Mathematics Education in the Digital Era

Dragana Martinovic
Viktor Freiman
Zekeriya Karadag *Editors*

Visual Mathematics and Cyberlearning

Springer

Visual Mathematics and Cyberlearning

MATHEMATICS EDUCATION IN THE DIGITAL ERA

Volume 1

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Introduction

Mathematics Education in the Digital Era (MEDEra) Series

The *Mathematics Education in the Digital Era* (MEDEra) is a new Springer book series co-edited by Dragana Martinovic, University of Windsor, Canada, and Viktor Freiman, Université de Moncton, Canada. With two annual volumes, it attempts to explore ways in which digital technologies change conditions for teaching and learning of mathematics. By paying attention also to educational debates, each volume will address one specific issue in mathematics education (e.g., visual mathematics and cyber-learning; inclusive and community based e-learning; teaching in the digital era) in an attempt to explore fundamental assumptions about teaching and learning mathematics in the presence of digital technologies.

This series aims to attract diverse readers including: researchers in mathematics education, mathematicians, cognitive scientists and computer scientists, graduate students in education, policy-makers, educational software developers, administrators and teachers-practitioners.

Among other things, the high quality scientific work published in this series will address questions related to the suitability of pedagogies and digital technologies for new generations of mathematics students who grew up with digital technologies and social networks. The series will also provide readers with deeper insight into how innovative teaching and assessment practices emerge, make their way into the classroom, and shape the learning and attitude towards mathematics of young students accustomed to various technologies.

The series will also look at how to bridge theory and practice to enhance the different learning styles of today's students, and turn their motivation and natural interest in technology into an additional support for meaningful mathematics learning. The series provides the opportunity for the dissemination of findings that address the effects of digital technologies on learning outcomes and their integration into effective teaching practices; the potential of mathematics educational software for the transformation of instruction and curricula; and the power of the e-learning of mathematics, as inclusive and community-based, yet personalized and hands-on.

Visual Mathematics and Cyberlearning – The First Book in the MEDEra Series

The first book in the MEDEra series, entitled *Visual Mathematics and Cyberlearning*, is co-edited by Dragana Martinovic, University of Windsor, Canada, Viktor Freiman, Université de Moncton, Canada, and Zekeriya Karadag, Bayburt University, Turkey. It offers a platform for dissemination of new ideas in visual mathematics and cyberlearning, addresses new developments in the field, and evokes new theoretical perspectives in mathematics education.

Recent studies describe the Net Generation as visual learners who thrive when surrounded with new technologies and whose needs can be met with the technological innovations. These new learners seek novel ways of studying, such as collaborating with peers, multitasking, as well as use of multimedia, the Internet, and other Information and Communication Technologies. How this can be used to present mathematics in new ways, as a contemporary subject that is engaging, exciting and enlightening?

For example, in the distributed environment of cyber space, mathematics learners play games, watch presentations on YouTube, create Java applets of mathematics simulations and exchange thoughts over the Instant Messaging tool. How should mathematics education resonate with these learners and technological novelties that excite them? How can educators make a meaningful use of dynamic, interactive, collaborative, and visual nature of new learning environments while having a deeper understanding of their potential advantages and limitations? Authors of nine chapters share their conceptual frameworks and research data that shed a light on innovative theories and practices in the field of visual mathematics and cyberlearning.

Jones, *Geraniou*, *and Tiropanis* study potential of Web 3.0 semantic tools that enhance mathematics discussion within collaborative, shared workspace by means of graphical argumentation and chat tools. Elementary students were given an opportunity to explore different patterns and combination of patterns by finding and augmenting an algebraic rule while working collaboratively in the *eXpresser* environment accompanied by visual support provided by *LASAD*. The authors reflect on innovative potential of cyberlearning to foster knowledge development and mathematical thinking.

Alagic and Alagic, on their turn, provide in-depth analysis of research mathematicians working together by means of large-scale computer supported collaborative learning tools that enrich networking opportunities and enhance self-regulated learning.

Çakır and Stahl describe socially situated interactional processes involved in collaborative online learning of mathematics. In the common online environment their Virtual Math Teams problem solve using chat, shared drawings and mathematics symbols and thus co-construct a deep mathematical understanding at the group level.

Güçler, *Hegedus*, *Robidoux*, *and Jackiw* investigate mathematical discourse of young learners involved in multi-modal mathematical inquiries. Using a context of dynamic geometry with haptic devices, authors claim that such integration fosters new learning experiences that lead to evolution of young learners' expression from informal to formal mathematical discourse.

Trninic and Abrahamson are interested in the role of embodied artefacts in the emergence of mathematical competence, viewed as independent from the physical world. By performing physically in the service of doing mathematics, students make observable what is otherwise hidden away 'in their heads'. In their chapter, the authors enrich investigations of embodied artefacts in light of increasingly ubiquitous monitor-sensor technologies, namely the Mathematical Imagery Trainer. Students work with proportions by moving their hands in an environment that changes its state in accord with the ratio of the hands' respective heights, then reflect on what they see on the computer screen, and analyze mathematically as particular case of proportionality.

Radford uses an approach where human cognition is conceptualized in non-dualistic, non-representational, and non-computational terms. The basic idea is that cognition is a feature of living material bodies characterized by a capacity for *responsive sensation*. As a result, human cognition can only be understood as a culturally and historically constituted sentient form of creatively responding, acting, feeling, transforming, and making sense of the world. In his chapter, the author presents classroom experimental data involving 7–8-year-old students dealing with pattern recognition that lead to suggesting that a sensuous-based materialistic monistic view of cognition needs to attend not only to the plethora of sensorial modalities that teachers and students display while engaging in mathematical activities, but also to the manner in which sensorial modalities come to constitute more and more complex psychic *wholes* of sensorial and artefactual units.

Gadanidis and Namukasa discuss a case study of online mathematics learning for teachers through the lens of four affordances of new media: democratization, multimodality, collaboration and performance, which help to rethink and disrupt existing views of mathematics for teachers and for students.

LeSage used web-based video clips on rational numbers to provide pre-service teachers with accessible and flexible learning opportunities to support their individual learning needs. According to research findings from participants' narratives, careful consideration must be paid not only to the instructional design of video clips but also to support the development of pre-service teachers' pedagogical content knowledge and content knowledge of mathematics.

In the final chapter, *Martinovic*, *Freiman*, *and Karadag* show that diverse examples and deep insights given by the authors of the book chapters extend our understanding of the features and complexity of virtual mathematics tools suitable for visualization and exploration in the light of Activity and Affordance Theories, thus opening new perspectives in researching mathematics education in the digital era that can be investigated further in next volumes of the series.

Viktor Freiman and Dragana Martinovic, MEDEra Editors

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Patterns of Collaboration: Towards Learning Mathematics in the Era of the Semantic Web

Keith Jones, Eirini Geraniou, and Thanassis Tiropanis

Abstract With current digital technologies there are a number of networked computer-based tools that provide ways for users, be they learners or teachers, to collaborate in tackling visual representations of mathematics, both algebraic and geometric. For learners, there are various ways of collaborating that can occur while the learners are tackling mathematical problems. In this chapter we use selected outcomes from recent innovative research on this aspect of learning and teaching mathematics with digital technologies to review the patterns of collaboration that can occur in terms of teacher and learner experience. Given that such patterns of collaboration are via current digital technologies, this chapter goes on to offer a view on the likely impact on the cyberlearning of mathematics of progress towards the next generation of Web technologies that seeks to make use of ideas related to the web of data and the semantic web. Such impact is likely to be in terms of enhancing the learning applications of digital technologies, improving ways of administrating the educational programmes that they support, and potentially enabling teachers to maintain involvement in technological development and use over the longer-term.

Keywords Argumentation • Algebra • Collaborative learning • Semantic web • Web 3.0

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Introduction

During the 30 years since the launch in 1981 of the IBM model 5150 personal computer (the PC that became the worldwide standard) there has been the implementation and enormous growth of communication between networked computers via the World Wide Web. The term Web 2.0 was coined by DiNucci (1999) to capture how Web technologies have developed since the beginning of the Web in 1991 such that users are able to interact and collaborate with each other in increasingly diverse ways. Since 2001, ideas about the nature of Web 3.0 (see, Berners-Lee, Hendler, & Lassila, 2001) have centred on features such as increasing personalisation and on the possible advent of what Berners-Lee calls the Semantic Web, a new vision of the Web where computers 'understand' the semantics (or meaning) of data and information on the World Wide Web.

Over the same time period governments across the world have been promoting digital technologies as powerful tools for education (for current information, see, for instance: Law, Pelgrum, & Plomp, 2008; EU Education, Audiovisual and Culture Executive Agency, 2011). As a result of commercial and Governmental initiatives, there are a range of computer-based networked tools that provide ways for users to interact and collaborate. In this chapter we use carefully-selected outcomes from recent innovative research involving digital technologies to review the various patterns of collaboration that can take place. The aim is to review the inter-person interaction and collaboration via digital technologies in terms of the experience of those involved. Given that such collaborations are based around current digital technologies and the current enactment of the World Wide Web, we use our research experience to offer a view on the likely impact on the 'cyberlearning' of mathematics of developments towards Web 3.0, especially developments relating to the notion of the Semantic Web.

Cyberlearning: From Web 1.0 to Web 2.0

The Web, from its beginnings in the early 1990s, has provided new and powerful ways for finding and accessing resources that could be used for learning. Regardless of the context (whether formal or informal), individuals have been able to use the Web to find content and software to support learning. The volume and types of resources that have become available on the Web have made it possible for people not only to find more content than ever before (and usually more efficiently too), but also collaboratively to publish additional content – and even to categorise it using taxonomies and tags. For many, this transition from a 'read-only' Web to a 'read-write' Web signifies the transition from Web 1.0 to Web 2.0 (although Berners-Lee, the inventor of the Web, always envisaged the Web as a means of connecting people; see, Laningham, 2006).

Web 2.0 technologies have enabled user-generated content, and powerful paradigms such as crowdsourcing (the outsourcing of tasks that might traditionally have been performed by employees, or a contractor, to an undefined group of people – a “crowd” – often via an open call). Such possibilities have been transformative by making the Web into a host for a large number of knowledge repositories (an example being Wikipedia) and by paving the way for a transition from a ‘Web of documents’ to a ‘Web of data’. In this way the Web is becoming a repository of data (in addition to documents) and people are able efficiently to aggregate the data that becomes available to create and provide new applications. Another key characteristic of the Web is that it has leveraged network effects, with resultant rapid growth. Such network effects have occurred because the more that content and data becomes available on the Web, the higher the value of the Web to the users and, in turn, the higher the volume of content and data that users are willing to contribute. At the same time, the Web has provided an environment for network effects to take place, examples being the growth of services like Wikipedia and of online social networks such as Facebook.

Key applications that enabled the transition to Web 2.0, according to Anderson (2007), include blogs, wikis, multimedia sharing, tagging and social bookmarking, audio blogging and podcasting, and RSS and syndication. These have been complemented by newer phenomena such as social networking, aggregation, data mashups, and collaboration services (Anderson). All these developments have meant that the use of Web 2.0 services for learning is becoming increasingly widespread. For example, the UK Higher Education sector has adopted technologies for publication repositories and wikis on an increasing scale; a survey in 2009 reported that 40 universities (out of 165) had adopted publication repository software systems and 14 had adopted wikis (see, Tiropanis, Davis, Millard, & Weal, 2009a).

The key value of Web 2.0 for education is in enhancing learning experiences, given its potential for personalisation, customisation and collaboration for knowledge creation (McLoughlin & Lee, 2007). Social software is increasingly an enabler for pedagogical innovation in terms of peer-to-peer learning, extended learning, cross-cultural collaborative work using student-generated content, and learner-centred instruction. The benefits of Web 2.0 technologies have been identified for a number of educational scenarios such as teacher-class communication and students’ participation in the collection and integration of learning material (Rollett et al., 2007). Most such benefits centre on the ease of reporting progress (e.g. through using blogs) and the efficiency of collaborative construction of complex reports (e.g. through using wikis for assignments). Criticism of Web 2.0 use in education often centres on the sometimes low quality of generated content and the way amateurishness can flourish; in addition, critics bemoan the time and knowledge investment that Web 2.0 technologies can require (see, Grosseck, 2009).

Overall, the unique value of Web 2.0 technologies seems to be in:

- Enabling information finding on a large scale, with the number of resources that learners can find on the Web growing every day.
- Supporting collaborative knowledge construction amongst a large number of people; using Web 2.0 services such as wikis and social software it is possible to mobilise communities across the world as Web 2.0 technologies can cope with the size and geographical distribution of these communities in efficient ways.

- Enabling collaboration among individuals for learning purposes; this is more easily and more efficiently accomplished using Web 2.0 software and, furthermore, when it comes to online collaboration, it is possible to achieve better matching among learners when geographical constraints are not a barrier.

These affordances that Web 2.0 technologies offer have also been incorporated into a number of areas of education including the teaching and learning of mathematics. It is to the findings of selected aspects of two pertinent current research projects that we turn next. In the next section we report on groups of lower secondary school pupils (aged 11–14) interacting and collaborating whilst tackling mathematical problems involving the visual and geometric representation of algebraic ideas. Our aim in doing this is to review the patterns of such collaboration in terms of learner and teacher experience.

Patterns of Collaboration: The Case of the MiGen and Metafora Projects

In this section, we illustrate how advances in technological tools can aid student collaboration by showing the patterns of collaboration in the cases of two such tools that were developed as part of the MiGen[1] and the Metafora[2] projects. Further below we elaborate on how these two tools, namely eXpresser and LASAD, impact on student collaboration. We begin by summarising the general possibilities for collaboration in exploratory learning environments.

Collaboration in Exploratory Learning Environments

Research (e.g. Cobb, Boufi, McClain, & Whitenack, 1997; Leonard, 2001; Linchevski & Kutscher, 1998) has revealed the considerable value of collaboration and classroom discourse towards students' cognitive development. When working in small groups, more students are likely to ask questions compared with whole class situations. In addition, students are more likely to reflect on their own work and attempt to make sense of the work of other students. Students who explain their ideas and solutions to their peers have greater success in their learning than those who do not (e.g. Cohen & Lotan, 1995; Lou et al., 1996). Through such

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[1] The MiGen project is funded by the ESRC/EPSRC Teaching and Learning Research Programme (Technology Enhanced Learning; Award no: RES-139-25-0381). For more details about the project, see http://www.migen.org

[2] The Metafora project is co-funded by the European Union under the Information and Communication Technologies (ICT) theme of the 7th Framework Programme for R&D (FP7). For more information, visit http://www.metafora-project.org

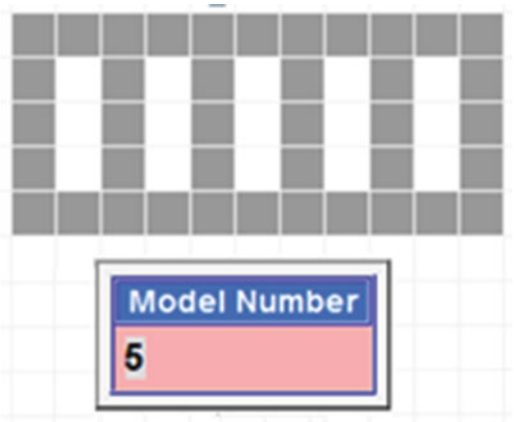
interactions, students participate actively in learning with their peers and tend to adopt metacognitive skills, all of which is beneficial for learning (Biggs, 1985; Maudsley, 1979; Schoenfeld, 1992).

Recent research (e.g. Geraniou, Mavrikis, Hoyles, & Noss, 2011; Healy & Kynigos, 2010) is showing how the use of exploratory learning environments can support, and moreover enhance, students' knowledge development and interactions through individual as well as collaborative activities. A pattern of collaboration is therefore emerging where exploratory learning environments such as microworlds (a "subset of reality or a constructed reality whose structure matches that of a given cognitive mechanism so as to provide an environment where the latter can operate effectively", Papert, 1980, p. 204) are increasingly being used in the classroom. Microworlds aim to embed "important ideas in a form that students can readily explore", with the best having "an easy-to-understand set of operations that students can use to engage tasks of value to them, and in doing so, they come to understanding powerful underlying principles" (diSessa, 2000, p. 47). As such, microworlds can empower learners to engage with abstract ideas and explore not only the structure of objects, but also the relationships through investigating the underlying representations that enforce these relationships (Hoyles, 1993; Thompson, 1987). This can happen through individual student interactions as well as through discussions with their peers. In the particular case of the MiGen system, we show in the next sub-section some of the forms of discussion between students that can be supported by visual artefacts and dynamic objects that students can interact with and explore.

The MiGen System and the Metafora Platform

The MiGen system provides digital tools that support students' collaboration by allowing them to interact with each other as they tackle algebraic generalisation problems. The Metafora platform, currently under development, is being designed to offer visual means (including pictorial symbols) for students to use to plan their learning together and visualise their sub-tasks, stages of work, and required roles. The Metafora platform is also being designed to provide an argumentation space where students can discuss their findings and emerge with an agreed solution. In what follows we illustrate the different patterns of collaboration of students while they interact in the mathematical microworld of the MiGen system and when their discussions and structured arguments are further supported by the Metafora platform. We start by giving some information regarding the two projects.

The MiGen project aimed to tackle a well-known issue in mathematics education – the difficulty that students in lower secondary school (when aged 11–14) can have in coming to terms with algebraic generalisation. Such students are generally able to verbalise algebraic rules in natural language but can struggle to use the appropriate mathematical language (Warren & Cooper, 2008). In addition, students can often fail to see the rationale, let alone the power, of algebraic generalisa-


Model Number

5

Fig. 1 An example of a TrainTrack in eXpresser

tion. In an effort to support such students in learning algebraic generalisation, a computational environment comprising a number of tools was developed. The core of the MiGen system is a microworld, named *eXpresser*, in which students build figural patterns of square tiles (as in Figs. 1, 2 and 3) and express the rules underlying the chosen patterns. The eXpresser is designed to provide students with a model for generalisation that could be used as a precursor to introducing algebra, one that helps them develop an algebraic 'habit of mind' (Cuoco, Goldenberg, & Mark, 1996). The sequence of student activity in eXpresser involves some free-play to explore the system, some introductory tasks to become familiar with its features, a generalisation task and a collaborative activity. The MiGen system also has an 'intelligent' component, namely *eGeneraliser*, which provides feedback to students throughout their interactions with the system (see, Gutierrez-Santos, Mavrikis, & Magoulas, 2010; Noss et al., 2012). A suite of tools, named the *Teacher Assistance Tools,* aim to help the teacher in monitoring students' progress, assisting with possible interventions and reviewing students' achievements to aid future lesson planning. One of these tools is the *Grouping Tool* which puts forward possible pairings of students for collaboration based on the similarities between students' constructions in the MiGen system (for details of the other tools, see, Gutierrez-Santos, Geraniou, Pearce-Lazard, & Poulovassilis, 2012; Pearce-Lazard, Poulovassilis, & Geraniou, 2010).

The Metafora platform, in comparison, includes a web-based argumentation tool called LASAD[3] that enables discussions to take place within groups of learners in a structured manner (Loll, Pinkwart, Scheuer, & McLaren, 2009; Scheuer, McLaren, Loll, & Pinkwart, 2009). This collaborative, shared workspace, together with graphical argumentation and chat tools, is used by students to share ideas, organise their thoughts, discuss and argue as they learn new concepts (Dragon, McLaren, Mavrikis, & Geraniou, 2011). In addition, other components of the Metafora platform analyse the students' work and provide feedback that supports collaboration

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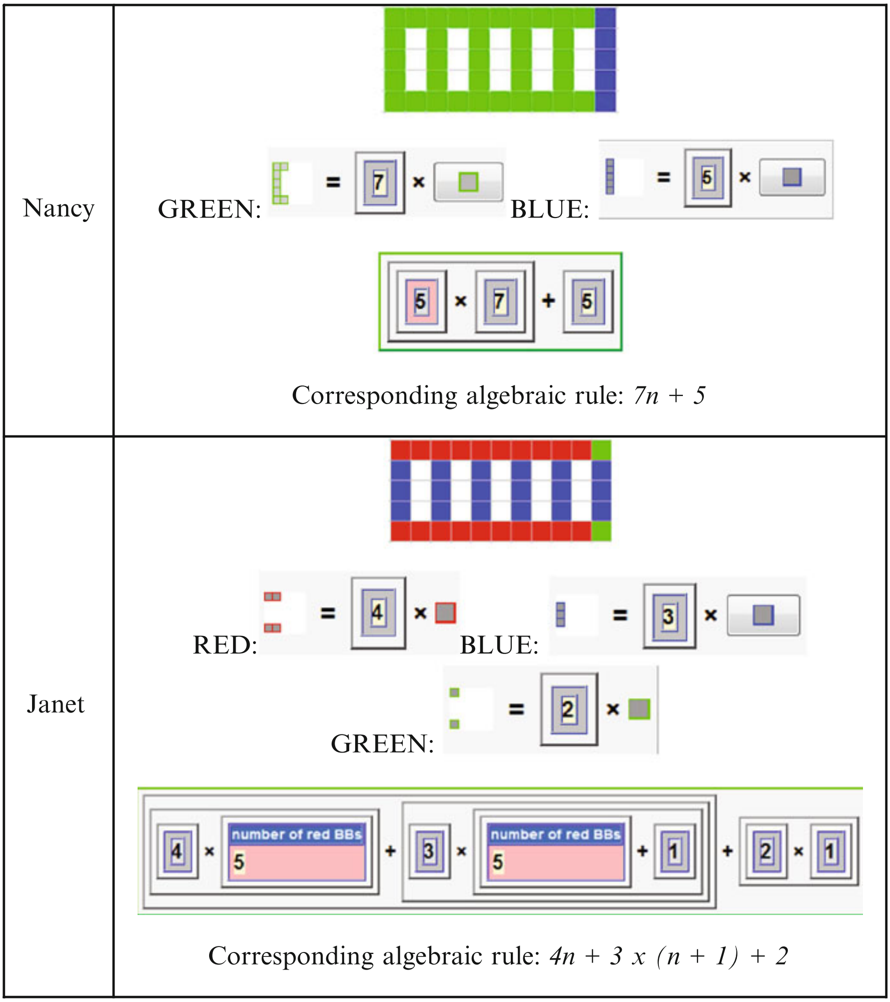
[3] http://cscwlab.in.tu-clausthal.de/lasad/



Corresponding algebraic rule: $7n + 5$

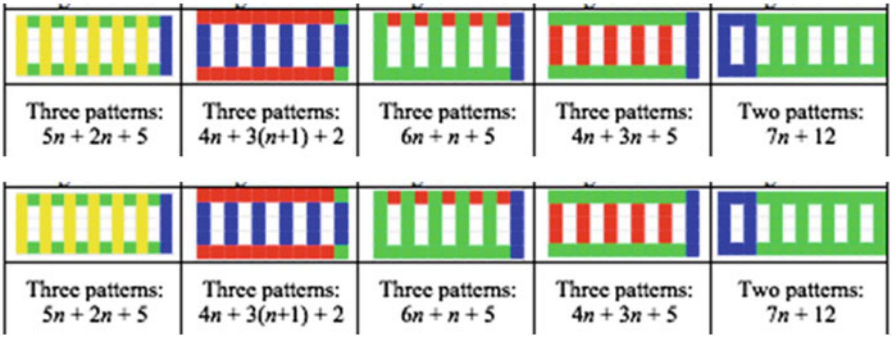
Corresponding algebraic rule: $4n + 3 x (n + 1) + 2$

Fig. 2 Nancy and Janet’s TrainTrack models

and helps students make progress while they grapple with the challenge. The system also identifies situations where the teacher might encourage peer support or shared knowledge evaluation.

Collaboration Within the MiGen System and the Metafora Platform

To illustrate what patterns of collaboration are possible with the MiGen and the Metafora systems, we analyse in this section some selected data from several learning episodes with these systems. For analyses of the wider pedagogical use of



Three patterns: $5n + 2n + 5$
Three patterns: $4n + 3(n+1) + 2$
Three patterns: $6n + n + 5$
Three patterns: $4n + 3n + 5$
Two patterns: $7n + 12$

Fig. 3 Students' different TrainTrack models

eXpresser, we refer readers to Geraniou, Mavrikis, Kahn, Hoyles, and Noss (2009), Mavrikis, Noss, Hoyles, and Geraniou (2012) and Noss et al. (2012).

The first scenario is from the work of two 12 year-old students, Janet and Nancy[4] from a UK school. The students were part of a class of 22 Year seven students (aged 11–12) who participated in a series of lessons during which they were introduced to eXpresser through a number of introductory and practice tasks and solved a linear pattern generalisation task, namely TrainTrack. In this task the students were presented with the TrainTrack model (see, Fig. 1) animated in the *Activity Document*, a tool of eXpresser that presents the task-model, the task-questions and the task-goals and in which students can type their answers. The students were asked to construct the TrainTrack model in eXpresser using different patterns and combinations of patterns depending on their perceptions of the TrainTrack's geometrical structure and to derive a general rule for the number of square tiles needed for any Model Number.

At the end of the TrainTrack activity, and to prepare for the collaborative activity, students were asked to use the *Activity Document* tool to record some arguments that would support the correctness of their general rule. Students were then paired by the system's *Grouping Tool* based on the dissimilarity of their models and asked to work on a new collaborative activity that involved discussing the correctness and equivalence of their rules. This new activity was presented to them in a new eXpresser window which was automatically generated by the system and included the two students' models and rules, and also the following two questions in the Activity Document: *(1) Convince each other that your rules are correct, (2) Can you explain why the rules look different but are equivalent? Discuss and write down your explanations.*

The models and rules developed by the two students (Janet and Nancy) are presented in Fig. 2 in the form that these are represented in eXpresser. To prepare for the collaborative activity, Janet and Nancy were asked to type any arguments

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[4] All names used for students are pseudonyms.

they had for the correctness of their rules (using the *Activity Document* tool). Their arguments were:

> Nancy: *"My rule is correct because each 'block' has 7 squares. So however many blocks there are, there are 7 squares for each one so you multiply the number of blocks by 7. But, at the end there is another block to finish the pattern off. In this block there are 5 squares so you add the number of squares (the blocks multiplied by 7) to the final block (the 5 squares). This rule should apply to this pattern each time."*

> Janet: *"I think my rule is correct, as it works every time and seems to make sense, as, because the number of red building blocks is unlocked, you can put any number in and it would work and is linked with the numbers of the blue and green building blocks".*

In the next lesson, Janet and Nancy were paired because they constructed the TrainTrack model in different ways (see, Fig. 2).

The two students worked together on the collaborative activity. They looked at each other's rules and compared them by interacting with each other's models in eXpresser (i.e. by changing the model number, animating the models, etc.). As a result they both stated that they understood each other's model:

> Nancy: *"I understand the rule so I don't see a reason why it shouldn't be correct"*

> Janet: *"Yeah, I understand yours too"*

In this way, both students were able to 'read' each other's rule and understand them. Yet it seems that the students viewed the 'correctness' of each other's models as so obvious and so 'understandable' that they failed to produce any justification during their collaboration. Even though they were prepared for this collaboration and had typed in their arguments during the previous lesson, the students failed to produce shared mathematically-valid arguments to justify the correctness of their rules. A possible reason for this is the limitation of the MiGen system in not drawing the students' attention to their written arguments.

After both of these students were convinced of the correctness of their rules, they continued by discussing their rules' possible equivalence. In this case, interacting with eXpresser and exploring each other's models acted as a catalyst to a constructive discussion. They benefitted from eXpresser's immediate feedback on their actions and were able to explore and validate their conjectures. After some debate, Nancy stated that:

> yeah, it's one red building block plus one blue building block so that would actually kind of make the . . .

and Janet interrupted to complete Nancy's chain of thought by saying

> yeah, it would make the same shape.

Nancy then added:

> because one red building block added to one blue building block

and Janet finished the argument:

> and that's the same as one of my green building blocks.

As a result, they reached agreement that their two seemingly different rules were in fact equivalent and they justified their conclusion by appealing to the structure of the models that they had constructed.

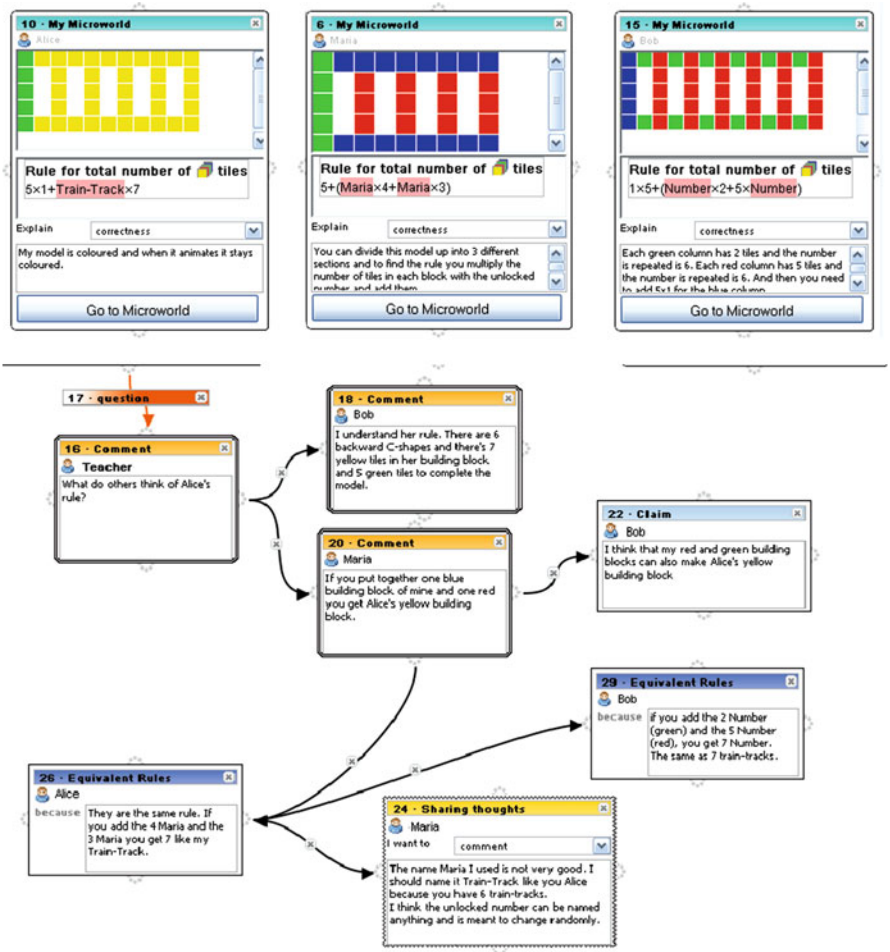
It is worth noting that there are many different ways of constructing the same model (see, Fig. 3) and pairing students with interestingly different constructions can lead to fruitful collaboration.

Collaboration in the context of the MiGen system entails students reading, deconstructing and matching their rule with their partner's by exploring, revisiting their actions, building on them and taking new actions using the tools available within the system. The eXpresser microworld provided the students with a visual means to express algebraic generalisation and through the manipulation of its entities, they were able to give meaning to algebraic concepts that are often elusive (such as constants, variables or the n-th term of a sequence). Such an expressive and exploratory tool proved to assist students in their development of complex mathematical ideas and this illustrates ways in which students can adopt an enquiry stance in making every effort to gain important mathematical skills (such as abstracting and generalising), as originally advocated by Papert (1980) and more recently by other researchers (e.g. Shaffer, 2007).

Although the potential of the MiGen system to support students' learning of algebraic generalisation, and of algebraic ways of thinking, was evident in the MiGen research (see, for example, Mavrikis et al., 2012; Noss et al., 2012), there was evidence of some inflexibility in terms of what collaborative actions the students could take (see, Geraniou et al., 2011). For example, the system is limited to groups of up to three students, they must work together on one machine, they have to store their shared answers on a local server and not on the web, all their collaboration is synchronous but offline, and while they can type their agreed answers they cannot easily post them for other students to see immediately. The latter action of sharing their final statements with fellow classmates could only be orchestrated by their teacher.

Taking into account the advances in digital media, and on the basis of relevant research on the affordances of new technology to support online collaboration (e.g. Stahl, 2006; Stahl, Zhou, Cakir, & Sarmiento-Klapper, 2011), the Metafora platform is innovative in integrating collaborative learning with microworlds that are extended for collaborative online use. A key technical and pedagogic innovation of the Metafora platform is that it gives students the opportunity to come together (not necessarily in the same time and space) using LASAD, an *argumentation tool*, to discuss the given challenge to solve, argue about their findings, and emerge with an agreed solution. In particular, the argumentation tool helps the students to organise their thoughts, discuss opinions, and display the relations between their arguments in graphical form. In this way, the students' discussions are structured and their learning scaffolded.

Taking further the example of students working on the TrainTrack task (the students who produced different models with equivalent rules and therefore were grouped together) we now demonstrate how LASAD, the argumentation tool, was used by the students to work on the collaborative activity of convincing each other of the correctness and possible equivalence of their algebraic rules. To do



10 · My Microworld
Alice
Rule for total number of tiles
5×1+Train-Track×7
Explain: correctness
My model is coloured and when it animates it stays coloured.

6 · My Microworld
Maria
Rule for total number of tiles
5+(Maria×4+Maria×3)
Explain: correctness
You can divide this model up into 3 different sections and to find the rule you multiply the number of tiles in each block with the unlocked

15 · My Microworld
Bob
Rule for total number of tiles
1×5+(Number×2+5×Number)
Explain: correctness
Each green column has 2 tiles and the number is repeated is 6. Each red column has 5 tiles and the number is repeated is 6. And then you need

17 · question

16 · Comment
Teacher
What do others think of Alice's rule?

18 · Comment
Bob
I understand her rule. There are 6 backward C-shapes and there's 7 yellow tiles in her building block and 5 green tiles to complete the model.

20 · Comment
Maria
If you put together one blue building block of mine and one red you get Alice's yellow building block.

22 · Claim
Bob
I think that my red and green building blocks can also make Alice's yellow building block

29 · Equivalent Rules
Bob
because if you add the 2 Number (green) and the 5 Number (red), you get 7 Number. The same as 7 train-tracks.

26 · Equivalent Rules
Alice
because They are the same rule. If you add the 4 Maria and the 3 Maria you get 7 like my Train-Track.

24 · Sharing thoughts
Maria
I want to: comment
The name Maria I used is not very good. I should name it Train-Track like you Alice because you have 6 train-tracks.
I think the unlocked number can be named anything and is meant to change randomly.

Fig. 4 Shared models, rules and arguments for correctness in LASAD

so, we analyse the collaborative process of three 12 year old students, Alice, Maria and Bob, while they interacted in the eXpresser mathematical microworld and simultaneously engaged in discussions and structured arguments using the argumentation tool LASAD.

After the students constructed their on-screen models and decided on the algebraic rules, they were directed not only to share their models and rules but also to prepare for the collaborative activity by stating arguments for their rule's correctness. All this is captured in Fig. 4.

From Fig. 4, we can see that Alice relied on the visual feedback from the microworld to validate her rule's correctness. Since her model remained coloured for different values of the Model Number (or, in other words, the unlocked number that she named as "Train-Track"), she was convinced that her rule was correct; providing further justification seemed unnecessary to her. In contrast, both the other two students, Maria and Bob, derived arguments for their rules' correctness based on the structure of their constructed models. As such, they both deconstructed their rules and models by matching each term in the rule to the corresponding component of the model. Throughout their preparation for the collaborative activity, all three students interacted with their models in eXpresser, explored their patterns' properties, and conjectured why their rules were correct. The combination of eXpresser and the argumentation tool LASAD provided them with the opportunity not only to reflect on their interactions within an exploratory learning environment, but also the opportunity to develop strategies to justify the correctness and equivalence of their rules. Additionally, the argumentation tool allowed them to share their way of thinking with each other and prompted their reflective thinking in terms of a valid argumentation and a mathematically-correct justification.

At this point the teacher intervened by prompting Maria and Bob to comment on Alice's rule. This triggered the students' reflective thinking and their discussion commenced. Maria and Bob tried to make sense of Alice's rule and compared it to theirs. They continued to think structurally as they focused on matching their building blocks to that of Alice's and recognised that their two building blocks formed Alice's yellow building block.

The students' discussion continued naturally until the issue arose of the equivalence of their rules. Alice immediately claimed that their rules were in fact the same rule. She supported her claim by explaining that if you add the terms "4 Marias and 3 Marias" in Maria's rule, you'll get the seven train-tracks she has. She recognised the unlocked number in Maria's rule and ignored the different name by focusing on the mathematical operations that would help her justify the rules' equivalence. Bob followed a similar approach. Maria, on the other hand, noticed that the main difference between their rules was in the name they chose for the unlocked number or, in other words, the variable. This triggered a conversation on the use of a meaningful name, like 'Train-Track', for the variable instead of 'Maria'. Their discussion revealed an appreciation of the notion of algebraic variable and what it represents in their model.

Throughout their discussions, the students used the language of the argumentation tool LASAD. As presented in Fig. 4, Bob and Maria, for example, added supportive comments about the correctness of Alice's rule and linked it to the teacher's prompt question; Bob made a claim reflecting on Maria's comment; Alice and Bob gave reasons for their rules being equivalent; and Maria shared her thoughts on the name she gave to her unlocked number and her view on its meaning. Such features allow students to go through stages in their argumentation process and form mathematically-valid arguments gradually.

In the above example of a use of the argumentation tool LASAD and the Metafora platform, the pedagogical benefits of allowing the interchange between the individual eXpresser workspace and the discussion space, i.e. LASAD, are evident. The students' collaboration encouraged them to recognise their different approaches to solving the same task as well as justifying the correctness and equivalence of their rules. Their reflective comments convey a mutual willingness to support their knowledge development and reach a consensus in terms of their collaborative task as well as recognise mathematically-valid arguments.

Student Collaboration and the Teacher

As mentioned earlier, research has documented the benefits of collaboration towards students' knowledge development (e.g. Cobb et al., 1997; Leonard, 2001). In presenting two different patterns of students' collaboration, we show students not only benefitting from being supported by the tool eXpresser (that provided them with visual feedback on their actions, and allowed them to share their solutions and their thoughts) but also another tool, LASAD, used in parallel to eXpresser, that provided a visual way of structuring their collaboration that scaffolded their knowledge development and mathematical thinking. As students' collaboration progresses from groupwork on paper, to groupwork with the assistance of digital tools (e.g. eXpresser and MiGen), to groupwork within a collaborative platform (e.g. LASAD and Metafora), attention needs to be paid to the integration of such tools in the mathematics classroom.

Through the cases of the MiGen and the Metafora projects (and their systems), and in parallel to the development of tools that support students' different collaboration patterns, we know that there is a need to encompass tools to support the teacher and move one step closer to successful integration of digital tools into the classroom. Both the MiGen and the Metafora projects foresaw the challenges for teachers in the digital era and aimed to provide assistance to teachers through the production of appropriate tools; ones that are able to provide feedback and draw the teacher's attention to prominent information regarding students' individual and collaborative work. Even though teacher support is not the focus of this section, we know that enabling the teacher to intervene when necessary is important in fostering students' collaboration. For example, when the teacher views a group's unproductive discussion, they could intervene in the argumentation space and remind students of their task (such a case was demonstrated in Fig. 4), give them hints to promote reflection on previous work, or extend their discussions to what they have learnt about collaborative learning. Our argument is that environments such as those created in the MiGen and the Metafora projects can offer the groundwork for the integration of digital technologies in the classroom by creating a collaborative workspace that can offer support to both students and teachers.

Cyberlearning: From Web 2.0 to Web 3.0

Having examined patterns of collaboration using current technologies, we now turn toward the next step in Web evolution that is the transition from the *Web* *of documents* to the *Web of data* (Berners-Lee et al., 2001). No doubt Web 2.0 technologies are enabling unprecedented growth in the volume of online content by enabling users not solely to be consumers of web content but also, if they chose, to be content producers. At the same time, the participation of people has been increasing significantly in intensity. The growth of online social networks in recent years has been phenomenal, giving rise to increased interaction among people on the Web. This includes the mechanisms of crowdsourcing that have evolved from content contribution on YouTube and collaborative knowledge construction on Wikipedia to the contribution of data and of applications that combine published data. This new era of Web 3.0 is not only that of the Web of data but also of the Web of online social networks.

This new stage of Web evolution provides significant opportunities for learning by leveraging the increasing amount of data that is getting published on the Web and by exploiting the connections that people form as part of their participation in online social networks. However, from a technological viewpoint, coping with the increased volume of content, data and people presents certain challenges when it comes of developing applications for learning. Certain questions arise, such as the following:

- How can one efficiently discover the most relevant content and data for learning on a Web that keeps increasing in size?
- How can one find the right people with whom one can collaborate and learn?
- How can one efficiently combine information that potentially comes from different data sources in order to provide new insights and new knowledge in a formal or informal learning context?
- What are the processes that transform online data to information and to knowledge and how can these processes be supported?
- What are the affordances of existing and emerging online social networks for learning on the Web?

The research community, and corresponding parts of the industry, have invested, and are continuing to invest, in technologies and operating standards in order to respond to these questions. In the emergent Web of data (or Web 3.0), a number of technologies for linked (open) data are available that enable both the publication of data on the Web in inter-operable formats (an example being RDF[5]) and the query and combination of those data (see, Bizer, 2009). The linked data movement has demonstrated on many occasions how these technologies are efficient enough to support ‘crowdsourcing’, not only of content production (as in classic Web 2.0 services) but also of linked data in a number of areas including e-government (see, Shadbolt, O’Hara, Salvadores, & Alani, 2011).

Regarding existing content, annotation (such as rating a Web resource) has always been central to efficient content discovery and aggregation. With the Web, the annotation process involves providing data about online content that will

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[5] Resource Description Framework http://www.w3.org/RDF/

describe what the content is about; this data might be contained within this content (in other words, hidden inside the source of a Web page) or in a separate document. Annotation can also be used to describe people and their learning background and objectives. In Web 2.0 applications, the process of annotation is often supported by simple tags that are searchable by users. Support for more advanced searching often requires more elaborate annotation where the tags are not just keywords but are concepts and relationships (with such concepts and relationships being rigorously described in an ontology).

In Web 3.0, semantic technologies are central in the discovery of data, content or people, and in the combination with Web resources, the provision of innovative applications. Semantic technologies make use of ontologies and annotations in order to support searching and matching as well as drawing conclusions based on available metadata. The vision of a Web in which content (documents or data) is described using ontologies can enable the realisation of a plethora of advanced applications, with such developments being part of the Semantic Web vision leading to Web 3.0 (Berners-Lee et al., 2001; Hendler, 2009).

The significance of semantic technologies for learning is being researched widely. A recent survey of the value of semantic technologies for Higher Education in the UK found that there is increasing adoption of semantic technologies in this sector of education (see, Tiropanis et al., 2009a). The most significant value of semantic technologies was identified as their support for well-formed metadata, something which can enable efficient resource annotation and discovery. In addition, semantic technologies were found valuable in providing inter-operability and support for data integration. Finally, the potential for improved data analysis and reasoning was another significant benefit. A classification of the tools and services on which the value of semantic technology was surveyed produced the following categories:

- Collaborative authoring and annotation tools (including semantic wikis and argumentation tools). Semantic technologies can help with the forming of collaboration groups based on the similarity among individuals and with efficiently discovering relevant resources or arguments. Such technologies also support argumentation and visualisation of arguments to enable critical thinking in that semantic technologies can help the learner to navigate to arguments online or to seek patterns relevant patterns of argumentation. In addition, semantic technologies can provide for precise representation of shared knowledge and recommendation of related content and people for collaborative activities related to learning.
- Searching and matching tools for discovering relevant content and individuals related to learning activities. Semantic technologies can enable searches across repositories and enable more efficient question and answer systems. At the same time, they can provide for better matching among people for learning activities (i.e. group formation). Learners can be grouped according to their background, the skills that they need to develop and their learning objectives.