital cellular (PDC)* system in Japan. GSM systems

normally operate in the 900 MHz frequency band. The GSM specification translated to higher frequency 1800 MHz band is known as the *digital communication system (DCS)* but also called as 1800 MHz GSM. IS-95/98 CDMA systems and IS-136 DAMPS usually operate in either 800 MHz cellular band or 1900 MHz PCS band. The second-generation mobile systems, as the first-generation systems, are still used mainly for voice communications. However, the efforts to bring the data capability to existing mobile systems in the wireless industry have not slowed down. The *general package radio system (GPRS)* and the *enhanced data rate for GSM evaluation (EDGE)*, so-called 2.5-generation technologies were then born one after another. An EDGE up to 384 kbps is an evolutionary technology for GSM systems. It allows these systems to provide voice, data, Internet, and other connectivity solutions.

The need for more voice capacity to accommodate more callers reliably and to generate more revenue through increased billable minutes of use of high-speed data services, 384 kbps mobile and 2 MHz fixed, to cater to the needs of an increasing mobile user community, corporate local network access, wireless Internet access, and global roaming have pushed the limits of existing 2G networks [2]. In late 1990s, the third-generation systems, *cdma2000\_1x* and *wide-band code division multiple access (WCDMA)* systems, emerged. The new-generation systems are able to support all these needs and to provide video and multimedia services as well as voice communications.

### 1.1.2. Wireless Local Area Network (WLAN)

*Wireless LAN* is seen as the technology that will enable the most convenient link between existing wired networks and portable computing and communications equipments, such as laptop computers and *personal digital assistants (PDAs)*, at the office, hotel, company, or campus level. Building company- or campuswide data communications through the WLAN can reduce the need for wiring among several buildings. In general, the applications of the WLAN systems can be simply between two computers or between a computer and a wired network, all the way up to a complete network with many users and a great number of data paths.

In mid 1997, the Institute of Electrical and Electronics Engineers (IEEE) finalized the initial standard for WLANs, IEEE 802.11. This standard specifies a license-free (ISM band) 2.4 GHz operating frequency with data rates of 1 and 2 Mbps using a direct sequence or a frequency-hopping spread spectrum. The 802.11 is not a single but a family of standards addressing WLAN. The IEEE 802.11a standard defines a WLAN

system based on the *orthogonal frequency division multiplexing (OFDM)* technology that splits an information signal across 52 separate subcarriers to provide a transmission-of-data rate up to 54 Mbps and throughputs over 24.3 Mbps, and the operating frequency of the system is in the license-free (UNII band) 5.15 to 5.35 GHz and 5.725 to 5.875 GHz bands. The 802.11a devices, as the other 802.11 based devices, share the channel on a *time-division multiple-access (TDMA)* basis. The 802.11b specifies a WLAN system using *direct sequence spread spectrum (DSSS)* to achieve the maximum 11 Mbps transmission-of-data rate in 2.4 GHz band. To benefit from the high data rates in 2.4 GHz band, a new standard, 802.11g, that is based on the OFDM and fully backward compatible with the 802.11b standard was developed in early 2000. The high-data-rate WLANs like the 802.11a and 802.11g systems satisfy requirements of multimedia applications including streaming HDTV-quality video in the home, high-speed Internet, and file transfer.

The supplement standards, 802.11e, 802.11f, 802.11h, and 802.11i are defined to enhance the capability of IEEE 802.11-based WLANs. The 802.11e is for enhancements of the *quality of service (QoS)*. The 802.11f provides a recommended practice for an interaccess-point protocol. The 802.11h extends the spectrum and transmit power management at 5 GHz for European operation. The 802.11i enhances MAC layer security.

The wireless local area networking emerged from the communication market in late 1990s and now has become one of major forces driving wireless communications technologies.

### 1.1.3. Bluetooth

*Bluetooth* is a *wireless personal area network (WPAN)* and was intended for the short distance ( $\sim 10$  m) connectivity and communications between devices in a room where cables had been used. Bluetooth operates in the unlicensed 2.4 GHz *industrial, scientific, and medical (ISM)* band. This band is 83.5 MHz wide, beginning at 2.4 GHz and ending at 2.4835 GHz. The *frequency-hopping spectrum spread (FHSS)* technique is used in Bluetooth. Signals of Bluetooth devices hop in a pseudo random manner among 79 defined channels spaced at 1 MHz. In this sense, Bluetooth occupies the entire ISM band, but at any instant only a small portion of the band ( $\sim 1$  MHz) is ever occupied. The frequency-hopping rate of the Bluetooth signal is 1600 hops per second.

The Bluetooth device should have low cost and low current consumption. It supports only up to a data rate of 720 kbps. To maintain its low current consumption and cost characteristics, its typical transmission

power is around 1 mW or 0 dBm. The Bluetooth receiver has sensitivity  $-70$  dBm or better. The minimum performance of a Bluetooth radio is defined in [2].

The main application of the Bluetooth is pervasive connectivity among a myriad of small office- or home-located wireless devices, including Bluetooth-enabled computers, telephones, home environment, security and small-office or home-management systems. Furthermore, some of these devices will have Bluetooth-enabled subdevices, such as wireless cameras, keyboards, mice, headset, and microphones. All these devices are wirelessly interconnected using the protocols of the Bluetooth.

A wireless personal area network (WPAN) standard, IEEE 802.15.1, has been developed based on the Bluetooth protocol. Since the Bluetooth, WPAN and the 802.11b/g WLAN operate in the same 2.4 GHz ISM frequency band, to alleviate potential problems with the coexistence interference, the coexistence solutions for these systems have been extensively discussed in the industry and proposed by the IEEE 802.15.2 coexistence task group, Bluetooth special interest group and 802.11 working group [3-4].

#### 1.1.4. Global Positioning System (GPS)

The *global positioning system (GPS)* is made up of 24 satellites, which orbit 12,000 miles above the earth, constantly transmitting the precise time and their position in spaces. GPS receivers use triangulation of signals from the satellites to determine their precise locations on earth. GPS satellites know their location in space and GPS receivers can determine their distance from a satellite by using the travel time of a radio message from the satellite to the receiver.

Commercial GPS operates at the frequency of 1575.42 MHz and a 1.023 Mchip/sec *pseudo noise (PN)* sequence spread spectrum is used to spread its ranging signal resulting in a bandwidth 2.046 MHz. The sensitivity of a stand-alone GPS receiver is  $-130$  dBm or better. The commercial GPS is commonly used in vehicle-tracking systems and navigation systems for automobile driving or boat sailing. Embedding GPS capability in mobile stations has become a Federal Communications Commission (FCC) requirement. In the mid-1990s, the FCC required that wireless mobile stations should embed GPS capability and the wireless service providers include features that would make the *Enhanced 911 (E911)* available to their customers. In this application, the wireless network assistant GPS receiver must have a close to  $-150$  dBm sensitivity [5]. The GPS may become a technology for everyday convenience. GPS-capable

mobile computers or PDAs and portable GPS devices will advise the user of the nearest hotel, restaurant, point of interest, or gas station [6].

### *1.1.5. Ultra Wide-band Communications*

The *ultra wide-band (UWB)* technology will be used in communications for the wireless personal area networks (WPAN) typically operating within ~10 meters [7]. The USB was defined as any signal that occupies more than 500 MHz in the 3.1 to 10.6 GHz band. The Federal Communications Commission's ruling limits UWB EIRP power to -41.25 dBm/MHz over the above band, and the emission limits vary according to frequency bands, such as rolling-down to -75 dBm/MHz in the GPS band.

In the UWB communications, two technical approaches are commonly used. One is based on 'impulses,' actually extremely short digital pulses in the subnanosecond range (1 to 1000 picosecond). To carry information, impulse trains can be modulated with either position, amplitude, or phase. The pulses can be modulated directly by the base-band signal instead of using a high-frequency carrier. Another is the multiband approach. The concept is to utilize multiple-frequency bands (say, 528 MHz for each band [8]) to efficiently use the UWB spectrum by frequency hopping over multiple UWB bands. These signals do not interfere with each other because they operate at different frequency bands within the UWB spectrum. Each of the UWB signals can be modulated to achieve a very high data rate. Several digital modulation techniques commonly employed in wireless communications, such as OFDM, can be used on each individual UWB signal. The advantage of the multibands approach is that they are extremely adaptive and scalable. The number of bands used in communications can vary. The bands can dynamically be adjusted to remove the affected bands with interference or to avoid using the band already utilized by another service.

The ultra wide bandwidth of UWB signals potentially has very high data rates of up to 480 Mbps. It is worth knowing that based on Shannon's information theory, the information-carrying capacity is linearly proportional to bandwidth and logarithmically with power, making it very attractive to increase the bandwidth for achieving a high data rate. Limiting UWB to a very short distance is the key as it isolates the wide-band transmission to a very local area. The UWB is being targeted at WPANs, audio and video distribution within the home, as well as a cable replacement option for USB and FireWire. The standard of the WPAN based on the UWB technology, IEEE P802.15.3a, requires a data rate of 110 Mbps at 10 meters with up to 4 bands and a power consumption of less than 100 mW.



The rates will be scaled up to 200 Mbps at 5 m, with rates increasing down past 4 m [9].

## 1.2. System Design Convergence

Convergence in the wireless worlds of communications, computing, global positioning systems, and consumer electronic devices is an irresistible trend today. The growing complexity of today's wireless products results largely from the rapid convergence of traditionally separate technologies. One inevitable consequence of the convergence is the impending integration of voice, data, image, video, music, the Internet, instant messaging, home automation, and GPS.

The design task in this converging world is tough to do because of the merging of once-distinct systems. The best way to overcome the challenges caused by the converging technologies is to successfully incorporate RF analog-digital, hardware-software, *system-on-a-chip* (SoC), and *printed-circuit-board* (PCB) designs. RF analog systems must be designed in sequence with digital systems, not as a separate exercise. The blurring of distinctions and proper partitioning between hardware and software make it critical to architect the system properly before deciding on the implementation. Increased complexity and the shrinking size of wireless products have resulted in designing and using highly integrated SoCs to reduce number of discrete devices on the PCBs as possible [10–11].

The age in which each of us lived in our own cozy design environment, caring little about what was going on in others, has passed [11]. Today, the key to success in our wireless system and product designs is knowledge proliferation across technical disciplines. There is no exception for the RF system design either. The design of the RF system in wireless products must cross technical boundaries to incorporate digital, DSP, and SoC designs.

The early cellular phone looked like and felt like a brick. Today's multiband and multimode mobile phone can be kept in a small pocket and has multiple functions — such as GPS-based E911, Bluetooth-enabled short distance connectivity, and digital camera — other than the voice and data communications. It is impossible to shrink the multi-function mobile phone into a pocket size without using highly *integrated circuits* (ICs), i.e., SoC solutions. The highly integrated RF analog chips that can be considered as SoCs came to the market later than digital BB ICs in the middle of 1990's. For instance, the so-called  $\alpha$  chip set consisting of one digital base-band chip and four RF analog chips for the *personal handy-phone system* (PHS) mobile stations was developed by Pacific Communication Science Inc. in

1994. Actually, the SoC not only makes a complex wireless product able to fit in a small-form factor but also is a unique enabler for implementation of the *direct conversion architecture receiver*. Since a transceiver is now built as a system of an IC chip, RF system design of the transceiver is tightly associated with the development of relevant RF analog ICs. The relevant ICs must be designed and developed according to the selected radio architecture, given system partitioning and the defined specifications of individual stages that resulted from the RF system design.

The RF system is actually a subsystem of the overall wireless digital transceiver. It is apparent that the performance of the digital base-band and the RF analog systems of the transceiver will affect each other. The digital base-band demodulator performance and processing gain determine the maximum allowed *noise figure* of the RF receiver for a defined sensitivity. The group delay distortion of channel filtering in the RF receiver chain impacts minimum requirement of the *signal-to-noise/interference ratio (SNIR)* to achieve a certain data *bit error rate (BER)*. The RF receiver *automatic gain control (AGC)* loop is closed in the digital *base-band (BB)* since the received signal strength is measured in the digital domain instead of using analog power detector as in old days. On the transmitter side, the transmission power level is entirely controlled by the digital base-band, and the DC offset and the imbalance between *in-phase (I)* and *quadrature (Q)* channels in the transmitter analog block are compensated by means of adjusting BB digital signal level in the I or Q channel. The interfaces between the RF and digital BB are the *analog-to-digital converter (ADC)* in the receiver and the *digital-to-analog converter (DAC)* in the transmitter, respectively. The dynamic range or resolution of the ADC and the DAC impacts the gain control range and filtering requirements of the RF receiver and transmitter. The RF system design, thus, together with the digital BB design (or what is referred to as RF-BB co-design), becomes necessary, and to be competent for this a qualified RF designer must possess enough knowledge in the digital base-band system and modern communications theory.

Another area that needs to be seriously considered in the RF system design is the *digital signal processing (DSP)* and associated software. Today, RF design demands more and more supports from the DSP. The RF analog block gain controls of transceiver AGC loops are usually executed by means of the DSP embedded in the digital base-band. Operating mode change, frequency band switch, and/or channel selection of a multimode and multiband wireless transceiver are carried out through software. In the wireless mobile station, the DSP is also heavily used in power management to dynamically control the power amplifier and IC biases according to transmission power. Some radio architectures, such as the direct conversion

and low IF receivers, need more supports from the DSP than ever requested by the superheterodyne receiver. These architecture receivers without the DSP cooperation to compensate DC offset, to correct I and Q channel imbalance, and/or to run more complicated AGC system cannot perform properly as expected in the best case or even may not work in the worst case. The DSP and software will play a much more important role in the future *software-defined radio (SDR)*, in which the ADC and DAC have been pushed as close to the antenna as possible and the RF front-end including RF filters and antennas are programmable as well as the digital base-band.

We are facing all these challenges in RF system-design convergence. In this book, the RF system of wireless transceivers will be developed based on highly integrated RF analog integrated circuits. The fundamental approaches and formulas presented in this book, however, can be definitely applied in discrete RF system design. To cope with the design convergence, fundamentals of the digital base-band system are introduced in the next chapter. The design incorporating the DSP will be described wherever it is needed.

### **1.3. Organization of This Book**

This book consists of six chapters. It systematically describes RF system design methodology for transceivers used in various wireless systems, particularly for those in mobile stations. Practically used radio architectures are discussed in detail. RF receiver and transmitter system designs are firmly backed by relevant formulas, and simple but efficient design tools for RF receiver and transmitter can be then developed in terms of the Excel spreadsheet or the Matlab program. Almost all examples in this book are based on mobile stations of different protocol wireless systems.

Fundamentals of the general system and the digital base-band are introduced in Chapter 2. Understanding them is very significant in today's RF system design. Linear system theory is the foundation for analysis and design of RF and digital base-band systems. The system analysis and design can be carried out either in the time domain or in the frequency domain in terms of Fourier transformations. Nonlinear modeling and simulation approaches, which are important for evaluating performance of power amplifier and other nonlinear devices in the transmitter, are addressed. Noise and random processes play substantial roles in communications, and they are extensively discussed in Chapter 2. The last section of this chapter presents digital base-band system related topics including the sampling theorem and process, jitter and quantizing noises, commonly used modulation schemes, pulse shaping technology and intersymbol interference, error probability of

detection and carrier-to-noise ratio estimation for achieving certain bit or frame error rate.

Chapter 3 mainly discusses different radio architectures and their design considerations. Selection of radio architecture is the first thing to do when developing an RF transceiver. The architecture discussions in this chapter are based on mobile stations adopted in wireless communication systems, such as GSM, CDMA, etc., but they are generally also applicable to other wireless transceiver. The most popularly used radio architecture was the superheterodyne transceivers. It has outstanding performance in receiver sensitivity, selectivity, and current consumption, and therefore it was commonly employed in all kinds of wireless systems. The direct-conversion or so-called zero IF, architecture transceiver is currently becoming more popularly used in wireless than the superheterodyne radio. Its extraordinary advantages and technical challenges are discussed. The modern RFIC and DSP technologies make the direct-conversion architecture implementation feasible. The third radio architecture addressed in this chapter is the low IF one. This architecture overcomes the technical issues of the direct conversion radio at a cost of more difficulty in obtaining needed image rejection. Today most GPS receivers are using low IF architecture, and some GSM mobile station receivers employ this architecture too under certain DSP supports. The last radio architecture is based on band-pass sampling, which steps further forward to the software-defined radio (SDR). In this architecture, the ADC and the DAC play the RF quadrature down- and up-converter roles, and most of analog function blocks are moved to the digital domain except the RF front-end, i.e., RF filters, *low noise amplifiers (LNA)* in the receiver, and power amplifiers in the transmitter.

Receiver RF system analysis and design are discussed in Chapter 4. Formulas for receiver performance evaluation are derived and presented in an individual section of the receiver key parameter analysis. They are the foundation of the receiver system design. Noise figure of a receiver is first analyzed since it determines one of the most important receiver parameters, sensitivity. Impacts of the transmitter power leakage and antenna mismatch on the receiver sensitivity are also analyzed. The relationship between the receiver linearity and the intermodulation spurious response attenuation is discussed in a generic way. In the CDMA mobile station receiver, there is a special requirement referred to as single-tone desensitization, which results from the AM transmission leakage cross-modulating a strong interference tone caused by the third-order nonlinearity of the receiver LNA. An approximate estimation approach of the single-tone desensitization is presented in the fourth subsection. The selectivity of adjacent and alternate channels is a commonly used specification in the wireless systems with channelization. They are determined mainly by the phase noise of local

oscillators at the adjacent and the alternate channel frequencies and at the receiver filtering characteristics as analyzed in Section 4.5. The automatic gain control (AGC) system has substantial importance for any configuration receiver. The basic design principles of a receiver AGC loop are discussed. The last section describes the receiver system design method and performance evaluation approach. A Matlab program for the receiver performance evaluation is provided in the appendix of this chapter.

Chapter 5 discusses transmitter system analysis and design. Similar to the previous chapter, formulas for transmitter performance evaluation are provided in individual sections of transmitter key parameter analysis. However, in the transmitter case, simulation instead of calculation may be necessary to evaluating nonlinear effects of devices in the transmitter chain on some transmitter parameters. Transmission power and spectrum are first discussed. In the successive sections, calculation approaches of modulation accuracy or so called waveform quality factor of a transmission signal are presented, and main factors that degrade the modulation accuracy are analyzed. Another key parameter of a transmitter is the adjacent/alternate channel power. The main contributor to the adjacent/alternate channel power is the nonlinearity of the transmitter chain. Simulation methods and approximate formulas for the adjacent/alternate channel power calculation are provided in the fourth section of this chapter. Out-of-channel band noise and spurious emissions of a transmitter are usually interference sources to adjacent channel mobile stations and other system equipments, and their level is tightly restricted. The calculation of noise and spurious emission of the transmitter is introduced in the fifth section. The transmitter AGC is usually incorporated with power-management controls through adjusting device bias or switching the device on or off. In Section 5.6, a preliminary discussion of the transmitter AGC and power management is presented. The final section of this chapter describes the transmitter system design considerations including architecture comparison and system block partitioning. A Matlab program for calculating adjacent/alternate channel power can be found in Appendix 5C.

The last chapter of this book provides RF system design examples of wireless mobile transceivers. A system design of multi-mode and multi-band superheterodyne transceiver is first discussed. The designed mobile transceiver is able to operate in GSM (GPRS), TDMA, and AMPS systems and to run in the 800 MHz Cellular band and 1900 MHz PCS band. The second application example is an RF system design dedicated to the CDMA direct conversion transceiver. Among wireless mobile communication systems, the CDMA system has the toughest performance requirements on its mobile stations as a result of its complexity and operation bandwidth allocations. It discusses in detail how the challenges of CDMA direct-

conversion transceiver can be efficiently overcome through a proper RF system design.

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# Chapter 2

## Fundamentals of System Design

### 2.1. Linear Systems and Transformations

#### 2.1.1. Linear System

A *system* refers to any entity that produces a unique output in response to a legitimate input. In wireless communications, communication channels, base stations, mobile stations, receivers, transmitters, frequency synthesizers, and even filters all are physical systems with different complexity. Mathematically, a system can be described in

$$y(t) = T[x(t)], \quad (2.1.1)$$

where  $x(t)$  is the input (or excitation),  $y(t)$  is the output (or response),  $t$  is an independent variable usually representing time, and  $T$  is the operation performed by the system. Then the system is also viewed as a transformation (or mapping) of  $x(t)$  into  $y(t)$ .

A system is linear if and only if the principle of superposition holds — i.e., its output can be expressed as a linear combination of responses to individual inputs:

$$T[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \alpha_1 T[x_1(t)] + \alpha_2 T[x_2(t)], \quad (2.1.2)$$

where  $\alpha_1$  and  $\alpha_2$  are arbitrary scalars. A system that does not satisfy the superposition relationship (2.1.2) is classified as nonlinear.

There is a class of linear systems called *linear time-invariant (LTI)* systems that play particularly important role in communication system theory and design. A system is referred to as time-invariant if a time shift,  $\tau$ , in the input signal  $x(t-\tau)$  causes the same time shift in the output signal  $y(t)$ . This is expressed as

$$y(t - \tau) = L[x(t - \tau)], \quad (2.1.3)$$



where  $L$  is LTI system operator. LTI systems provide accurate models for a large number of building blocks in a communication system. Important examples of LTI systems include many of the basic devices used in base and mobile stations, such as filters, isolators, duplexers, and amplifiers operating in their linear region.

The LTI system can be completely characterized by what is referred to as the impulse response of the system, which is defined as the response of the system when the input is an impulse signal  $\delta(t)$  — i.e.,

$$h(t) = L[\delta(t)]. \quad (2.1.4)$$

The impulse signal  $\delta(t)$  also known as impulse function or Dirac delta function, is defined by its effect on a test function  $\phi(t)$  as follows:

$$\int_{-\infty}^{\infty} \phi(t) \delta(t) dt = \phi(0), \quad (2.1.5a)$$

and in general, a delayed delta function  $\delta(t-t_o)$  is defined by

$$\int_{-\infty}^{\infty} \phi(t) \delta(t-t_o) dt = \phi(t_o). \quad (2.1.5b)$$

The  $\delta(t)$  function possesses the following properties:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad (2.1.6)$$

and

$$\int_{-\varepsilon}^{\varepsilon} \delta(t) dt = 1, \quad (2.1.7)$$

$\varepsilon \rightarrow 0$

where  $\varepsilon$  is a small value approaching zero.

The time response of LTI systems to their inputs can be simply derived by performing convolution of the input and the impulse response of the system. From (2.1.5b), the input  $x(t)$  can be expressed as

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau. \quad (2.1.8)$$

The response  $y(t)$  of a LTI system to an input  $x(t)$  is

$$\begin{aligned} y(t) = \mathcal{L}[x(t)] &= \mathcal{L}\left[\int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau\right] = \int_{-\infty}^{\infty} x(\tau)\mathcal{L}[\delta(t - \tau)]d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau = x(t) * h(t). \end{aligned} \quad (2.1.9)$$

In (2.1.9),  $*$  denotes the convolution.

It is worth while to note an example of the response of a LTI system with an impulse response  $h(t)$  to a complex exponential input signal  $x(t) = e^{j\omega t}$ , where  $j = \sqrt{-1}$  and  $\omega = 2\pi f$  is referred to as *angular frequency*. From (2.1.9), we have

$$\begin{aligned} y(t) = \mathcal{L}[e^{j\omega t}] &= \int_{-\infty}^{\infty} h(\tau)e^{j2\pi f(t-\tau)}d\tau \\ &= e^{j2\pi ft} \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}d\tau = H(f)e^{j2\pi ft}, \end{aligned} \quad (2.1.10)$$

where

$$H(f) = |H(f)|e^{j\angle H(f)} = \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}d\tau. \quad (2.1.11)$$

In (2.1.11),  $|H(f)|$  and  $\angle H(f)$  are amplitude and phase of function  $H(f)$ , respectively. (2.1.10) tells us that the response of a LTI system to a complex exponential signal with a frequency  $f = \omega/2\pi$  is still a complex exponential signal with the same frequency. Thus,  $e^{j\omega t}$  is the eigenfunction of the LTI system and  $H(f)$  is the eigenvalue of  $\mathcal{L}$  associated with  $e^{j\omega t}$ . This result provides a foundation of Fourier analysis.

### 2.1.2. Fourier Series and Transformation

Fourier series and transformation convert time-domain signals to frequency domain (or spectral) representations. This conversion, in many cases, provides better insight into the behavior of certain types of systems. Based on Fourier series and transformation, a frequency-domain approach of analyzing LTI systems called Fourier analysis has been developed. This

approach makes analyzing some LTI systems in the frequency domain easier than directly in the time domain.

A periodic signal  $x(t)$  with a period  $T_o$  can be expanded into a complex exponential Fourier series as follows if it satisfies the Dirichlet conditions:<sup>§</sup>

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jk \cdot 2\pi f_o t}, \quad f_o = \frac{1}{T_o}, \quad (2.1.12)$$

where  $x_k$  are known as the *complex Fourier coefficients* and are given by

$$x_k = \frac{1}{T_o} \int_{T_o} x(t) e^{-jk \cdot 2\pi f_o t} dt \quad (2.1.13)$$

where  $\int_{T_o}$  denotes the integral over any one period. The complex coefficients  $x_k$  can be expressed as

$$x_k = |x_k| e^{j\varphi_k}. \quad (2.1.14)$$

where  $|x_k|$  and  $\varphi_k$  are amplitude and phase of the  $k$ th harmonic of  $x(t)$ . A plot of  $|x_k|$  versus the frequency is called *amplitude spectrum* of the periodic signal  $x(t)$ , and a plot of  $\varphi_k$  versus frequency is referred to phase spectrum of  $x(t)$ . When  $x(t)$  is real, from (2.1.1.3) we have

$$x_{-k} = x_k^* \quad (2.1.15)$$

In (2.1.14),  $x_k^*$  means the complex conjugate of  $x_k$ .

The average power of a periodic signal  $x(t)$  over any period is defined as

$$P = \frac{1}{T_o} \int_{T_o} |x(t)|^2 dt \quad (2.1.16)$$

---

<sup>§</sup> 1.  $x(t)$  is absolutely integrable over its period — i.e.,  $\int_{T_o} |x(t)| dt < \infty$ ; 2. The number of maximum and minimum of  $x(t)$  is finite and 3. The number of discontinuities of  $x(t)$  in each period is finite.

Substituting (2.1.12) into the above equation, we obtain the following expression:

$$\frac{1}{T_o} \int_{t_o} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |x_k|^2. \quad (2.1.17)$$

This is called *Parseval's identity*. It says that the average power of a period signal  $x(t)$  is equal to the sum of the power contents of its harmonics or its spectral lines.

For a nonperiod signal  $x(t)$ , we can imagine that the signal has a  $T_o \rightarrow \infty$ . Assuming  $x(t)$  satisfies the Dirichlet conditions given in the foot note of the previous page, we can introduce new variables  $df = 1/T_o$  and  $2\pi f = \omega = k\omega_o$  to (2.1.12) and (2.1.13) and obtain

$$x(t) = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f \tau} d\tau \right) e^{j2\pi f t} df. \quad (2.1.18)$$

Let us define the Fourier transform of  $x(t)$  as

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = F[x(t)]. \quad (2.1.19)$$

Then (2.1.18) becomes the inverse Fourier transform of  $X(f)$ :

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df = F^{-1}[X(f)], \quad (2.1.20)$$

where notations  $F$  and  $F^{-1}$  denote the Fourier transform and inverse Fourier transform, respectively. Equations (2.1.19) and (2.1.20) are referred to as a *Fourier transform pair*. In general,  $X(f)$  is a complex function, and it is also referred to as voltage spectrum of the signal  $x(t)$ .

Comparing equations (2.1.8) and (2.1.18), we derive that

$$\delta(t - \tau) = \int_{-\infty}^{\infty} e^{j2\pi f(t-\tau)} df \quad (2.1.21a)$$

or in general

$$\delta(t) = \int_{-\infty}^{\infty} e^{j2\pi f t} df . \quad (2.1.21b)$$

Equation (2.1.21) expresses that the impulse signal  $x(t) = \delta(t)$  can be resulted from the inverse Fourier transformation of a spectrum  $X(f) = 1$  — i.e., the spectrum of the impulse signal  $\delta(t)$  is a constant equal to 1, as shown in Fig. 2.1.

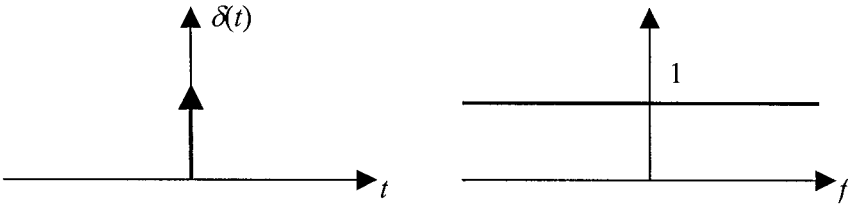


Figure 2.1. Impulse signal and its spectrum

In the following, we present some Fourier transformation properties, which are useful in the RF system design, and the other properties can be found in many textbooks [1–2].

- *Linearity.* The Fourier transform operation is linear. If  $x_1(t)$  and  $x_2(t)$  possesses Fourier transforms  $X_1(f)$  and  $X_2(f)$ , respectively, then for any two scalars  $\alpha$  and  $\beta$ , results in

$$F[\alpha x_1(t) + \beta x_2(t)] = \alpha X_1(f) + \beta X_2(f) . \quad (2.1.22)$$

- *Convolution.* If the time functions  $x(t)$  and  $h(t)$  both have Fourier transforms, then

$$F[h(t) * x(t)] = F[h(t)] \cdot F[x(t)] = H(f) \cdot X(f) . \quad (2.1.23)$$

The Fourier transform of the convolution of two time functions is the product of their Fourier transforms in the frequency domain.

- *Modulation.* The Fourier transform of a modulated carrier,  $x(t)e^{j2\pi f_o t}$ , is

$$F[x(t)e^{j2\pi f_o t}] = X(f - f_o) . \quad (2.1.24)$$

The Fourier transform of an amplitude modulated signal,  $x(t) \cos(2\pi f_o t)$ , has a form

$$\begin{aligned} F[x(t) \cos(2\pi f_o t)] &= \frac{1}{2} F[x(t)(e^{j2\pi f_o t} + e^{-j2\pi f_o t})] \\ &= \frac{1}{2} [X(f + f_o) + X(f - f_o)]. \end{aligned}$$

- *Time shifting.* A time shift of  $t_o$  from its origin results in a phase shift of  $-2\pi f t_o$  in the frequency domain — i.e.,

$$F[x(t - t_o)] = X(f)e^{-j2\pi f t_o} \quad (2.1.25)$$

- *Parseval's relation.* If  $X(f) = F[x(t)]$  and  $Y(f) = F[y(t)]$ , then

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df. \quad (2.1.26)$$

Note that if  $y(t) = x(t)$ , we have Parseval's identity for the Fourier transform

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df. \quad (2.1.27)$$

This expression is similar to (2.1.17) of the periodic signal. It explains that the energy content of the signal  $x(t)$  can be computed by integrating the energy spectral density  $|X(f)|^2$  of  $x(t)$  over all the frequency.

### 2.1.3. Frequency Response of LTI Systems

Comparing (2.1.11) with the Fourier transform (2.1.19),  $H(f)$  is the Fourier transform of the impulse response  $h(t)$  of a LTI system. The Fourier transform of the time response  $y(t)$  for a LTI system with an input  $x(t)$  is the product of  $X(f) = F[x(t)]$  and  $H(f)$  as given in (2.1.23) — i.e.,

$$Y(f) = F[h(t) * x(t)] = H(f)X(f). \quad (2.1.28)$$