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Haijun Li
Xiaohu Li
*Editors*

# Stochastic Orders in Reliability and Risk

## In Honor of Professor Moshe Shaked

Springer

# Lecture Notes in Statistics 208

*Edited by*
P. Bickel, P. Diggle, S. Fienberg, U. Gather, I. Olkin, S. Zeger

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Editors

# Stochastic Orders in Reliability and Risk

## In Honor of Professor Moshe Shaked

Springer

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# Preface

## Introduction

In summer of 2010, the first author (HL) visited the second author (XL) at Lanzhou University, China, and chaired the dissertation defense for XL's two graduating doctoral students. During the visit, we discussed that a large reliability meeting (MMR2011) was scheduled to be held in Beijing in the summer of 2011 and that the meeting would attract some stochastic inequality people, including Professor Moshe Shaked, to visit Beijing. XL then initiated the idea of organizing a small academic gathering for these people at Xiamen University, China, focusing specifically on stochastic inequalities in honor of Moshe Shaked—our common academic mentor, our coauthor, and our good friend. A stochastic order workshop was immediately planned to promote close collaboration in honor of Moshe. The people whom we have contacted with were overwhelmingly enthusiastic about the idea. Some people couldn't come but sent us their suggestions about the workshop. The funding for this workshop was provided by XL's NNSF research funds with support from the School of Mathematical Sciences and Center for Actuarial Studies at Xiamen University.

Xiamen is situated on the southeast coast of China, to the west of Taiwan Strait. Known as a "Garden on the Sea," Xiamen is surrounded by ocean on three sides. The International Workshop on Stochastic Orders in Reliability and Risk Management, or SORR2011, was held in Xiamen City Hotel from June 27 to June 29, 2011. SORR2011 featured 11 invited speeches and nine contributed talks, covering a wide range of topics from theory of stochastic orders to applications in reliability and risk/ruin analysis. Professor Moshe Shaked delivered the opening keynote speech. A social highlight of SORR2011 was a surprise banquet party for Professor Moshe Shaked and Ms Edith Shaked.

This volume is based on the talks presented at the workshop and the invited contributions to this special occasion to honor Professor Moshe Shaked, who has made fundamental and widespread contributions to theory of stochastic orders and its applications in reliability, queueing modeling, operations research, economics, and risk analysis. All the papers submitted were subjected to reviewing, and all the accepted papers have been edited to standardize notations and terminologies. The volume consists of 19 contributions that are organized along the following five categories:

Part I: Theory of Stochastic Orders

- "A Global Dependence Stochastic Order Based on the Presence of Noise" by Moshe Shaked, Miguel A. Sordo, and Alfonso Suárez-Llorens
- "Duality Theory and Transfers for Stochastic Order Relations" by Alfred Müller
- "Reversing Conditional Orderings" by Rachele Foschi and Fabio Spizzichino

Part II: Stochastic Comparison of Order Statistics

- "Multivariate Comparisons of Ordered Data" by Félix Belzunce
- "On Stochastic Properties of Spacings with Applications in Multiple-Outlier Models" by Nuria Torrado and Rosa E. Lillo
- "On Sample Range from Two Heterogeneous Exponential Variables" by Peng Zhao and Xiaohu Li

Part III: Stochastic Orders in Reliability

- "On Bivariate Signatures for Systems with Independent Modules" by Gaofeng Da and Taizhong Hu
- "Stochastic Comparisons of Cumulative Entropies" by Antonio Di Crescenzo and Maria Longobardi
- "Decreasing Percentile Residual Life Aging Notion: Properties and Estimation" by Alba M. Franco-Pereira, Jacobo de Uña, Rosa E. Lillo, and Moshe Shaked

- "A Review on Convolutions of Gamma Random Variables" by Baha-Eldin Khaledi and Subhash Kochar
- "Allocation of Active Redundancies to Coherent Systems: A Brief Review" by Xiaohu Li and Weiyong Ding
- "On Used Systems and Systems with Used Components" by Xiaohu Li, Franco Pellerey, and Yinping You

Part IV: Stochastic Orders in Risk Analysis

- "Dynamic Risk Measures Within Discrete-Time Risk Models" by Hélène Cossette and Etienne Marceau
- "Excess Wealth Transform with Applications" by Subhash Kochar and Maochao Xu

Part V: Applications

- "Intermediate Tail Dependence: A Review and Some New Results" by Lei Hua and Harry Joe
- "Second-Order Conditions of Regular Variation and Drees Type Inequalities" by Tiantian Mao
- "Individual and Moving Ratio Charts for Weibull Processes" by Francis Pascual
- "On a Slow Server Problem" by Vladimir Rykov
- "Dependence Comparison of Multivariate Extremes via Stochastic Tail Orders" by Haijun Li

We thank all the authors and workshop participants for their contributions. This volume is dedicated to Professor Moshe Shaked to celebrate his academic achievements and also intended to stimulate further research on stochastic orders and their applications.

## Professor Moshe Shaked

Moshe Shaked has been for the past 31 years a professor of mathematics at the University of Arizona, Tucson, AZ. He received his B.A. and M.A. degrees from Hebrew University of Jerusalem in 1967 and 1971, respectively. Moshe pursued his graduate studies in mathematics and statistics under Albert W. Marshall at the University of Rochester from 1971 to 1975. Moshe received his Ph.D. in 1975 and his dissertation was entitled "On Concepts of Positive Dependence."

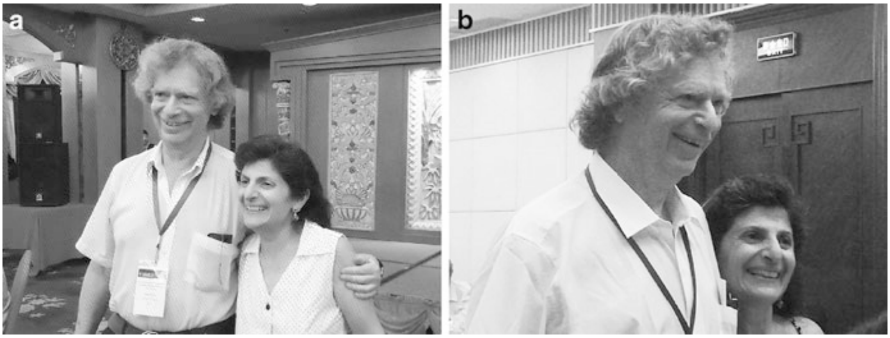


Figure 1: Moshe Shaked and Edith Shaked. (**a**) Beijing, China, June 2011. (**b**) Xiamen, China, June 2011

After short stays at the University of New Mexico, University of British Columbia, and Indiana University, Moshe became an associate professor of mathematics at the University of Arizona in 1981. Since 1986, he has been a full professor at Arizona (Fig. 1).

Moshe has made fundamental contributions in various areas of probability, statistics, and operations research. He has published over 180 papers and many of his papers appeared in the top journals in probability, statistics, and operations research. Coauthored with George Shanthikumar, Moshe published one of the two popular books on stochastic orders [426] (the other book was written by Alfred Müller and Dietrich Stoyan [335]). Moshe's contribution is extremely broad; for example, Moshe made seminal contributions to the following areas:

- Dependence analysis, positive and negative dependence notions, dependence by mixture of distributions, distributions with fixed marginals, and global dependence
- Comparison of stochastic processes, aging properties of stochastic processes, and aging first passage times
- Stochastic variability orders, dispersive ordering of distributions, and excess wealth order
- Accelerated life tests—inference, nonparametric approach, and goodness of fit
- Multivariate phase-type distributions

- Multivariate aging notions and multivariate life distributions
- Multivariate conditional hazard rate functions
- Linkages as a tool for construction of multivariate distributions
- Inventory centralization costs and games
- Stochastic convexity and concavity and stochastic majorization
- Stochastic comparisons of order statistics
- Total time on test transform order
- Use of antithetic variables in simulation
- Scientific activity and truth acquisition in social epistemology

In recognition of his many contributions, Moshe Shaked was elected as a Fellow of the Institute of Mathematical Statistics in 1986. He has been serving in editorial boards of various probability, statistics, and operations research journals and book series.

Moshe enjoys collaborations and has been working with more than 60 collaborators worldwide. Moshe is a stimulating, accommodating, and generous collaborator with colleagues and students alike. Moshe and Edith travel a lot professionally, so the concepts of "vacation" and "conference" often have the same meaning for them. Changing a routine in Tucson, visiting different places in other parts of the world, and meeting new friends (potential collaborators?) are all both relaxing and rewarding for Moshe and Edith. In coffee breaks of several conferences, we have witnessed that Moshe still worked on problems with collaborators one by one. It seems to us that Moshe values collaborating itself as much as he values possible products (i.e., papers) resulting from collaboration. This reminds us of Paul Erdős, a great mathematician, who strongly believed in scientific collaboration and practiced mathematics research as a social activity.

On the personal side, it was Moshe who helped HL get his academic job in the USA and it was Moshe who mentored XL in launching his academic career. Collaborating with Moshe has been a real treat for both of us, and by working with Moshe, we learned and became greatly appreciative to the true value of professionalism.

## Stochastic Orders: A Historical Perspective

Stochastic ordering refers to comparing random elements in some stochastic sense and has evolved into a deep field of enormous breadth with ample structures of its own, establishing strong ties with numerous striking applications in economics, finance, insurance, management science, operations research, statistics, and other fields in engineering, natural, and social sciences. Stochastic ordering is a fundamental guide for decision making under uncertainty and an essential tool in the study of structural properties of complex stochastic systems.

Take two random variables $X$ and $Y$, for example. One way to compare them is to compare their survival functions; that is, if

$$P\{X > t\} \leq P\{Y > t\}, \text{ for all real } t, \tag{1}$$

then $Y$ is more likely to "survive" beyond $t$ than $X$ does, and we say $X$ is stochastically smaller than $Y$ and denote this by $X \leq_{\text{st}} Y$. Using approximations, the path-wise ordering Eq. (1) can be showed to be equivalent to

$$E[\phi(X)] \leq E[\phi(Y)], \text{ for all nondecreasing functions } \phi : \mathbb{R} \to \mathbb{R}, \tag{2}$$

provided that the expectations exist. That is, $X \leq_{\text{st}} Y$ is equivalent to the comparisons with respect to a class of increasing functionals of random variables. If a system performance measure can be written as an increasing functional $E[\phi(X)]$, where $\phi(\cdot)$ is increasing, then the system performance comparison boils down to the stochastic order Eq. (1).

The stochastic order $\leq_{\text{st}}$ enjoys nice operational properties (see [335, 426]), and its utility can be greatly enhanced via coupling [444]. For any two random variables $X$ and $Y$, $X \leq_{\text{st}} Y$ if and only if there exist two random variables $\hat{X}$ and $\widehat{Y}$, defined on the same probability space $(\Omega, \mathcal{F}, P)$, such that $\widehat{X}$ and $X$ have the same (marginal) distribution, $\hat{Y}$ and $Y$ have the same (marginal) distribution, and

$$P\{\hat{X} \leq \hat{Y}\} = 1. \tag{3}$$

That is, one can work with almost-sure inequalities on the coupling space $(\Omega, \mathcal{F}, P)$ and move back to the original random variables using marginal distributional equivalence.

The stochastic order $\leq_{\text{st}}$ is also mathematically robust; namely, the order $\leq_{\text{st}}$, as described in Eqs. (1)–(3), can be extended to probability measures defined on a partially ordered Polish space [220] (i.e., a complete separable metric space endowed with a closed partial ordering). For example, the stochastic order $\leq_{\text{st}}$ on $\mathbb{R}^\infty$ can be applied to comparing two discrete-time stochastic processes. The stochastic order $\leq_{\text{st}}$ is also extended to nonadditive measures [138]. The models that involve nonadditive probability measures have been used in decision theory to cope with observed violations of expected utility [412] (e.g., the Keynes–Ellsberg paradox). These models describe such distortions using different transforms of usual probabilities and have been applied to insurance premium pricing [116, 465, 466].

The stochastic order $\leq_{\text{st}}$ is just one example that illustrates the deep stochastic comparison theory with widespread applications [335, 426]. The stochastic order $\leq_{\text{st}}$, however, is one of *strong orderings*, and many stochastic systems can only be compared using *weak orders*. One example of weak integral stochastic orders is the increasing and convex order $\leq_{\text{icx}}$ that uses the set of all increasing and convex functions in Eq. (2). The idea of seeking various weaker versions of a problem solution has been used throughout mathematics (e.g., in the theory of partial differential equations), and indeed various weak stochastic orders and their applications add enormous breadth to the field of stochastic orders.

The studies on stochastic orders have a long and colorful history. To the best of our knowledge, the studies on inequalities of type (2) for convex functions $\phi(\cdot)$ can be traced back to Karamata [223]. Known as the dilation order, the comparison Eq. (2) for all continuous convex functions $\phi(\cdot)$ is closely related to the notion of majorization. The theory of stochastic inequalities based on majorization is summarized in Marshall and Olkin [308] and its updated version [312].

Historically, stochastic orders have been used to define and study multivariate dependence. Some strongest dependence notions can be defined in terms of *total positivity* [224]. Earlier studies have been focused on dependence structures of multivariate normal distributions and multivariate distributions of elliptical type (see Tong [449]). For analyzing dependence structures of non-normal multivariate distributions, stochastic orders have been substantially used in Joe [211] and Nelsen [355], in which dependence structures of copulas, especially extreme value copulas, have been systematically investigated using orthant and supermodular orders.

Stochastic orders have been applied to various domain fields and especially to reliability theory. Both of us first learned stochastic orders from the 1975 seminal book on reliability and life testing by Barlow and Proschan [38], where Erich L. Lehmann's earlier contributions to the field are highlighted. To show how stochastic orders can be used in reliability contexts, let us consider the following example.

There are a few aging notions and three of them, IFR (increasing failure rate), IFRA (increasing failure rate average), and NBU (new better than used), are particularly useful. IFR implies IFRA, which in turn implies NBU. We now illustrate how the IFRA and NBU can naturally arise from Markov chains with stochastically monotone structures. We consider only the discrete case to ease the notations and a more complete survey can be found in [237].

Let $\{X_n, n \geq 0\}$ be a discrete-time, homogenous Markov chain on $\mathbb{R}_+$. The chain is said to be stochastically monotone if

$$[X_n|X_{n-1} = x] \leq_{\text{st}} [X_n|X_{n-1} = x'], \text{ whenever } x \leq x'. \tag{4}$$

Consider the discrete first passage time $T_x := \inf\{n : X_n > x\}$. In such a discrete setting,

1. $T_x$ is IFRA if either $\mathrm{P}\{T_x = 0\} = 1$ or $\mathrm{P}\{T_x = 0\} = 0$ and $[\mathrm{P}\{T_x > n\}]^{1/n}$ is decreasing in $n \geq 1$
2. $T_x$ is NBU if $[T_x - m|T_x > m] \leq_{\text{st}} T_x$ for all $m \geq 0$.

**Theorem.** *Assume that $\{X_n, n \geq 0\}$ is stochastically monotone.*

*1.* (Brown and Chaganty [79]) *$T_x$ is NBU for any $x$.*

*2.* (Shaked and Shanthikumar [419]) *If, in addition, $\{X_n, n \geq 0\}$ has increasing sample paths, then $T_x$ is IFRA for any $x$.*

That is, the aging properties NBU and IFRA emerge from Markov chains with stochastic order relation (4). The continuous-time version of this theorem can also be obtained. The comparison method used here is again robust and this theorem can be extended to a Markov chain with general partially ordered Polish state space.

It is well known that an IFRA life distribution arises from a weak limit of a sequence of coherent systems of independent, exponentially distributed components. The method used to establish such a result, however, is restricted to the continuous case (see, e.g., [38], page 87). In contrast, this result can be reestablished using a sequence of stochastically monotone Markov chains along the lines of the above theorem. More importantly, the stochastic order approach used in this theorem sheds structural insight on the fact that aging properties arise in a very natural way from stochastically monotone systems.

Many stochastic systems used in reliability and queueing modeling are indeed stochastically monotone in the sense of Eq. (4). The English edition of Dietrich Stoyan's book ([443], 1977 version in German, 1979 version in Russian) attracted quite a few queueing theorists in the 1980s and early 1990s to apply stochastic comparison methods to queueing modeling and analysis. The 1994 book by Moshe Shaked and George Shanthikumar included several chapters (written by some leading queueing and reliability theorists) that highlight research on stochastic orders in queueing and reliability contexts.

The comparison methods of stochastic processes have been discussed in detail in Szekli [446]. The studies on dependence and aging via stochastic orders are presented in Spizzichino [440]. An early study of stochastic orders in risk contexts is documented in Mosler [329] and more recent applications of stochastic orders to analyzing actuarial risks are discussed in Denuit et al. [117].

The most up-to- date, comprehensive treatments of stochastic orders are given by Müller and Stoyan [335] and Shaked and Shanthikumar [426].

## Looking Forward

In the late 1980s and early 1990s, there were several international workshops focusing exclusively on stochastic orders and dependence. We mention some of them below.

- Symposium on Dependence in Probability and Statistics [62], Hidden Valley Conference Center, Pennsylvania, August 1–5, 1987. Organizers: H.W. Block, A.R. Sampson, and T.H. Savits

- Stochastic Orders and Decision Under Risk [330], Hamburg, Germany, May 16–20, 1989. Organizers: K. Mosler and M. Scarsini

- Stochastic Inequalities [431], Seattle, WA, July 1991. Organizers: Moshe Shaked and Y. L. Tong

- Distribution with Fixed Marginals and Related Topics [398], Seattle, WA, August, 1993. Organizers: L. Rüschendorf, B. Schweizer, and M. D. Taylor

These workshops and their proceedings enhanced communication and collaboration between scholars working in different fields and simulated research on stochastic orders and dependence. It is our hope that at the time we honor Professor Moshe Shaked, the Xiamen Workshop and this volume will revive the community workshop tradition on stochastic orders and dependence and strengthen research collaboration.

Last but not least, we would like to thank the School of Mathematical Sciences of Xiamen University for the support to the SORR2011. We would also like to express our sincere thanks to XL's graduate students Jianhua Lin, Jintang Wu, Yinping You, Rui Fang and Chen Li. Without their effort in organizing the Xiamen workshop, we would not have had such a wonderful academic meeting. Our special thanks go to Mr. Rui Fang, who helped us edit and revise the Latex source files of all submitted papers. Due to his enthusiasm and quiet efficiency, we finally present this nice volume (Fig. 2).

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Figure 2: SORR 2011, Xiamen, China, June 27–29, 2011

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