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# Real-Time Detection of Lines and Grids

By PClines and Other  
Approaches



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# Chapter 1

## Introduction

Detection of straight lines is an important basic task of computer vision. It is receiving attention of researchers since the “early days” till today. Solving this task introduced an important concept—the Hough transform. Recently, with the increasing popularity of two dimensional barcodes and augmented reality markers, straight lines are also interesting as a building block of perpendicular grids and chessboard-like structures.

A planar chessboard-like grid is characterized by two groups of lines. Each of the groups of lines has a common vanishing point and, as we discuss in the text, it can be characterized as a whole. A proper representation of such two groups of lines can be used for efficient detection of perpendicular grids. This text deals with suitable representations of grids composed of straight lines and captures the recent development in the Hough transform for straight line detection.

First, Chap. 2 reviews the basics of the Hough transform for line detection. The Hough transform is formalized for 2D curves and some important theoretical properties and relationships to the Radon and Fourier transforms are derived. Chapter 2 also reviews the existing parameterizations of lines which can be used by the Hough transform.

Chapter 3 gives an overview of the existing literature on Hough transform for line detection and it provides a comprehensive overview and taxonomy of the algorithm’s variants. The taxonomy is based on a survey article by Kälviäinen et al. [1] but it is updated and extended.

Chapter 4 presents our recent parameterization of lines PCLines [2]. This chapter discusses the point-to-line mappings as a class of line parameterizations for the Hough transform. The properties of the point-to-line mappings are further described in Chap. 5 about pencils of lines, regular grids, and possibilities of their detection by the Hough transform.

Chapter 6 presents several efficient implementations of the Hough transform based on the point-to-line-mapping parameterizations. These implementations are using the graphics processors both in the regime of shader computing and in the GPGPU manner. The algorithms presented in this chapter can also be ported to embedded

processors and reconfigurable chips, because the algorithmic modifications allow for avoiding any floating-point or fractional numbers and they are memory efficient.

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