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Statistics for SPATIO-TEMPORAL DATA

Noel Cressie • Christopher K. Wikle

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Statistics for Spatio-Temporal Data

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Statistics for Spatio-Temporal Data

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Preface

Nothing puzzles me more than time and space; and yet nothing troubles me less . . . .

These words, by the English essayist Charles Lamb in his 1810 letter to Thomas Manning, provide a concise summary of this book. Of course, he was not thinking of Statistics, nor Science, nor Statistical Science, rather the more ephemeral notion of space and time. But here, in the physical world, there are scientific questions to resolve and predictions to make, and understanding the effects of dependencies across time and space is a crucial part. Those dependencies are in there somewhere, and we too are puzzled by them.

Up until the mid-twentieth century, Statistics' response to this puzzle was often to ignore the complicated structure, or to find clever ways to remove it. Interestingly, Charles Lamb seemed to be in tune with this, as he finished his sentence by saying, "as I never think about them." As Statistics has progressed through the twentieth century and moved into the twenty-first, spatial and temporal statistical methodology has been incorporated more and more into the scientific models of our world and indeed of our universe. As technology has improved, Statistics has been given a Rosetta Stone to begin to unlock Science's mysteries, from the molecular to the global to the cosmological.

In the beginning, there were data. Then, there were theories, formed from data, and those theories rose and fell according their agreement with new data. Data are central, and they should be cared for accordingly. To do good Science, databases should be fully documented and algorithms ("black boxes") involved in creating them should be in the public domain.

However, data alone do not tell us all that much about our world. When we look at data, how do we know if what we are seeing is "signal" as opposed to "noise"? How do we compare two sources of data—what is the basis of a comparison?

Similarly, theories (or models) by themselves are not often the best descriptions of the real world. What can be said about processes on scales at which

there were no observations? What about uncertainties in parameters, or forcings, or interactions with other processes? Indeed, the key is the proper blending of such models and the data. Sometimes this might be done informally, for example, by taking a simulated field, simulated from a mathematical (say) model, and visually comparing it to a field of actual data. From the visual comparison, a deficiency in the model might be obvious, which might lead to a parameter adjustment, or even a new parameter. Then a *new* simulation might be implemented and a new visual comparison made, and so forth. This is one way to combine data and model, but it is not a very efficient way to deal with either. Indeed, the power of Statistical Science is that it provides several frameworks in which to combine data and model, in optimal ways, for the purpose of scientific inference. It might seem strange that there is not just one framework in which to carry out inferences, but even Statistics has its tribes. However, the element that is common to all is an attempt to partition variability and to quantify uncertainty.

In this book, we take the firm stand that the best paradigm (to date) in which to partition variability and quantify uncertainty is the *hierarchical statistical model.* Such a model explicitly acknowledges uncertainty in the data, different from that in the process and parameters, and then it accommodates the uncertainty in the process (and finally in parameters, if necessary). We have used color in key places in our exposition, to distinguish between the parts of the hierarchical model concerned with data (green), with processes (blue) and with parameters (purple).

Hierarchical thinking (i.e., hierarchical modeling) is intimately tied to conditional thinking (i.e., modeling with conditional probabilities). Indeed, it is our perspective that conditional thinking is the aforementioned Rosetta Stone: It allows us to separately partition the effects of measurement error and scales of variability below the resolution of our data, conditioned on the process at possibly some other scale. Similarly, conditional thinking allows us to model a spatio-temporal process as it actually *evolves* through time, as opposed to just accounting for its marginal dependencies. Equally important, it allows us to use spatio-temporal dependencies in errors and/or parameters as a proxy for unknown and unknowable processes. Its impact goes even deeper, in that it allows parameters themselves to be dependent on other processes or other sources of data. Finally, it makes clear that if some components of variability are not interesting for a particular question, then the end result should *not* look like the data. Any unwanted components (e.g., measurement error) are filtered out: What you see (data) is not always what you want to get (process).

Critically, in the presence of data, conditional thinking allows sensible trade-offs to be made between data availability, process complexity, and computational complexity. Combined with the ever-expanding computational tools of the twenty-first century, the hierarchical statistical framework provides the structure to tackle important questions in Energy, Climate, Environment, Food, Finance, and so on. This will be a century of massive (spatio-temporal) datasets

collected to answer Society's dominant questions. In this book, we are particularly interested in inference for Climate and the Environment, where processes at small spatio-temporal scales influence those at larger scales, and vice versa. The questions to be resolved are fundamental to sustaining our planet, they involve complex spatio-temporal phenomena, *and* they are inherently statistical.

Our lives are spent marching through a space–time continuum, but space is different from time. We can (and often do) visit the same place over and over but always at different, ordered time points. We can go north, south, east, and west, up and down, but only ever forward from the past to the present and into the future. Any study of a spatio-temporal phenomenon needs to respect this difference. In this book, we have proposed spatio-temporal statistical methodologies that align with the underlying science, and we have found that hierarchical thinking is a natural way to achieve this alignment.

In the pages that follow, we have deliberately tried to build a bridge between the twentieth and twenty-first centuries in our presentation of spatio-temporal statistics. There are strong and powerful traditions that have developed in the last few decades and, even if they are not hierarchical, they provide the tools and motivation that can be used in hierarchical thinking. In some cases, hierarchical approaches that may be appealing in principle may be out of our reach in terms of timely implementation.

The official ftp site associated with the book can be found at:

ftp://ftp.wiley.com/public/sci_tech_med/spatio_temporal_data

In addition to posting errata, we will post supplemental material that we hope will be helpful to the reader.

As mentioned above, we have questions to resolve and predictions to make, so let's get started....

NOEL CRESSIE
CHRISTOPHER K. WIKLE

Columbus, Ohio
Columbia, Missouri
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Acknowledgments

There are many people to thank, but two are missing. I wish my parents were alive to read this and know how important they are to what I do. I finally have a good answer to Ray's question, "How's the book going?"

My children, Amie and Sean, were only just present for my last book project, and now, as young adults, they are happily cheering on the completion of this one. Everyone has a book in them, and I hope they write theirs.

My dear friends and sibs may not understand all that is on these pages, but they know how important it was for me to "write it down." I would like them to know how important they are to me.

My co-author Chris has been a generous colleague and friend, and I admire enormously his intelligence and erudition. Together, we have been able to write a book that is so much more than what was in each of us; I have learned much from him. The order of authors is simply determined by the alphabet.

I have been taught by co-authors, colleagues, and students, who have shaped the material in this book. Their contributions are there in many, many ways. Some deserve special mention, which can be found in the joint acknowledgment.

The Department of Statistics at The Ohio State University has provided a nurturing environment for the whole lifetime of this book project. As part of my role as Director of the Department's Program in Spatial Statistics and Environmental Statistics, I have relied on my Program Assistant, Terry England, in many ways. This book has been a major project we have shared, and her efforts (which are described in the joint acknowledgment) are deeply appreciated and warmly acknowledged. Paul Brower, our Department Administrative Manager, has been a great supporter of this book project and, more generally, of my attempts to "save the planet." The office staff, the computer-support personnel, and the custodial staff are all warmly thanked for making my working environment in Cockins Hall a place where things get done . . . with good cheer.

Finally, it is not lost on me that this book will appear exactly 20 years after an earlier book I authored on spatial data. A lot can happen in two decades . . . .

N. C.

My rather unorthodox training in Statistics and Meteorology would not have been possible without the tremendous support of my advisors, Tsing-Chang (Mike) Chen and Noel Cressie, both of whom set a great example of having mastered the art of teaching and the more elusive skill of mentoring. Mike Chen opened my eyes to the beauty of dynamics through his courses in Dynamic Meteorology and Geophysical Fluid Dynamics (in particular) and his patient one-on-one expositions; he is a master of explaining the "real" meaning behind the mathematics. In addition to teaching these subjects, he taught me the *philosophy of science* and to appreciate the importance of knowing the history of your field. Noel has been an inspirational mentor, collaborator, and friend. In addition to opening my eyes to the beauty of spatial statistics, he has, more than anyone, taught me to think like a statistician, and to stand up for the things in which I believe scientifically. At the time I was choosing an advisor in Statistics, some students warned me that I did not want to work with Noel because he was too "hard." One of the best decisions I ever made professionally was to ignore that advice! If hard means having high expectations, and providing generous advice and support, than I guess he was. Such a mentor has turned into a good friend. Noel mentioned at the beginning of this project how it can be very trying on a friendship to write a book. To be sure, we've had some disagreements, but we have managed quite well and I would not trade this journey for any other professional endeavor. I have learned a tremendous amount from Noel in writing this book, and I believe our friendship is stronger today than it was when we started.

I also want to thank two wonderful mentors with whom I started working when I was a postdoc at the National Center for Atmospheric Research (NCAR), and with whom I have continued to work along the way: Ralph Milliff and Mark Berliner. Ralph is a friend, a mentor, and an inspiration. He is a true scientist and is one of the most generous individuals I have met professionally. On every project in which we work, Ralph reminds me by the example he sets of what it means to be a scientist with integrity. Ralph saw the potential of Bayesian hierarchical modeling (BHM) as a way to manage uncertainty in oceanography before anyone else and, more importantly, he has stuck with it through good times and bad. Although he probably wouldn't admit it, Ralph is indeed an expert in BHM. Mark Berliner probably deserves a special chapter in this book. In fact, the book owes its philosophy to the ideas about hierarchical modeling and science that Mark began promoting when he was the director of the Geophysical Statistics Project (GSP) at NCAR in the mid-1990s. My time in that group was the most stimulating intellectual environment of my career. Mark had a vision of how BHM could be used as a paradigm for Science. Indeed, Mark is a master of weaving Science into the fabric of Statistics; he is always an inspiration! Perhaps more importantly, Mark is a friend and mentor and is one of the truly generous people in Statistics. There is no doubt that I would not be the statistician I am today if it were not for Mark's mentoring and collaborations.

A special thanks to Andy Royle, who was also part of the GSP group. Andy is an amazing statistician, and our interactions and collaborations during my time at NCAR and immediately thereafter were very important in shaping the way in which I envisioned Mark's philosophy playing a role in spatio-temporal statistics. In fact, Andy and I talked about writing a lower-level book on spatio-temporal statistics while we were at NCAR, and we even drafted an outline and a couple of chapters. We both moved in different directions (Andy to ecological statistics in the Fish and Wildlife Service and then to the U.S. Geological Survey, and me to academia), which prevented us from pursuing that project. But, there is no doubt that there is much of Andy in this book.

My path to Statistics was certainly not direct. In fact, the first classes I had in the subject as an undergraduate did little to impress upon me its importance. However, along the way I have had the great fortune of having some fantastic teachers and collaborators. In particular, I would like to thank Peter Sherman and Rol Madden, for introducing me to the power of spectral analysis and its scientific interpretation; and Mark Kaiser, for introducing me to hierarchical modeling in the early 1990s through a wonderful experimental course at Iowa State University. Many other people have influenced me along the way, either through conversations or collaborations. In particular, knowing that I am sure to unintentionally leave someone off of this list, I would like to thank: Chris Anderson, Thomas Bengtsson, Jim Clark, Bob Dorazio, William Dunsmuir, Dave Higdon, Tim Hoar, Dave Larsen, Andy Moore, Doug Nychka, Nadia Pinardi, Yanyan Sheng, Jon Stroud, Joe Tribbia, and Jay Ver Hoef. Of course, there are many others who have influenced me with the quality of their research, their integrity, and through brief discussions at meetings.

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At Wiley, the acquisition, production (particularly Lisa Van Horn), and marketing teams have all played a crucial role in taking our idea to write a book on spatio-temporal statistics, all the way to a four-color volume in the Wiley Series in Probability and Statistics. This is a publisher that is committed to getting our ideas into the offices, labs, and libraries of scientists and engineers. We want to change the way they practice Statistics . . . particularly for spatio-temporal data.

N. C.
C. K. W.

CHAPTER 1

Space–Time: The Next Frontier

This book is about the statistical analysis of data . . . *spatio-temporal* data. By this we mean data to which labels have been added showing where and when they were collected. Good science protocol calls for data records to include place and time of collection. Causation is the "holy grail" of Science, and hence to infer cause–effect relationships (i.e., "why") it is essential to keep track of "when"; a cause always precedes an effect. Keeping track of "where" recognizes the importance of knowing the "lay of the land"; and, quite simply, there would be no History without Geography.

We believe that in order to answer the "why" question, Science should address the "where" and "when" questions. To do that, spatio-temporal datasets are needed. However, spatial datasets that do not have a temporal dimension can occur in many areas of Science, from Archeology to Zoology. The spatial data may be from a "snapshot" in time (e.g., liver-cancer rates in U.S. counties in 2009), or they may be taken from a process that is not evolving in time (e.g., an iron-ore body in the Pilbara region of Australia). Sometimes, the temporal component has simply been discarded, and the same may have happened to the spatial component as well. Also, temporal datasets that do not have a spatial dimension are not unusual, for analogous reasons. For example, two time series, one of monthly mean carbon dioxide measurements from the Mauna Loa Observatory, Hawaii, and the other of monthly surface temperatures averaged across the globe, do not have a spatial dimension (for different reasons).

Spatio-Temporal Data

Spatio-temporal data were essential to the nomadic tribes of early civilization, who used them to return to seasonal hunting grounds. On a grander scale, datasets on location, weather, geology, plants, animals, and indigenous people were collected by early explorers seeking to map new lands and enrich their kings and queens. The conquistadors of Mesoamerica certainly did this for Spain.

Statistics for Spatio-Temporal Data, by Noel Cressie and Christopher K. Wikle
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The indigenous people also made their own maps of the Spanish conquest, in the form of a *lienzo*. A lienzo represents a type of historical cartography, a painting on panels of cloth that uses stylized symbols to tell the *history* of a *geographical* region. The *Lienzo de Quauhquechollan* is made up of 15 joined pieces of cotton cloth and is a map that tells the story, from 1527 to 1530, of the Spanish conquest of the region now known as Guatemala. It has been restored digitally in a major project by Exploraciones sobre la Historia at the Universidad Francisco Marroquín (UFM) in Guatemala City (see Figure 1.1). This story of the Spanish conquest in Guatemala is an illustration of complex spatio-temporal interactions. Reading the lienzo and understanding its correspondence with the geography of the region required deciphering; see Asselbergs (2008) for a complete description. The original lienzo dates from about 1530 and represents a spatio-temporal dataset that is almost 500 years old!

In a sense, we are all analyzers of spatial and temporal data. As we plan our futures (economically, socially, academically, etc.), we must take into account the present and seek guidance from the past. As we look at a map to plan a trip, we are letting its spatial abstraction guide us to our destination. The philosopher Ludwig Wittgenstein compared language to a city that has evolved over time (Wittgenstein, 1958): "Our language can be seen as an ancient city: A maze of little streets and squares, of old and new houses, and of houses with additions from various periods; and this surrounded by a multitude of new burroughs with straight and regular streets and uniform houses!"

Graphs of data indexed by time (time series) and remote-sensing images made up of radiances indexed by pixel location (spatial data) show variability at a glance. For example, Figure 1.2 shows the Missouri River *gage-height* levels during the 10-year period, 1988–1997, at Hermann, MO. Figure 1.3 shows two remotely sensed images of the river taken in September 1992, before a major flood event, and in September 1993, after the highest crest ever recorded at Hermann (36.97 ft on July 31, 1993). The top panel of Figure 1.3 shows the town of Gasconade in the middle of the scene, situated in the "V" where the Gasconade River joins the Missouri River; Gasconade is at mile 104.4 and eight miles downstream is the river town of Hermann, visible at the very bottom of the scenes. Notice the intensive agriculture in the river's flood plain in September 1992. The bottom panel of Figure 1.3 shows the same region, one year later, after the severe flooding in the summer of 1993. The inundation of Gasconade, the floodplain, and the environs of Hermann is stunning. There is a multiscale process behind all of this that involves where, when, and how much precipitation occurred upstream, the morphology of the watershed, microphysical soil properties that determine run-off, the U.S. Army Corps of Engineers' construction of levees upstream, and so on. However, by looking only in the spatial dimension, or only in the temporal dimension, we miss the dynamical evolution of the flood event as it progressed downstream. Spatio-temporal data on this portion of the Missouri River, which shows how the river got from "before" to "after," would be best illustrated with a movie, showing a temporal sequence of spatial images before, during, and after the flood.

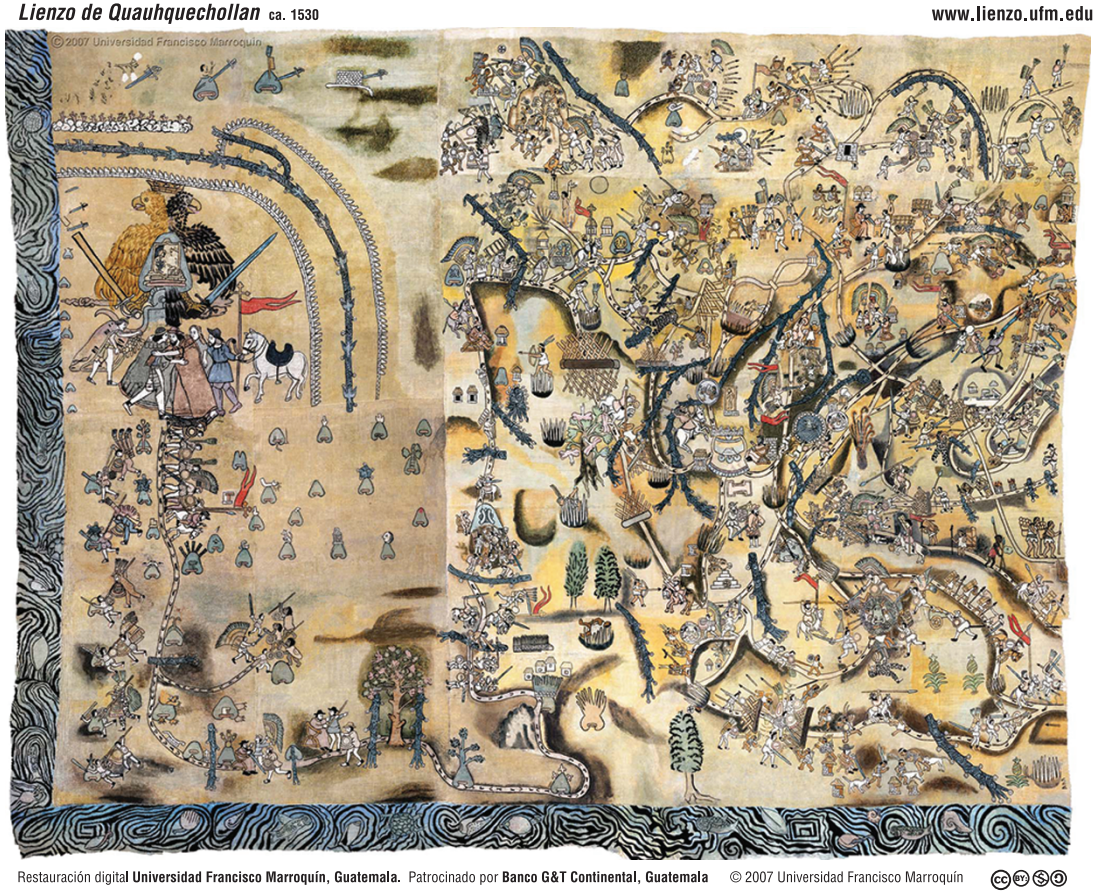

Figure 1.1 Digitally restored Lienzo de Quauhquechollan, whose actual dimensions are 2.45 m in height by 3.20 m in width. [Image is available under the Creative Commons license Attribution-Noncommercial-Share Alike © 2007 Universidad Francisco Marroquín.]

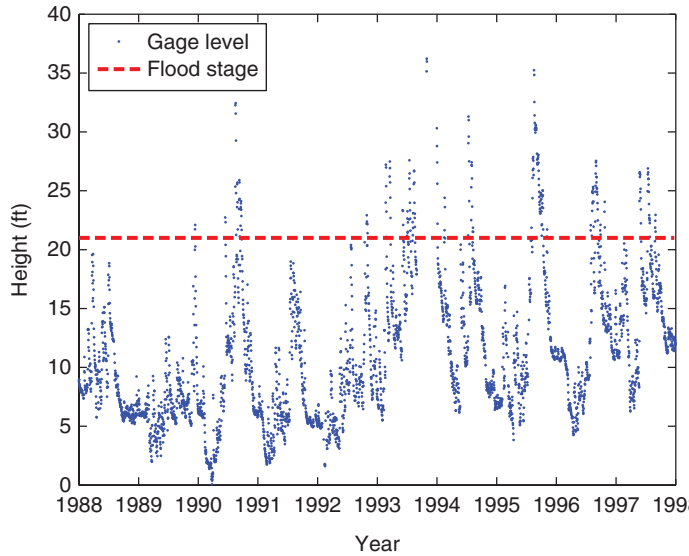

Figure 1.2 Time-series levels of gage height at Hermann, MO (mile 96.5 on the Missouri River) from January 1, 1988 through December 31, 1997. Flood stage is given by the horizontal dashed line. The highest recorded gage height in the 10-year period was 36.97 ft on July 31, 1993.

There is an important statistical characteristic of spatio-temporal data that is very common, namely that nearby (in space and time) observations tend to be more alike than those far apart. However, in the case of "competition," the opposite may happen (e.g., under big trees only small trees can grow), but the general conclusion is nevertheless that spatio-temporal data should *not* be modeled as being statistically independent. [Tobler (1970) called this notion "the first law of Geography."] Even if spatio-temporal trends are used to capture the dependence at large scales, there is typically a cascade of smaller spatio-temporal scales for which a statistical model is needed to capture the dependence. Consequently, an assumption that spatio-temporal data follow the "independent and identically distributed" (*iid*) statistical paradigm should typically be avoided. Paradigms that incorporate dependence are needed: The time series models in Chapter 3 and the spatial process models in Chapter 4 give those paradigms for *temporal data* and *spatial data*, respectively. From Chapter 5 onwards, we are concerned directly with Statistics for *spatio-temporal data*.

Uncertainty and the Role of Statistics

Uncertainty is everywhere; as Benjamin Franklin famously said (Sparks, 1840), "In this world nothing can be said to be certain, except death and taxes." Not only is our world uncertain, our attempts to explain the world (i.e., Science) are uncertain. And our measurements of our (uncertain) world are uncertain. Statistics is the "Science of Uncertainty," and it offers a coherent approach to

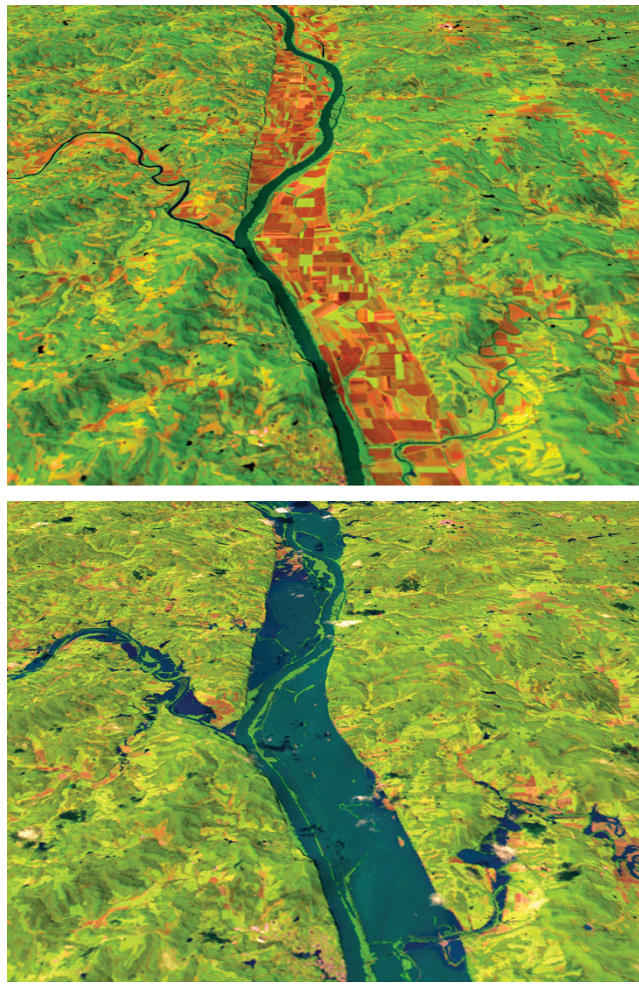

Figure 1.3 Images from NASA's Landsat Thematic Mapper. Each image shows a segment of the Missouri River near Hermann, MO (mile 96.5, at the bottom of the scene), and Gasconade, MO (mile 104.4, in the "V" in the middle of the scene). The river flows from west (top of the scene) to east (bottom of the scene). **Top panel:** September 1992, before a major flood event. **Bottom panel:** September 1993, after a record-breaking flood event in July 1993.

handling the sources of uncertainty referred to above. Indeed, in our work we use the term *Statistical Science* interchangeably with *Statistics* (with a "capital" S); we use *statistics* (with a "small" s) to refer to summaries of the data.

In most of this book, we shall express uncertainty through variability, but we note that other measures (e.g., entropy) could also be used. Just as the physical and biological sciences have the notions of mass balance and energy balance, Statistical Science has a notion of variability balance. The total variability is

modeled with variability due to *measurement*, variability due to using a (more-or-less uncertain) *model* of how the world works, and variability due to uncertainty on *parameters* that control the measurement and model variabilities.

Although real-world systems may in principle be partially deterministic, our information is incomplete at each of the stages of observation, summarization, and inference, and thus our understanding is clouded by uncertainty. Consequently, by the time the inference stage is reached, the lack of certainty will influence how much knowledge we can gain from the data. Furthermore, if the dynamics of the system are nonlinear, the processes can exhibit *chaos* (Section 3.2.4), even though the theory is based on *deterministic* dynamical systems. (In Chapters 3 and 7, we show how model uncertainty in these systems naturally leads to *stochastic* dynamical systems that incorporate *system*, or *intrinsic,* noise.)

Data can hold so much potential, but they are an entropic collection of digits or bits unless they can be organized into a database. With the ability in a database to structure, search, filter, query, visualize, and summarize, the data begin to contain *information*. Some of this information comes from judicious use of statistics (i.e., summaries) with a "small s." Then, in going from information to *knowledge*, Science (and, with it, Statistics with a "capital S") takes over. This book makes contributions at all levels of the data–information–knowledge pyramid, but we generally stop short of the summit where knowledge is used to determine policy. The methodology we develop is poised to do so, and we believe that at the interface between Science, Statistics, and Policy there is an enormous need for (spatio-temporal) decision-making in the presence of uncertainty.

In this book, we approach the problem of "scientific understanding in the presence of uncertainty" from a probabilistic viewpoint, which allows us to build useful spatio-temporal statistical models and make scientific inferences for various spatial and temporal scales. Accounting for the uncertainty enables us to look for possible associations within and between variables in the system, with the potential for finding mechanisms that extend, modify, or even disprove a scientific theory.

Uncertainty and Data

Central to the observation, summarization, and inference (including prediction) of spatio-temporal processes are *data*. All data come bundled with error. In particular, along with the obvious errors associated with measuring, manipulating, and archiving, there are other errors, such as discrete spatial and temporal sampling of an inherently continuous system. Consequently, there are always scales of variability that are unresolvable and that will further "contaminate" the observations. For example, in Atmospheric Science, this is considered a form of "turbulence," and it corresponds to the well known aliasing problem in time series analysis (e.g., see Section 3.5.1; Chatfield, 1989, p. 126) and the microscale component of the "nugget effect" in geostatistics [e.g., see the introductory remarks to Chapter 4 and Cressie (1993, p. 59)].