Systems & Control: Foundations & Applications

Serdar Yüksel
Tamer Başar

Stochastic Networked Control Systems

Stabilization and Optimization under Information Constraints

Birkhäuser

Birkhäuser

Systems & Control: Foundations & Applications

For further volumes:
http://www.springer.com/series/4895

Serdar Yüksel • Tamer Başar

Stochastic Networked Control Systems

Stabilization and Optimization under Information Constraints

Birkhäuser

Serdar Yüksel
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario
Canada

Tamer Başar
Department of Electrical
and Computer Engineering,
Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
Urbana, Illinois, USA

ISSN 2324-9749 ISSN 2324-9757 (electronic)
ISBN 978-1-4614-7084-7 ISBN 978-1-4614-7085-4 (eBook)
DOI 10.1007/978-1-4614-7085-4
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013936940

Mathematics Subject Classification (2010): 91B44, 93A14, 93E03, 93E15, 93E20, 94A15, 94A17, 94A24, 94A29, 60J05, 60J20, 60B10, 62B05, 62B10, 62C10

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.birkhauser-science.com)

To
Kate, Sait, and Saibe Yüksel (S.Y.)
and
Tangül, Gözen, Elif, Altan, and Koray (T.B.)

Preface

Our goal in writing this book has been to provide a comprehensive, mathematically rigorous, but still accessible treatment of the interaction between information and control in multi-agent decision making in the context of networked control systems. These are systems where different decision units (or equivalently decision makers or agents, which could be sensors, controllers, encoders, or decoders) are connected over a real-time communication network, where the communication medium is heterogeneous, information is decentralized and distributed, and its acquisition is not instantaneous. The questions we address are all performance driven, and entail the issues of what data to pick and how to shape and transmit them for control purposes under various resource constraints as well as how to design optimal control policies with partial information. We deal specifically with the issues of quantization and encoding, design of optimum channels, effects of decentralization on control performance, stability, learning, signaling, and relationships between team performance (of a group of agents) and various information structures.

The book draws and utilizes a diverse set of tools (of both conceptual and analytical nature) from various disciplines, including stochastic control, stochastic teams, information theory, probability theory and stochastic processes, and source-coding and channel-coding theory, and amalgamates them into a unified, coherent, applicable theory. It could be used as a textbook or as an accompanying text in a graduate course on networked control or multi-agent decision making under informational constraints.

Acknowledgements We would like to thank several individuals whose work, either independently or through collaboration, has shaped the contents of the book. Of particular mention are Tamás Linder (Chaps. 4 and 10); Sean Meyn (Chaps. 6 and 8); Andrew Johnston (Chaps. 7 and 9); Sekhar Tatikonda (Chap. 11); Ali Zaidi, Tobias Oechtering, and Mikael Skoglund (Chap. 11); and Aditya Mahajan, Giacomo Como, and Nuno C. Martins, general collaborations with whom have been reflected in the book.

Abhishek Gupta, Andrew Johnston, Naci Saldi, Charalambos Charalambous, Maxim Raginsky, Emrah Akyol, and Daniel Quevedo read different versions of the book and provided substantial technical comments which this final version has greatly benefitted from.

The book has also benefitted from technical discussions with many of our colleagues and students, including Nevroz Şen, Marcos M. Vasconcelos, Debasish Chatterjee, Ather Gattami, Yücel Altuğ, Orhan Arikan, Demos Teneketzis, Alexander Condello, Adina Goldberg, Anant Sahai, Gürdal Arslan, R. Srikant, Chris Hadjicostis, Venu Veeravalli, Cédric Langbort, Todd Coleman, Orhan Çağri Imer, Atilla Eryilmaz, Akshay Kashyap, Tansu Alpcan, Abdol-Reza Mansouri, Andrew Lewis, and Fady Alajaji, to whom we also convey our thanks.

An earlier version of the book was used in an intensive course offered by one of us (S.Y.) at KTH, Stockholm, and the discussions there also helped shape this final version. We thank the hosts, Tobias Oechtering, Mikael Skoglund, and Ali Zaidi, for providing this opportunity.

Finally, we acknowledge the stimulating, conducive environment of the Coordinated Science Laboratory, University of Illinois, where the initial seeds of this book project were sown. The bulk of the work was carried out after the first author joined Queen's University, which provided another stimulating environment for the project to be completed. We thank both institutions and many of our colleagues there.

Kingston, ON — Serdar Yüksel
Urbana, IL — Tamer Başar

Contents

Part II Stabilization of Networked Control Systems

Acronyms and Notations

trace(A)	Trace of a square matrix A
det(A)	Determinant of a square matrix A
1_E	Indicator function for event E
$\mathbb{X}$	A space of vectors
$\langle x, y \rangle$	Inner product between x and y on a Hilbert space
$\mathcal{B}(\mathbb{X})$	The Borel σ-field on $\mathbb{X}$
$\sigma(y)$	σ-field generated by a random variable y
$\mathcal{P}(\mathbb{X})$	Set of probability measures on $\mathcal{B}(\mathbb{X})$
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Vector space of n-dimensional real vectors
$\mathbb{Z}$	Set of integers
$\mathbb{Z}_+$	Set of nonnegative integers
$\mathbb{N}$	Set of positive integers
$\mathcal{Q}$	A space of quantizers
Q	Quantizer or channel depending on context
$\Pi^{comp,i}$	Composite quantization policy for encoder i
Q_t^i	Quantizer used by agent i at time t
DM i or **A** i	decision maker i or agent i
$\underline{\gamma}^i$	Policy of DM i, that is, $\{\gamma_t^i, t \geq 0\}$
$\underline{\gamma}$	Ensemble of policies for all decision makers, that is, $\{\underline{\gamma}^i\}$
I_t^i or $\mathcal{I}_t^i$	Information variable at agent i at time t
$\underline{\eta}$	Information structure inducing map $\{\eta^1, \ldots, \eta^N\}$
$E_P^{\underline{\gamma}}\{\cdot\}$	Expectation under policy $\underline{\gamma}$, with initial condition measure P
E_x	Expectation conditioned on an initial condition realization x, or with respect to a random variable x, depending on the context
$H(\cdot)$	Discrete entropy
$h(\cdot)$	Differential entropy
$I(\cdot;\cdot)$	Mutual information
$D(P_1 \|\| P_2)$	Kullback–Leibler divergence between P_1 and P_2

$\|x\|$	Euclidean norm of a finite-dimensional real vector x
A' or A^T	Transpose of matrix A
$\|S\|$	Cardinality of a set S
$A \setminus B$	Set difference: $\{x : x \in A, x \notin B\}$
$A \triangle B$	$(A \setminus B) \cup (B \setminus A)$
$\ln(x)$ or $\log(x)$	Natural logarithm of positive real x
$\underline{o}$	Zero vector
$\mathcal{T}$	Time/stage index set, $\{1, 2, \ldots, T\}$ or $\{0, 1, \ldots, T-1\}$
$\mathcal{N}$ ($\mathcal{L}$)	Decision maker (DM) index set, $\{1, 2, \ldots, N\}$ ($\{1, 2, \ldots, L\}$)
$u_{[k,s]}$	Action (decision) variables from $t = k$ to $t = s$ for $s > k$, $\{u_k, u_{k+1}, \cdots, u_s\}$
$\mathbf{u}$	Collection of actions in $\mathcal{N}$ (or $\mathcal{L}$): $\{u^1, u^2, \ldots, u^N\}$

Chapter 1
Introduction

This chapter provides an introduction to the field of networked control and thereby to this *book*. It highlights the main approaches taken to address issues unique to networked control and describes the scope of coverage and the contents of the book.

1.1 Information and Control

The *interaction between information and control* is a phenomenon that arises in every decision and control problem. On the one hand, any performance-driven controller requires information on the unknowns that affect the operation of the underlying system; on the other hand, the *quality* of the relevant information itself is typically affected by the choice of the control action in a closed-loop system. Further, the transmission of information over communication channels with high fidelity and the process of shaping the source output and recovering the transmitted signal at the other end can themselves be viewed as controller design problems. This book is a comprehensive undertaking aimed at furthering our understanding of this interaction in the context of decentralized and networked control systems.

Networked control refers to a decentralized control system in which the components are connected through real-time communication channels or a data network. Thus, there may be a data link between the sensors (which collect information), the controllers (which make decisions), and the actuators (which execute the controller commands); and the sensors, the controllers, and the plant themselves could be geographically separated.

With such a networked structure, many modern control systems are decentralized. Such systems feature multiple decision makers (e.g., sensors, controllers, and encoders) which have access to different and imperfect information, either cooperate with or compete against each other. Such systems are becoming ubiquitous, with applications ranging from automobile and inter-vehicle communications designs, control of surveillance and rescue robot teams for access to hazardous environments,

S. Yüksel and T. Başar, *Stochastic Networked Control Systems*, Systems & Control: Foundations & Applications, DOI 10.1007/978-1-4614-7085-4_1,

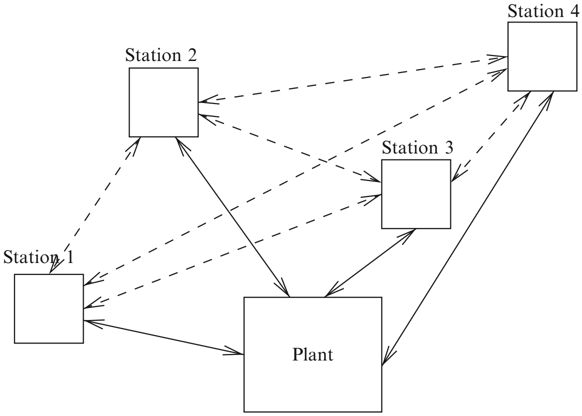

Fig. 1.1 A decentralized networked control system. *Solid lines* show the interaction between the control stations and the plant *Dashed lines* depict the possible communication links between the stations

remote surgery, space exploration and aircraft design, and control of economic systems, among many other fields of applications most of which involve *remote control*.

In such decentralized networked control problems, one major concern is the characterization of the *minimum* amount of information transfer needed for a satisfactory performance and particularly for *stability* of the overall system. This information transfer would be between various components of the networked control system. One necessity for satisfactory control performance is the ability for the controllers to track the plant state under various constraints on the communication (see Fig. 1.1). Another set of challenges is the determination of the data rate required for the transmission of control signals, and the construction of dynamic encoding, decoding, and control policies meeting some selected design criteria. Another important problem is the establishment of effective coordination among multiple sensors or multiple controllers/decision makers using minimum possible information exchange. Even in cases when communication resources are not scarce, a strong understanding of the fundamentals can be useful in constructing the system architecture, and finally, such an insight can help reduce the computation requirements and complexity.

Various forms of system architectures have been introduced and studied in the networked control literature. To be able to analyze different scenarios, it is important to identify and formalize the probabilistic description of the system and characterize the underlying information structure. Further, it is equally important to precisely pin down the objective in the system design, whether it is *stochastic stability* (in some appropriate sense) or *optimization*. Learning and identifying an unknown system through observations and actions is another important issue. Finally, the notion of *signaling*–that controllers could communicate through actions–is another aspect which has to be taken into account. In all this, it is essential to understand both the qualitative and the quantitative values of information for achieving different objectives in such networked settings.

The above description of a networked control system lies at the intersection of three disciplines of applied mathematics and engineering, namely, *decentralized control* (in view of information structures and decision making under measurability constraints), *stochastic control* (in view of decision making under uncertainty), and *communication, information, and quantization theories* (in view of information exchange among decision makers). Finally, *probabilistic analysis* (in view of being the ultimate mathematical tool needed to conduct a study in all of these disciplines) plays an essential role in the understanding, analysis, and synthesis of such systems.

Our aim in this book is to bridge these three disciplines in a precise and rigorous manner, while also conveying practical messages to systems designers and controller architects. The field of networked control has had accelerated growth during the past decade, but many of the problems and challenges that arise were actually obstacles identified already in the 1960s and the 1970s. What makes the situation different today is the accumulation (since that time) of an arsenal of more powerful results and tools from information theory, source coding theory, and the theory of Markov chains, directly applicable to the problems at hand. Furthermore, we have a richer pool of computational algorithms, and the computers of the current generation have significantly more processing power. The field has reached some state of maturity, and it seems timely to collect and present the main results in a book form. Furthermore, it is also useful to revisit many of the results of the 1970s and the 1980s and blend them in such a treatise with the current developments, as they have by no means lost their relevance. And this is another one of our goals here.

1.2 Coverage and the Intended Audience

Within the framework of networked control systems and in discrete time, five essential concepts are visited recurrently in the book:

- The characterization of information structures in team problems defined in terms of measurability relations in a given probability space. Comparison and topological properties of information structures for stochastic team optimization problems and identification of information structures which may lead to a systematic program for generation of optimal policies.
- Stochastic stability of systems (state, controller, and encoders). Converse results through information theoretic analysis and constructive algorithms via stochastic drift equations. Stochastic stability corresponds to the existence of an equilibrium distribution, ergodicity of a process, or existence of finite moments.

- The operational differences between information theoretic settings (which, in the classical sense, requires an infinite ensemble of messages to be transmitted or encoded) and real-time settings in control which do not tolerate delay. Use of information theory in establishing fundamental bounds on information requirements.
- Optimal information transmission under causality and delay constraints, and jointly optimal channel and controller design for real-time systems, under a variety of information structures. Structural results as well as existence results on optimal policies.
- The notion of signaling, its utilization in decentralized stabilization, and the technical issues that are associated with it, such as the lack of convexity, the dual effect of control, and non-neutrality.

These concepts and notions are interweaved throughout the book, constituting the backbone of a comprehensive theory of networked control. Specific results are built on that foundation and seamlessly presented throughout the book.

What is Not Covered

As indicated earlier, networked control systems, and multi-agent systems in general, entail multiple decision makers that provide input into the system using only local (decentralized) information. Throughout the book the underlying assumption is that the agents act in unison, toward a common goal, that is, as members of a team, even though they do not necessarily share the information they acquire. An extended framework would be one where the agents' goals are not aligned and may even be conflicting, which then cannot be cast as a decentralized team problem. Such problems belong to the realm of *dynamic noncooperative stochastic games*, where appropriate solution concepts are the Nash equilibrium or Stackelberg equilibrium, or a blend of the two, depending on whether there is a hierarchy in decision making or not [32]. Such problems entail other intricate issues and their analysis requires a different set of tools, beyond the scope of the coverage here.

Another direction in which the framework of this book can be broadened is as follows: The treatment in the book is restricted to settings where there is an underlying probability space and all the variables in the system are well-defined random variables on this probability space given the policies of the decision makers. Such a *Bayesian* setting does not include the probability-free settings occasionally used in control theory (as in robust control with distribution-free disturbances with norm constraints [28]) and in information theory and machine learning (as in coding and learning for individual sequences [90]). As objective or loss functionals, such settings typically admit a min-max type formulation instead of minimization of the expected value of a loss function with respect to a probability measure. This book does not consider such settings explicitly; however, the approaches presented here are applicable to many such scenarios.

Intended Audience for the Book

The intended audience is broad, including academic as well as industrial researchers interested in control theory, information theory, statistics, and applied mathematics. As indicated earlier, the book adopts a probability theory, information theory, and decentralized stochastic control theory view to networked control problems. To comfortably follow the material in the book, the reader should be familiar with linear systems (at the first-year graduate level), basics of information theory, and measure-theoretic stochastic processes (again at the first-year graduate level). The reader is also expected to have a basic understanding of Markov chains and martingale theory. For those who do not have the requisite background, appropriate references are provided throughout the development in the book, and four appendices are included, covering some of this material as well as others.

1.3 Contents of the Book

The book is comprised of twelve chapters, organized into three parts, as described below. It also has four appendices, providing background material.

1.3.1 Part I. Information Structures in a Networked Control System

This part is primarily concerned with the mathematical description of a networked control system as a stochastic dynamic team. It provides a treatise on stochastic dynamic teams and a detailed investigation of information structures. Comparison of different information structures is also covered.

In Chapter 2, a general probability theoretic framework for stochastic team decision problems is established, by defining and classifying information structures, interaction dynamics, policy spaces, and objective functions. A number of examples are included to provide a gentle introduction to the concepts. Team problems and information structures are classified according to various criteria. Solution methods for static teams are presented, with particular emphasis on convex cost functions. Dynamic teams are considered further in Chap. 3.

Chapter 3 focuses on comparison of information structures and solution approaches to a class of dynamic team problems. Under nonclassical information structures, the notion of signaling is introduced and thoroughly discussed. Witsenhausen's counterexample is studied, along with its generalizations and a class of dynamic team problems involving Gaussian sources and channels. Expansion of information structures is presented as a general recipe for studying dynamic teams with nonclassical information patterns. Witsenhausen's characterization of information structures is also presented in this chapter.

Chapter 4 investigates the optimal design of information structures and studies a number of topological properties of information structures modeled as observation channels under various topologies. Continuity, compactness, concavity, and existence properties are studied for single-stage and multistage optimal stochastic control problems. An introduction to quantizers is given. Quantizers are viewed as a special class of measurement channels, and existence of optimal quantizers is established. Furthermore, a partial ordering on the value of information channels for the minimization of cost functions is studied (known as Blackwell ordering). Applications to empirical consistency and learning are discussed.The results presented in this chapter are used extensively later in Part III.

1.3.2 Part II. Stabilization of Networked Control Systems

This part focuses on the stabilization of networked control systems, for both single-sensor/controller and multi-sensor/controller systems, and comprises five chapters.

The chapters in this part introduce fundamental criteria that need to be satisfied for stochastic stabilization. Constructive methods are presented which meet the fundamental (converse) bounds. The constructive proofs utilize a drift approach offered in a number of recent papers by us and our collaborators. Toward further understanding the value of information channels in stochastic control, it is shown that *Shannon capacity* provides a total ordering on the set of channels for the existence of policies for stochastic stability and ergodicity properties.

Chapter 5 introduces policies and actions regarding the selection of quantizers and controllers in networked control. It reviews fundamentals of information theoretic notions. The chapter exhibits the important differences between the real-time communication formulation and the traditional Shannon theoretic setup which allows for large blocks of data (with unbounded block length) to be encoded and transmitted. This distinction is highlighted in the context of distortion-constrained quantizer design and the rate-distortion theory. The chapter also establishes fundamental lower bounds on information rates needed for various forms of stochastic stabilization. These lower bounds are further studied in Chap. 9.

Chapter 6 is an important one for the general program of Part II, where random-time state-dependent stochastic drift criteria for stabilization of Markov chains are established together with a class of application areas in networked control systems. Criteria for transience and other forms of stochastic stability are presented. Related background material is reviewed in Appendix C.

In the context of stochastic stabilization of linear sources driven by noise with unbounded support, controlled over information channels, Chap. 7 focuses on finite-rate noiseless channels and provides the architectural setup for coding and control policies. Chapter 8 investigates stabilization over erasure channels, discrete memoryless channels with and without feedback as well as a class of continuous-alphabet channels (Gaussian channels are further discussed in Chap. 11). A common theme in these chapters is the relationship between Shannon capacity and the ergodicity of the controlled Markov process: For ergodicity (under additional technical assumptions), Shannon capacity (with feedback) provides a boundary condition in the space of communication channels for stochastic stabilization of unstable linear systems. For finite moment stability, however, further conditions are required both on the channels and on the tail distributions of the system noise. The results in these chapters also include extensions to multidimensional and partially observed settings.

Chapter 9 considers stabilization under a decentralized information structure for multi-sensor and multi-controller systems. Existence results on stabilizing policies under the decentralized information structure are obtained. In the absence of noise, it is shown that multi-controller systems, unlike multi-sensor systems with a centralized controller, entail a rate loss due to decentralization. The noisy cases are also investigated and rate conditions are established for multi-sensor systems.

1.3.3 Part III. Optimization in Networked Control: Design of Optimal Policies Under Information Constraints

The third, and final part of the book, comprising three chapters, studies simultaneous design of optimal encoding and control policies for networked control systems.

Chapter 10 establishes the structure of optimal quantization policies under various information structures for general cost functions. The coverage includes both single decision maker and multiple decision maker formulations, with partial as well as full observation. A dynamic programming approach is presented building on classical results by Witsenhausen, and Walrand and Varaiya. The chapter also presents optimal solutions for encoders and controllers under quadratic performance measure for linear Gaussian systems controlled over discrete noiseless channels.

Chapter 11 obtains optimal solutions for encoders and controllers under quadratic cost functions for linear Gaussian systems controlled over Gaussian channels, proving also the existence of optimal solutions. Furthermore, the chapter identifies conditions under which optimal coding and control policies are linear. Counterexamples on sub-optimality of linear policies are also presented.

Chapter 12 presents the notions of agreement and common knowledge and addresses the question of how to achieve common knowledge. The chapter presents a general framework for obtaining solutions to dynamic team problems under decentralized information structures based on dynamic programming and an evolving common knowledge, and applies this primarily in the context of the belief sharing information pattern. Information rates required for tractability of optimal solutions are also presented. Finally, the chapter introduces a team cost-rate function, which provides the minimum cost subject to a rate constraint on the information exchange among members of a team.

1.4 A Guide for the Reader or the Instructor

For students who have a background in stochastic processes at the first-year graduate level and who are also familiar with the basics of information theory, the book can be used as a textbook in a course on *networked control systems* or *stochastic control* or *multi-agent decision making*. It could also be used in a special topics course or as an independent resource.

For a graduate-level course on *decentralized control* where the students do not have any background on communication theory at the graduate level, Chaps. 2, 3, 4, 9, 10, 11, and 12 together with the Appendices can be used as primary or supporting material. A basic information theory background can be acquired from standard textbooks, such as [103] or [151]. For further advanced topics on information theory, the reader is referred to advanced texts such as [107] or [153].

Chapters 2, 3, 4, 6, 10, and 12 together with the Appendices can be used as primary or supporting material in a graduate-level course on *Stochastic Control* with limited information theory content.

For students who do not have a background at a graduate-level stochastic process course, all chapters except Chaps. 4, 6 should be accessible.

For students who are familiar with information theory, but not stochastic control and optimization, all chapters should be accessible with some further reading. The stochastic control and optimization fundamentals could be supplemented by resources such as [84, 194, 225, 269], and [14] on stochastic control and [55] and [242] on optimization. The material in Chap. 6 can be supplemented by [271]. A further useful reference is [189].

A useful related book in the literature on networked control systems is the text by Matveev and Savkin [266], which has partial overlap with the material in Chaps. 7, 8, and 9 of this book, even though the approaches are different. The two books can be used as complementary resources.

Part I
Information Structures in Networked Control

Chapter 2
Networked Control Systems as Stochastic Team Decision Problems: A General Introduction

2.1 Introduction

Networked control systems can be viewed as stochastic decision problems with dynamic decentralized information structures or as stochastic dynamic teams, with each subcontroller viewed as an *agent* in a dynamic team. The goal of this introductory chapter is accordingly to introduce the reader to a general mathematical formulation of stochastic teams, first with static and then with dynamic information structures, and to discuss some salient features of these decision problems and associated solution concepts through some simple but illustrative examples.

The chapter discusses both *static* stochastic teams (i.e., team decision problems where the information signals received by the decision makers are not affected by actions) and *dynamic* stochastic teams (where the information of at least one decision maker is affected by action). Sections 2.2, 2.3, and 2.6 deal with static teams, whereas Sects. 2.4 and 2.5 discuss dynamic teams. Section 2.2 provides a general formulation for static teams, which is followed by a complete analysis of a finite stochastic team problem under various information patterns, in Sect. 2.3. Section 2.6 provides some general explicit results on existence, uniqueness, and characterization of optimal solutions first for general static teams and then for special classes of teams with Gaussian statistics: those with quadratic and exponentiated quadratic costs.

Sections 2.4 and 2.5 can be viewed as the counterparts of Sects. 2.2 and 2.3 for dynamic teams. First a precise mathematical formulation for dynamic team decision problems is given, in Sect. 2.4, along with various dynamic information structures and appropriate solution concepts, and then an illustrative example of a finite dynamic team is provided in Sect. 2.5, within the framework of which some important features of optimal solutions in teams are discussed. The chapter concludes with Sect. 2.7 which provides some bibliographical notes and guidelines for further reading on the topics covered herein.

S. Yüksel and T. Başar, *Stochastic Networked Control Systems*, Systems & Control: Foundations & Applications, DOI 10.1007/978-1-4614-7085-4_2,

2.2 A Mathematical Framework For Static Decision Problems

Multiple person stochastic decision problems could be formulated with varying degrees of generality, abstraction, and rigor, depending on the types of problems to be solved (*i.e*., the scope of coverage) and the level of mathematical sophistication to be expected from the reader. Common to all possible formulations, however, is the specification of **five** basic **ingredients** which are essential for a well-founded mathematical treatment of *decision making under uncertainty*. These are:

1. The number of **decision makers** (synonymously, *agents* or *controllers*) and the sets of alternative **actions** (synonymously, *decisions* or *controls*) available to them
2. The **uncertainty** and its probabilistic description
3. The **information** acquired by each decision maker on the uncertainty and the previous actions
4. The **payoff** (or *loss*) that accrues to each decision maker as a result of joint actions (over the decision period) and realization of uncertainty
5. A **solution concept** whereby "best" or "satisfactory" decision rules can be chosen

Before going into further specification of these entities, let us pause to introduce some terminology and notation which will be needed in the sequel. We will refer to a decision problem as **static** if the information available to each decision maker is independent of the actions of other decision makers (this statement will be made precise later in the section as well as in Sect. 3.8); otherwise, the decision problem is said to be **dynamic**. We will refer to decision makers interchangeably as *agents* or *controllers*, with the *i*th one denoted $\mathbf{A}i$, where i takes values in the set $\mathcal{N} := \{1, \ldots, N\}$ which is called the *agent* (*decision maker*) *set*. The variable under the control of each decision maker will be called the *action* (synonymously, *decision* or *control*) *variable* and will be denoted by u^i for $\mathbf{A}i$. Each u^i will take values in a given *action set* to be denoted by U^i. Finally, the N-tuple $(u^1, \ldots, u^N)$ will be denoted by $\mathbf{u}$ and the product action space $U^1 \times \cdots \times U^N$ by $\mathbf{U}$.

Basic Ingredients of Static Decision Problems

In the static framework we will initially study the class of problems where the action sets, $U^i, i \in \mathcal{N}$, are either (finitely or infinitely) countable or uncountable but finite dimensional. In the latter case, we take the action set (space) to be isomorphic to the Euclidean[1] space $\mathbb{R}^{m_i}$, for some integer $m_i, i \in \mathcal{N}$; furthermore, if there are any

[1]Some background material on sets and topological notions can be found in Appendix A.