Studies in Universal Logic

Jean-Yves Béziau
Dale Jacquette
Editors

Around and Beyond the Square of Opposition

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Studies in Universal Logic

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Around and Beyond the Square of Opposition

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Preface

The theory of inferences and oppositions among categorical propositions, based on Aristotelian term logic, is pictured in a striking square diagram. The graphic representation of contradictories, contraries, subcontraries and subalterns intended as a foundation for syllogistic logic can be understood and applied in many different ways with interesting implications for various disciplines, notably including epistemology, linguistics, mathematics, psychology. The square can also be generalized in other two-dimensional and multi-dimensional graphic depictions of logical and other relations, extending in breath and depth the original Aristotelian theory. The square of opposition is accordingly a very attractive theme which has persisted down through the centuries with no signs of disappearing or even diminishing in fascination. For the last 10 years, there has been a new growing interest for the square due to new discoveries and challenging interpretations. This book presents a collection of previously unpublished papers by well-regarded specialists on the theory and interpretation of the concept and application of the square of opposition from all over the world. We thank all the authors who have contributed a paper to this book, and the referees who have analyzed, commented on, and made invaluable recommendations for improving the essays.

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Part I
Historical and Critical Aspects of the Square

The New Rising of the Square of Opposition

Jean-Yves Béziau

Abstract In this paper I relate the story about the new rising of the square of opposition: how I got in touch with it and started to develop new ideas and to organize world congresses on the topic with subsequent publications. My first contact with the square was in connection with Slater's criticisms of paraconsistent logic. Then by looking for an intuitive basis for paraconsistent negation, I was led to reconstruct S5 as a paraconsistent logic considering $\neg\Box$ as a paraconsistent negation. Making the connection between $\neg\Box$ and the O-corner of the square of opposition, I developed a paraconsistent star and hexagon of opposition and then a polyhedron of opposition, as a general framework to understand relations between modalities en negations. I also proposed the generalization of the theory of oppositions to polytomy. After having developed all this work I have begun to promote interdisciplinary world events on the square of opposition.

Keywords Square of opposition · Paraconsistent logic · Negation · Modal logic

Mathematics Subject Classification Primary 03B53 · Secondary 03B45 · 03B20 · 03B22

One can never tell when a old, discarded theory will suddenly burst into renewed life.

Brendan Larvor [35]

1 The Square Adventure

In June 2007 I organized the first world congress on the square of opposition in Montreux, Switzerland. In June 2010 was organized a second edition of this event in Corsica and presently a third edition is on the way for June 2012 in Beirut, Lebanon. After the first event was published a special issue of the journal *Logica Universalis* dedicated to the square of about 200 pages with 13 papers [15] and the book *The Square of Opposition—A General Framework for Cognition* of about 500 pages with 18 papers [19]. Most of the papers in the present book *Around and Beyond the Square of Opposition* are related to the talks presented at the second congress in Corsica and a double special issue of *Logica Universalis* [18] of about 250 pages with 10 papers has also been released following this event. Thus about 1500 pages have been recently published based on the square of opposition, a very productive tool.

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The aim of this paper is to tell how all this square animation did arise. In this paper I will speak about my first encounter with this creature and how our relation has developed changing the shape of both of us. There are mainly four stages in this square adventure: Los Angeles, CA, 1995; Fortaleza, Brazil, 1997; Stanford, CA, 2001; and Neuchâtel, Switzerland, 2003.

2 The Slater Affair (Los Angeles 1995)

In 1995 I was spending some time in Los Angeles, CA, as a Fulbright scholar at the Department of Mathematics of UCLA (invited by Herb Enderton). I was 30 years old and had never yet met the square face to face. Like any logician I had heard about it, but didn't know exactly what it was. For me it was connected with Aristotelian logic and syllogistic, something out-of-date, of historical interest, if any, similar to astrology.

I was then asked to write a *Mathematical Review* of a paper by Hartley Slater entitled "Paraconsistent logics?" [49] challenging the very existence of paraconsistent logics, a bunch of organisms I had been working on since a couple of years. I was quite interested to know up to which point I had been wasting time studying non-existent beings.

The weapon used by Slater to annihilate the paraconsistent ghosts was related to the square of opposition, a good reason to study this magic stick. In his paper the notions of contradictories and subcontraries, essential features of the square, are used to discuss the question whether a paraconsistent negation is a negation or not. Slater recalls that Priest and Routley did argue at some point that da Costa's negation is not a negation because it is a subcontrary functor but not a contradictory functor: in the paraconsistent logic C1 of da Costa, p and $\sim p$ cannot be false together, but sometimes can be true together, this is exactly the definition of subcontrariety.

An essential feature of a paraconsistent negation $\sim$ is to allow the rejection of the *ex-contradictio sequitur quod libet*: $p, \sim p \not\models q$. If we interpret this semantically, this means that there is a model (valuation, world) in which both p and $\sim p$ are true and in which q is false, for one p and for one q. This means therefore that $\sim$ is not a contradictory forming functor, but a subcontrary forming functor. So what applies to da Costa's paraconsistent negation seems to apply to any paraconsistent negation. That is on this basis that Slater argues that there is no paraconsistent negation at all, using the very idea of Priest and Routley that a negation should be a contradictory forming functor.

How can then the two pseudo-Australians argue that their paraconsistent negations are negations but not the Brazilian ones? That sounds quite impossible, unless there is a play on words, that's what Slater emphasizes. Priest has developed a paraconsistent logic LP with three-values, the difference with Łukasiewicz's logic is that the third value is considered as designated, that is the reason why in LP, we have $p, \sim p \not\models q$, since p and $\sim p$ can both have this third value. Now how can Priest argue that the paraconsistent negation of LP is a contradictory forming functor? It is because he is not calling the third value "true", despite the fact that it is designated. In my paper "Paraconsistent logic!" I comment the situation as follows:

> Priest's conjuring trick is the following: on the one hand he takes truth to be only 1 in order to say that his negation is a contradictory forming relation, and on the other hand he takes truth to be $\frac{1}{2}$ and 1 to define LP as a paraconsistent logic. However it is reasonable to demand to someone to keep his notion of truth constant, whatever it is. ([13], p. 21)

The subtitle of the quoted paper is "A reply to Slater". In this paper I argue that a contradictory forming functor is necessarily a classical negation, with a proof of a general theorem sustaining this fact. This entails that: (1) a paraconsistent negation cannot be a contradictory functor but not that: (2) a paraconsistent negation is not a negation—unless we admit that a negation has to be a classical negation. For me Slater is wrong arguing that:

> If we called what is now 'red', 'blue', and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn't change, only the mode of expression of them. Likewise if we called 'subcontraries', 'contradictories', would that show that 'it's not red' and 'it's not blue' were contradictories? Surely the same points hold. And that point shows that there is no 'paraconsistent' logic. ([49], p. 451)

A paraconsistency negation like the one of C1 is not definable in classical logic, so in this logic there is a new operator, it is not just a question of shifting names:

> It is important to note, against Slater, that the paraconsistent negations in the logics of da Costa and Priest (...) cannot be defined in classical logic. Thus paraconsistent logic is not a result of a verbal confusion similar to the one according to which we will exchange "point" for "line" in geometry, but rather the shift of meaning of "negation" in paraconsistent logic is comparable to the shift of meaning of "line" in non-Euclidean geometry. ([4], this idea was further developed in [8])

Nevertheless one should have good reasons to sustain that such new operators are indeed negations, one cannot define a paraconsistent negation only negatively by the rejection of the *ex-contradictio*. This is what I have explained at length in my papers "What is paraconsistent logic?" [5] showing also that it was not easy to find a bunch of positive criteria for positively define a paraconsistent negation. The square can help to picture a positive idea of paraconsistent negation: the very notion of subcontrariety can be used to support the existence of paraconsistent negations rather than their non-existence. I was led to this conclusion not by a mystical contemplation of the square, nor by a philological study of this legendary figure, but indirectly by trying to find a meaningful semantics for paraconsistent logics.

3 S5 Is a Paraconsistent Logic (Fortaleza/Rio de Janeiro 1997)

Paraconsistent logic was my point of departure for exploring the logic world. I did a Master thesis in mathematical logic on da Costa's paraconsistent logic C1 at the university of Paris 7 under the supervision of Daniel Andler in 1990 (see [3]). As I have explained elsewhere [14] my interest for paraconsistent logic was based on questions regarding the foundations of logic rather than by a childish attraction to paradoxes. If the principle of (non)contradiction is not a fundamental principle, what are the fundamental principles of logic, if any? How is it possible to reason without the principle of contradiction? Those were the questions I was exploring.

Investigating these questions I did study in details various paraconsistent logics and I noticed that generally they were *ad hoc* artificial constructions or/and had some problematic features. I was wondering if there was any intuitive idea corresponding to the notion of paraconsistent negation nicely supported by, or expressed through, a mathematical framework. In August 1997 I was taking part to the 4th WOLLIC (Workshop on Logic, Language, Information and Computation) in Fortaleza, Brazil and I was discussing the question of intuitive basis for paraconsistent negation with my colleague Arthur de Vallauris Buchsbaum. Influenced by *The Kybalion* [34], he was telling me that something can be considered true from a certain point of view and false from another viewpoint.

I was sympathetic to this idea having worked on the philosophy of quantum mechanics during my *Maîtrise de Philosophie* at the Sorbonne under the supervision of Bernard d'Espagnat (see [2]). I had been in particular studying the work of Heisenberg, Bohr and Bohm. From a certain viewpoint an object o is a particle, from another viewpoint it is a wave. Since a particle is something that cannot be a wave, this means that from a viewpoint it is true that o is a particle and from another viewpoint it is true that o is not a particle. Using classing negation this means that from this other viewpoint it is false that o is a particle. Bohr developed *complementarity* to explain this paradoxical situation, saying that there is no contradiction here since these two viewpoints are distinct visions that are not conflicting since the two contradictory features do not manifest in the same experiment. But what is then the reality beyond these two viewpoints? The Copenhagen interpretation eludes the problem, rejecting the idea of an objective reality beyond experiments. This can lead to a relativism not in the sense of Einstein, but in the sense of post-modernism, as caricatured by Sokal in his hoax quoting in fact Bohr: "A complete elucidation of one and the same object may require diverse points of view which defy a unique description" and Heisenberg: "As a final consequence, the natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them. Nor is it any longer possible to ask whether or not these particles exist in space and time objectively" ([52], p. 218).

David Bohm, like d'Espagnat, had a different philosophical idea, more Kantian, in the sense that for him these contradictory viewpoints are the expression of the limitations of our thought to capture the unthinkable noumenal reality. Bohm uses metaphors to explain paradoxes of quantum physics, for example he explains inseparability by the metaphor of an aquarium in a room with a fish filmed by two perpendicular cameras. In another room someone may see the two movies on two different screens trying to understand the strange interaction between the two fishes without knowing that it is in fact the same fish (see [24]).

Discussing with Arthur, I was disagreeing with him saying that everything can be considered as true form a certain viewpoint and false from another viewpoint. This seems to me too easy, too trivial. I had then the following idea: the negation of p is false if and only if p is considered true from all viewpoints. If p can be considered as true from a viewpoint and false from another viewpoint, then $\sim p$ can be considered also as true. What does this mean exactly, true from which viewpoint? This can be clarified using a Kripkean semantics. Viewpoints are considered as worlds and we say:

$$\sim p \text{ is false in a world } w \text{ iff } p \text{ is true in all worlds accessible to } w.$$

To simplify I decided to consider a universal accessibility relation, then we have:

$$\sim p \text{ is false in a world } w \text{ iff } p \text{ is true in all worlds.}$$

If p is not true in all worlds, this trivially means that there is a world w in which p is false, but this also means that $\sim p$ is true in all the worlds and we may have a world w in which p is true, so, in this world, p and $\sim p$ are both true.

Back to Rio de Janeiro where I was living I started to work the details of this semantics. I decided to call Z the logic with such a paraconsistent negation by reference to LeibniZ, because it was connected to the semantics of possible worlds. I developed this paper in view of the projected conference in July 1998 in Torun commemorating the 50 years of Jaśkowski's paper on paraconsistent logic [32]. Jaśkowski's logic is called *discussive*

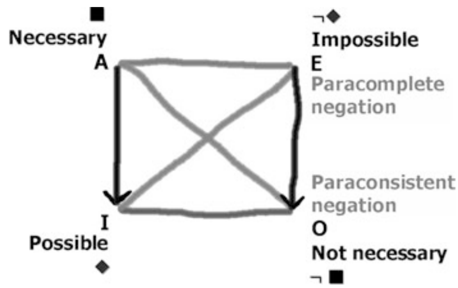

Fig. 1 Paraconsistent negation as a modality

logic because the motivation was to formalize the logic of people discussing together. Different people may have different ideas, according to someone the proposition p may be true, according to someone else the proposition p may be false. The worlds of the logic Z can be considered as different opinions people may have. According to Z if everybody agrees that the proposition p is true, then everybody has to agree that the proposition $\sim p$ is false, but if someone thinks that the proposition p is false, then the proposition $\sim p$ is true for everybody and someone may also think that the proposition p is true, then for him p and $\sim p$ are both true. So my paper was called "The paraconsistent logic Z—a possible solution to Jaśkowski's problem" and presented at the Jaśkowski's commemorative conference. For a reason I ignored the paper was not subsequently published in the double issue of *Logic and Logical Philosophy* related with the conference, but much later in a issue of this same journal [12] and is having a certain success (cf. [38, 43–45]).

The main result of this paper is to axiomatize the logic Z which has, as primary connectives, classical conjunction, disjunction and implication and the negation $\sim p$ defined according to the above possible worlds semantical condition. In this paper I give a Hilbert's style axiomatization for Z, proving the completeness theorem generalizing the method of completeness for modal logic. I also show that it is possible to translate Z into S5, by interpreting $\sim p$ as $\neg\Box p$. This is easily shown by checking that:

$$\neg\Box p \text{ is false in a world } w \text{ iff } p \text{ is true in all worlds.}$$

I wrote this paper in December 1997 and then discussed these questions with Claudio Pizzi, who was visiting Rio at this time. The results of this discussion are described as follows in my paper "Adventures in the paraconsistent jungle":

> I discussed my discovery with the Italian logician Claudio Pizzi, who has a little castle in Copacabana and uses to come there frequently. Pizzi told me two important things: one right and one wrong. The wrong was that the operator $\neg\Box$ corresponds to contingency, the right was that S5 and Z are equivalent since it is possible in S5 to define classical negation and necessity with $\neg\Box$, conjunction and implication. ([14], p. 71)

This discussion will lead to my paper "S5 is a paraconsistent logic and so is first-order logic" [9], where I show how possible worlds semantics can be used to develop paraconsistent logics. Despite the fact that Z is the same logic as S5, it is a quite interesting construction, since it gives a very different picture of the situation: we have a logic with conjunction, disjunction, implication and a paraconsistent negation as the only primitive connectives. And after discussing with Pizzi I was on the way to develop a connection between the modal square and an intuitive interpretation of paraconsistent negation, with the picture shown in Fig. 1. The modality $\neg\Diamond$, i.e. impossible, is considered as a paracomplete negation, whose intuitionistic case is a particular case. This is connected with

Gödel's result: it is possible to translate intuitionistic logic into the modal logic S4 [26]. I presented a talk on this subject in March 1998 at the PUC-Rio before presenting it in Poland in July 1998 at the Jaśkowski's event. The following year I was again in Poland taking part to the 11th International Congress of Logic, Methodology and Philosophy of Science (Kraków, August 20–26, 1999), where I had the opportunity to meet Slater in flesh and blood, being convinced of his existence, but this didn't convince me of the non-existence of paraconsistent negations.

4 The O-Corner (Stanford 2001)

In 2000 and 2001 I was a visiting scholar at the CSLI/EPGY at Stanford University invited by Patrick Suppes. During this time I had the opportunity to discuss the question of $\neg\Box$ and the square with various people at Stanford and also circulating around the world.

At Stanford University I discussed the question of $\neg\Box$ with Johan van Benthem during Spring 2001, who oriented me, as Pizzi did it before, to contingency in particular the work of Lloyd Humberstone from Melbourne. So at the beginning of May 2001 I wrote to Humberstone. Discussing with him I was led to completely clarify the relation between $\neg\Box$ and contingency. Here is the e-mail exchange I had with Humberstone (May 8, 2001):

> JYB: I have been interested recently on the logic of contingency and Johan van Benthem told me that perhaps you can give me some information about the subject. I was led to the study of the operator of contingency defined standardly as $\neg\Box$, because it has some interesting properties from the point of view of paraconsistent logic. It is in fact dual of the intuitionistic negation considered as $\Box\neg$.
> LH: Now in fact as far as I know, nobody has ever proposed that contingency is definable as $\neg\Box$, since $\neg\Box$ is consistent with (indeed follows from) being impossible $\Box\neg$ and so is not a notion of contingency. The standard definition is rather: $\neg\Box \wedge \neg\Box\neg$ or alternatively $\Diamond \wedge \Diamond\neg$.

So $\neg\Box$ was not contingency. What it was, what was a good name for it, other than the pure negative name "not necessary"? This question would be clarified a couple of months later when still at Stanford I attended a talk by Seuren. Anyway at this time I just had learned more about contingency, had a look at Humberstone's work, who has also been working on non-contingency [29]—the contingent theme being a long time Australian topic, Routley was one of the first to work on the subject [48]. At this time I remembered the work of Blanché about the hexagon [20] that I read several years ago before. I was thus facing the picture shown in Fig. 2. This picture replaces the wrong picture that can be found in many different papers or books, and that I had in mind (see Fig. 3). The mistake has been cleared up in the 1950s by Robert Blanché reconstructing the square within two triangles forming an hexagon (see [20–22]). Jean-Louis Gardies, a follower of Blanché, explains that during the history of logic since Aristotle there was a confusion about modalities, people having in mind rather a triangle but trying to express the relation between the modalities with a square similar to the square of quantifiers (see [25]). Blanché was nevertheless able to keep the parallel between quantifiers and modalities by elaborating a hexagon of modalities which also clarifies some confusions about the theory of quantifiers (making the difference between "at least one" and "some but not all"). The triangle of modalities is shown in Fig. 4. According to this triangle, contingent is the same as possible and means not necessary, where "not" is interpreted as a contrary negation: "it is necessary that god exists" and "it is contingent that god exists" cannot be true together, but they can be false together, in this case "it is impossible that god exists" is true.

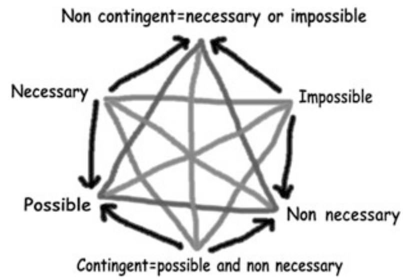

Fig. 2 Hexagon of modalities

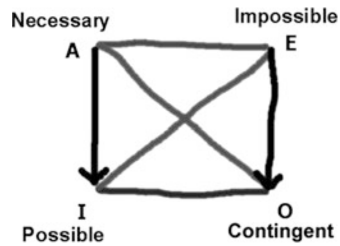

Fig. 3 Wrong square of modalities

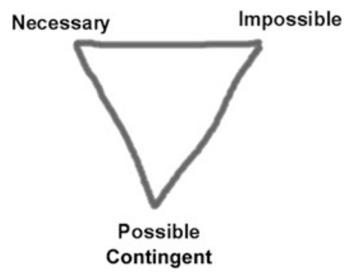

Fig. 4 Triangle of contrary modalities

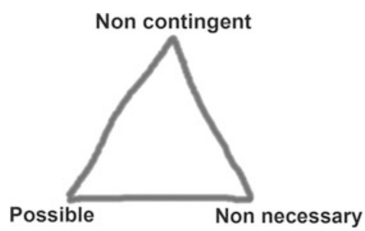

Fig. 5 Triangle of subcontrary modalities

But "not necessary" as ¬□ appears as one of the vertices of the dual triangle, the triangle of subcontrariety (Fig. 5), where possible here has the standard meaning given by the modal square, not excluding necessity. My idea was to go on studying ¬□, a primitive and fundamental connective of the paraconsistent logic Z. The Australians, like Routley and Humberstone, have been working on modal logic based on contingency as primitive, but have they been working with ¬□ as primitive? Humberstone wrote me: "I don't know of any work specifically on taking ¬□ as a primitive" (e-mail 18.05.2001). Bob Meyer that I met at the Annual Meeting of the Society for Exact Philosophy (Montreal, May 11–14, 2001) also told me that he had never heard about this or/and the fact that ¬□ could be considered as a paraconsistent negation.

June 25–28, 2001, I organized, jointly with Darko Sarenac a workshop on paraconsistent logic in Las Vegas, part of the International Conference on Artificial Intelligence (IC-AI'2001). I wrote for this event the paper "The logic of confusion" and presented it in Las Vegas. This paper is a generalization of the techniques used in my Z paper and I decided to talk about viewpoints rather than possible worlds, here is a summarize of it:

> The logic of confusion is a way to handle together incompatible "viewpoints". These viewpoints can be information data, physical experiments, sets of opinions or believes. Logics of confusion are obtained by generalizing Jaśkowski-type semantics and combining it with many-valued semantics. In this paper we will present a general way to handle together incompatible viewpoints which promotes objectivism via paraconsistent logic. This general framework will be called logic of confusion. The reason for the name is that we want to put (*fusion*) together (*con*) different viewpoints. ([7], p. 821)

I then went to the Annual Meeting of the Australasian Association of Philosophy in Tasmania (Hobart, July 1–6, 2001) and to Melbourne for a one day workshop (July 12, 2001) at the philosophy department with Graham Priest, Ross Brady, Otávio Bueno, and where I met Humberstone. I gave in Melbourne a talk about modality and paraconsistency based on the square which I presented few days later in Perth at the University of Western Australia where I was visiting Slater.

Back to Stanford via South-Africa and Brazil presenting talks about this theme, I attended in the Fall 2001 an interesting lecture by Pieter Seuren from Max Planck Institute for Psycholinguistics (Language and Cognition Group), Nijmegen, The Netherlands. It was a talk entitled "Aristotelian predicate calculus restored" presented at CSLI CogLunch on 25 October 2001.

This was the first time I heard about the non-lexicalization of the O-corner. "O-corner" is the traditional name for the corner where $\neg\Box$ is positioned in the square of modality. As A, E, I, this is an artificial name, all being mnemotechnic abbreviations of some Latin expressions describing the corners of the square of quantification. Larry Horn, author of the famous book *A Natural History of Negation* [28], has shown that there was no known languages in the world where there is a primitive word for the O-corner of the quantificational square. Horn's analysis is summarized as follows by Hoeksema in a paper dedicated to JFAK for his 50th birthday:

> Striking is the observation in [Horn 1989, p. 259] that natural languages systematically refuse to lexicalize the O-quantifier, here identified with "not all". There are no known cases of natural languages with determiners like "nall"; meaning "not all". Even in cases that look very promising (like Old English, which has an item nalles, derived from alles, "all"; by adding the negative prefix ne- the same that is used in words like never, naught, nor, neither), we end up empty-handed. Nalles does not actually mean "not all" or "not everything", but "not at all" [Horn 1989, p. 261]. Jespersen [1917] suggested that natural language quantifiers form a Triangle, rather than a Square. ([27], p. 2)

Based on the non-lexicalization of the O-corner, Seuren was arguing, following the Danish linguist Jespersen, that the O-corner didn't make sense, that the square was wrong. I was no really convinced by such linguistical argument telling him that the equation

$$\text{exists} = \text{has a natural name}$$

seems wrong to me, and that a concept may exist even if he has no name in natural language, like many mathematical concepts, for example the number 0 (funny because of the similarity with the "O" of the O-corner which can be seen as a zero corner). Mathematics is an intelligent way to develop new concepts. The square is a simple mathematical construction and the O-corner makes sense within this mathematical construction. But of course it is always interesting to make the bridge between mathematics and reality, to turn mathematics reality. For me the fact that paraconsistent negation was exactly at the location of the O-corner in the modal square was interesting: these two attacked concepts could mutually be strengthened by sitting together.

At this time I was also working on a four-valued modal logic influenced by Łukasiewicz who in his last paper [36] presents a four-valued system of modal logic starting with the modal square of opposition. Łukasiewicz's logic has several defects, I started to rethink the problem with the same starting point: the square of opposition. I present this work at the Annual Meeting of the Australasian Logic Society, November 28–30, 2001, in Wellington, New Zealand. I have developed this work along the years and it was finally published in 2011 [16]. Before that I wrote another paper entitled "Paraconsistent logic from a modal viewpoint" [11] mixing the idea of this four modal logic and the square vision of paraconsistent logic, contents of which was presented at a paraconsistent workshop organized by João Marcos at the 14th European Summer School in Logic, Language and Information (ESSLLI), August 5–16, 2002 in Trento, Italy (João Marcos has been extending my work in different directions, see [39] and [40]).

5 Polyhedron and Polytomy of Oppositions (Neuchâchel 2003)

I settled down in August 2002 in my country of origin, Switzerland, where I would be a SNF (Swiss National Science Foundation) Professor for six years at the University of Neuchâchel. This is during this period that I fully expanded the square activities:

- raising the ideas of polyhedron of oppositions and polytomy of oppositions (2002–2003),
- directing Alessio Moretti's PhD thesis "The geometry of logical opposition" (2004–2008),
- organizing the first world congress on the square of opposition in Montreux in June 2007.

Arriving in Neuchâchel, I finished to elaborate the ideas that will lead to the paper "New light on the square of oppositions and its nameless corner" [10], published in *Logical Investigations* in 2003, the logic journal of the Russian Academy of Sciences where had been previously published in 2002 my paper on S5 as a paraconsistent logic.

In this paper I systematically developed the parallel between the three oppositions of the square (contradiction, contrariety, subcontrariety) and three kinds of negations (classical, paracomplete and paraconsistent) and I argued that subalternation is not really an opposition (since the square is about three oppositions, I decided to use the expression "square of oppositions", rather than the standard "square of opposition").

For this reason the square diagram seemed to me quite artificial and that was one more reason for me to emphasize the hexagon. I started to use colors for a better visual representation, deciding to have contradiction in red, contrariety in blue, subcontrariety in green, and choosing black for subalternation (see Fig. 6).

Considering subalternation as secondary I was most of the time rather picturing the Star of David (Fig. 7).

In the Fall 2002 I gave a series of lectures in Italy presenting a talk entitled "The square of oppositions, modal logic and paraconsistent logic":

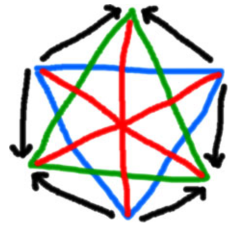

Fig. 6 Colored hexagon of oppositions

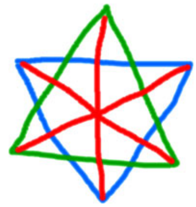

Fig. 7 Star of David of oppositions

Fig. 8 Octagon of oppositions

Fig. 9 Paraconsistent star of oppositions

Fig. 10 Paracomplete star of oppositions

- on November 12, 2002 in Cagliari, invited by Francesco Paoli,
- on November 15, 2002 in Napoli, invited by Nicola Grana,
- on November 20, 2002 in Siena, invited by Claudio Pizzi, with whom I had discussed these questions in Rio in 1999.

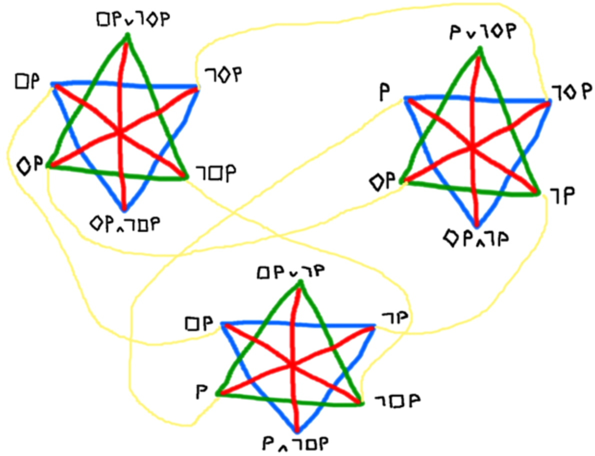

Fig. 11 Linking the stars

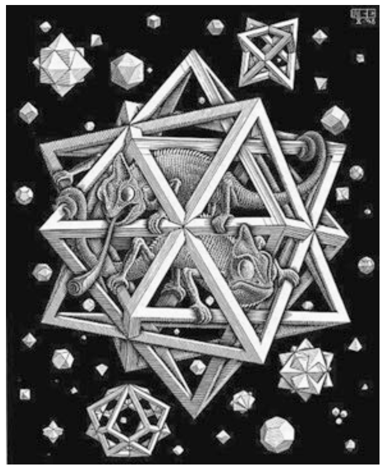

Fig. 12 Stellar dodecahedron of oppositions

Back to Switzerland, during the winter 2002–2003, I started to conceive a polyhedron of oppositions. I was led to this polyhedron by trying to systematically picture the relations between the 4 modalities and p and $\neg p$. It is nice to say that the three notions of opposition of the square correspond to three notions of negation, but in the modal square there is not the proposition p itself that can be confronted to these three negations. The simplest idea is to picture the octagon (Fig. 8), where $\neg p$ also appears. But I was dissatisfied with this picture mainly for esthetic reasons.

I thought that stars were more beautiful and had the idea to draw the two stars shown in Figs. 9 and 10. I saw then that a way to put together the three stars according to the links—as shown in Fig. 11— was to construct a polyhedron. My intuitive idea was that this polyhedron was the stellar dodecahedron shown in Fig. 12.

I was still in touch with Humberstone and at this time he sent me a preliminary draft of [30] where two other hexagons corresponding to these two stars are presented. The difference with my work is that no paraconsistent and/or paracomplete interpretations are provided and one cannot find here the idea to put the three hexagons together building a three-dimensional object (influenced by my work Humberstone wrote later on [31]).

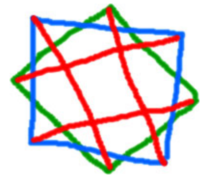

Fig. 13 Double square of oppositions

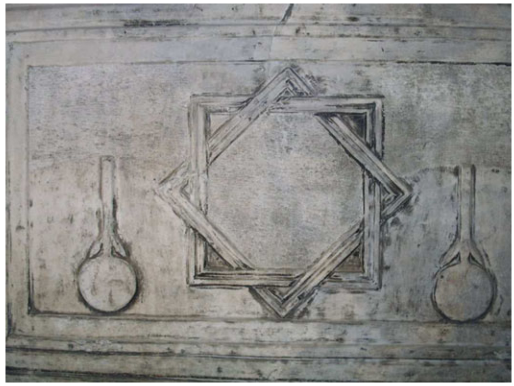

Fig. 14 Double square in Hagia Sofia

Not a specialist of polyhedra, I was looking on the web and I found a webpage on this topic produced by Hans Smessaert, a linguist from the Catholic University of Leuven, Belgium. I e-mailed him, explaining my discovery, he kindly replied to me, describing a fourth hexagon. At the same time I discussed this question with my friend Alessio Moretti who independently arrived at the same conclusion. And both Hans and Alessio were led to consider a tetradecahedron rather than a stellar dodecahedron in particular because they were figuring subalternations. An improvement of this polyhedron was constructed later on by Régis Pellissier (see details of the story in [42] and see the references of the work of Smessaert, Pellissier and Luzeaux in the bibliography [37, 46, 47, 50, 51]).

I had met Alessio at the ESSLLI 2000 in Birmingham where he was attending my tutorial on paraconsistent logic and we met again at the ESSLLI in Trento in 2002. He was quite interested by the square and I proposed him to supervise a PhD on this theme at the University of Neuchâchel. The title of his PhD is: "The geometry of logical opposition" (see [42]), but he generally prefers to use the expression "Theory of n-opposition" (see e.g. [41]) to name the work he is developing around the square (an ambiguous terminology, as pointed out by Alexandre Costa-Leite, since this theory is not about multiplication of oppositions). This is related with an idea I had at the same time I was developing the polyhedron of oppositions early in 2003. My idea was to generalize the hexagon of Blanché into the double square shown in Fig. 13.

Later on in 2003 I would have the pleasure to see plenty of double squares when visiting Turkey. I was taking part to the 21st World Congress of Philosophy at the Congress center in Istanbul (August 10–17, 2003) and the floor was covered by such squares and I also saw such squares visiting the famous temple Hagia Sophia. A picture I took is shown in Figure 14.

Later on Alessio told me that it is better to figure the idea beyond this double square by a bi-simplex, since we have then the same distances between the vertices of each "square". The generalization from the hexagon to a double square or a bi-simpex is quite natural

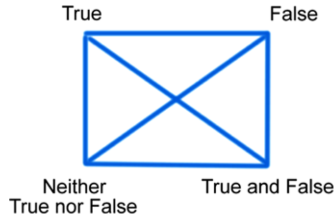

Fig. 15 Truth-valued square of contrariety

from a mathematical viewpoint, it is like going from trivalent logic to quadrivalent logic. And we are facing the same problem: does this make sense? Can we find interesting philosophical motivations and applications of these generalizations? In his logic book Kant argues that only dichotomy is *a priori*, polytomy is *a posteriori* (see [33]). But Blanché's masterpiece *Structures intellectuelles. Essai sur l'organisation systématique des concepts* gives us good reasons to think that trichotomy can be put on the *a priori* side: a ternary structure is ruling conceptualization (see [23]). However it is difficult to find some systematic n-tomic ($n > 3$) structures and we can agree with Kant that at some point polytomy is rather empirical, maybe though the case of quadritomy can reasonably put on the *a priori* side, considering for example the square of contrariety (Fig. 15) corresponding to the famous four-valued logic of Dunn-Belnap [1].

Having developed these new ideas about the square, I gave in 2003 a series of lectures on this topic around the world:

- March 6, in Brussels (Belgium),
- March 10, in Lausanne (Switzerland),
- April 11, in Rio de Janeiro,
- April 24, in Florianópolis (Brazil),
- May 17, in Vancouver (at the 31st Annual Meeting of the Society for Exact Philosophy),
- May 29, in Moscow at the 4th Smirnov's Readings,
- August 10, in Oviedo (Spain) at the 12th International Congress of Logic, Methodology and Philosophy of Science,
- September 9, at the Second Principia International Symposium, Santa Catarina, Brazil,
- September 16, at the Academy of Science of Buenos Aires, Argentina,
- October 15, in Geneva,
- October 18, in Amsterdam.

6 Squaring the World (2007–2011)

In 2003, 2004 and 2005 I was working intensively to develop universal logic. After having organized an international workshop on universal logic In Neuchâtel in October 6–8, 2003, I started the preparation of a big world congress and school on universal logic in Montreux in 2005 (UNILOG'05) and then launched the journal *Logica Universalis* and the book series *Studies in Universal Logic* with Birkhäuser. Universal logic is a general theory of logical systems trying in particular to study concepts that are beyond specific systems of logic and that can be therefore applied to a huge variety of logics (see my paper

[6] that can be seen as a complement to the present one). The square of opposition is typically a universal tool, as explained in the preface of a special issue of *Logica Universalis* on the square I edited with Gillman Payette:

> The square of opposition is something which is very much in the spirit of universal logic. It is a general framework than can be used to appraise existing logics and develop new ones (...) It can be applied to logical notions such as: quantifiers, negation and all kinds of modalities (ontic, deontic, alethic, epsitemic).
> The square of opposition helps to turn logic more universal. The square can not only be used as a tool for a systematic study of classes of logics and for developing bridges between logics but also as a central logical key to semiotics, ethnology, linguistics, psychology, artificial intelligence, computer science. ([15], p. 1)

UNILOG'05 was a great success with more than 200 participants from about 50 different countries. Everybody enjoyed very much the atmosphere of Montreux and the friendly hotel Helvétie in which the event took place. I decided therefore to organize the first world congress on the square of opposition in the same location. However the event was projected in a different way, a small interdisciplinary 3 day congress: the event took place June 1–3, 2007, in Montreux. We had 10 invited speakers, among them: Larry Horn, Pascal Engel, Alessio Moretti, Pieter Seuren, Hans Smessaert, Terence Parsons, Jan Woleński, and we had plenty of good contributors (about 60 people from all around the world). I wanted to really develop interdisciplinarity, gathering people from philosophy, linguistics, mathematics, computer science (the basic square), but also form psychology, ethnology and art. Since we were in Montreux, jazz was natural. We had a jazz show with compositions based on the square. This was organized by my friend Michael Frauchiger, from the Lauener Foundation in Bern.[1] We also presented a movie we had produced especially for the event, a remake of the Biblical story of Salomé, the square being used to articulate the relationships between the four main characters: Herod, Herodias, Salomé, Saint John the Baptist.

[1]Henri Lauener was a Swiss philosopher who liked very much music, after his death a philosophy prize was created and during the award ceremony there is always a band playing music.

The trailer of the movie and extract of the square jazz show can be seen in the DVD which was produced after the event by Catherine Chantilly, included in the book *The Square of Opposition—A General Framework for Cognition* [19] that I have edited with Gillman Payette after the event.

At the final round square table of the first congress in Montreux it was discussed if there would be future editions of the event, where and when. Among the participants there was a colleague, Pierre Simonnet, working at the University Pascal Paoli in Corté, Corsica, who suggested to organize the event there. I liked the idea since I had been living in Corsica as a child and had wonderful time. I visited Pierre in Corsica in March 2008 and again in October 2009 to prepare the event. The event took place in Corsica in June 17–20, 2010, with the same format as in Montreux. Among the participants, there were Damian Niwinski, Stephen Read, Dale Jacquette and Pierre Cartier—the famous Bourbachic mathematician, and many old and new contributors. At the end of this second event we discussed the organization of the third world congress on the square of opposition, and it seems that this series of events is now launched for many years. The third square event is scheduled in June 2012 at the American University of Beirut and the fourth square congress will probably happen in Munich in 2014.

Working on the organization of these events, I have also been continuously working on the square, always having new ideas, applying the square to various fields. Among many talks on the square around the world I presented "Extensions of the square of opposition and their applications" at ECAP08 (6th European Conference on Computing and Philosophy, June 16–18, 2008, Montpellier, France), "The logical geometry of economy" at a congress on Epistemology of Economical Sciences (October 1–2, 2009, Buenos Aires, Argentina). I also applied the square to semiotics (talk at Dany Jaspers's CRISSP seminar in Brussels, October 28, 2010) and music (talk at the seminar *mamuphi*—mathematics, music and philosophy—organized by François Nicolas at the ENS Ulm in Paris, November 5, 2011).

I see the square as a way to go beyond dichotomy. Dichotomy is a thinking methodology, promoted by the Pythagoreans, which was essential for the development of science. But dichotomy is limited and many dichotomic pairs are quite artificial. Blanché has rightly pointed out that trichotomy is more interesting and this is the basis of the hexagon of oppositions. In the hexagon, trichotomy appears at two levels: there are the two trichotomic triangles, but there are also the three notions of oppositions from the square of oppositions: contrariety which is the basis of the blue triangle, subcontrariety, which is the basis of the dual green triangle and contradiction linking these two trichotomic triangles. Blanché's hexagon is a really powerful tool based on trichotomy (see [17]).

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Logical Oppositions in Arabic Logic: Avicenna and Averroes

Saloua Chatti

Abstract In this paper, I examine Avicenna's and Averroes' theories of opposition and compare them with Aristotle's. I will show that although they are close to Aristotle in many aspects, their analysis of logical oppositions differs from Aristotle's by its semantic character, and their conceptions of opposition are different from each other and from Aristotle's conception. Following Al Fārābī, they distinguish between propositions by means of what they call their "matter" modalities, which are determined by the meanings of the propositions. This consideration gives rise to a precise distribution of truth-values for each kind of proposition, and leads in turn to the definitions of the logical oppositions. Avicenna admits the four traditional oppositions, while Averroes, who seems closer to Aristotle and especially to Al Fārābī, does not mention subalternation, but admits subcontrariety. Nevertheless, we can find that Averroes defends what Parsons calls SQUARE and [SQUARE], because he holds **E** and **I**-conversions and the truth conditions he admits are just those that make all the relations of the square valid, while Avicenna defends SQUARE and [SQUARE] only for the waṣfī reading of assertoric propositions. They also give a special attention to the indefinite which in Averroes' view is ambiguous, while Avicenna treats it as a particular. Some points of their analysis prefigure the medieval concepts and distinctions, but their opinion about existential import is not as clear as the medieval one and does not really escape the modern criticisms.

Keywords Logical oppositions · Matter necessity · Possibility and impossibility · Existential import · Indefinites · waṣfī vs ḍātī readings of propositions

Mathematics Subject Classification Primary 03A05 · Secondary 03B10

1 Introduction

In this paper, I will examine Avicenna's and Averroes' views on the specific topic of the so-called logical oppositions in order to compare between them both on the one hand and between them and Aristotle on the other hand. As is well known, Aristotle identifies opposition with incompatibility, since the only logical relations that he holds to be oppositions are contradiction and contrariety. But can one say the same thing about Avicenna and Averroes? Are their respective accounts of the notion of opposition distinct from Aristotle's one? Are they distinct from each other? How are logical oppositions characterized in their respective systems? And how is the notion of opposition itself viewed in these systems? In order to answer these questions, I will start by analyzing Avicenna's system then

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I will turn to Averroes' one and finally, I will compare between them both and Aristotle in order to determine the way they define this notion. This comparative analysis shows that the notion of opposition is different in the three systems, and gives rise to three quite distinct figures.

2 Avicenna's Analysis of the Logical Oppositions

The logical oppositions are analyzed by Avicenna (980–1037) in his book entitled *al-shifā'* (*The Cure*), more precisely in *al-Maqūlāt* (*Categories*) and *al-'Ibāra* (*Peri Hermeneias*). In *al-'Ibārā*, he defends his views on oppositions and presents what we would consider as a square of oppositions since he analyzes the four traditional relations which are: contradiction, contrariety, subcontrariety and subalternation. But before turning to his analysis of these relations let us first see how he defines the notion of opposition itself. This definition is presented in *al-Maqūlāt* (*Categories*) where Avicenna considers many types of oppositions which could concern the terms or the propositions. He defines opposition in general by saying that "The opposites do not conjoin in the same subject by any aspect in any time" [12, p. 241].[1] The opposites could be terms or propositions expressed by sentences, and opposition itself could be expressed by means of negation or by other means; as examples, he gives the following pairs: "horse/non-horse" and "even/odd". Avicenna follows here Aristotle's *Categories* (10) when he distinguishes between oppositions by virtue of correlation such as "double" and "half", of possession and privation, of contrariety such as "sick" and "healthy" and finally the opposition of truth-values. This last one affects propositions and is expressed by means of a negation; thus singular propositions such as "Zayd is a man" and "Zayd is not a man" are opposed because they do not have the same truth-value. The real opposition between propositions is, then, the opposition between truth-values. This opposition in truth-values is specifically made by the negation since Avicenna, like Aristotle, thinks that sentences that contain opposite predicates are not contradictories when the subject does not exist, because in that case, they are both false. Avicenna gives the following example: "Zayd who does not exist [*al ma'dūm*] is seeing" does not contradict "Zayd who does not exist is blind", but it does contradict "Zayd who does not exist is not seeing" [12, p. 259], because this last sentence is true. In the same vein, the sentence "Stones are sick" does not contradict "Stones are healthy" but it does contradict "Stones are not sick" [12, p. 258], because the first two sentences are false while the last one is true. But the notion of opposition is more general than contradiction, contrariety or correlation. It can be seen as a genus that includes several species [12, p. 245].

How can we apply this to the different kinds of propositions that are expressed by singular, quantified or non quantified sentences? To answer this question let us see what Avicenna says in *al-'Ibāra*. In this book, he classifies propositions into three kinds: (1) Singulars, (2) Indefinites (i.e. not quantified) and (3) Quantified, i.e. Universal and Particular propositions. We have to notice here that Avicenna uses a specific term to indicate the presence of a quantification. The quantifier is expressed by the word '*sūr*' [14, p. 52] and

[1] Wilfrid Hodges in [19, p. 13] translates this passage in the following way: "We say: opposing pairs are those which don't combine in a single subject from a single aspect at a single time together."