

Donald A. Nield
Adrian Bejan

Convection in Porous Media

4th Edition

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Fourth Edition



Donald A. Nield
Department of Engineering Science
University of Auckland
Auckland, 1142, New Zealand
d.nield@auckland.ac.nz

Adrian Bejan
Department of Mechanical Engineering
and Materials Science
Duke University
Durham, NC 27708, USA
abejan@duke.edu

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To our wives

Rachel Nield and Mary Bejan

Our children

Cherry, Alexandra, and Peter Nield

Cristina, Teresa, and William Bejan

Our grandchildren

Michael and Rachel van der Mark

Charlotte and Susan Nield

Elizabeth and John Hayman

Preface to the Fourth Edition

Papers on convection in porous media continue to be published at a rate that is now over 250 per year. This indication of the continued importance of the subject, together with the wide acceptance of the first, second, and third editions of the book, has encouraged us to prepare an expanded fourth edition. We have retained the basic structure and most of the text of the third edition. We have not attempted to be exhaustive in our choice of references, but nevertheless there are approximately 1,750 new citations to the literature! Again, we have made an effort to highlight new conceptual developments and engineering applications.

We found that it was possible to fit most of the new material under the existing section headings. However, we now have new sections on nanofluids, carbon dioxide sequestration, and the reaction scenarios that arise in a geological context.

Once again we decided that, except for a brief mention, convection in unsaturated media was beyond the scope of this book. Also, we are aware that there are some topics in the area of hydrology that could be regarded as coming under the umbrella of the title of our book but are not treated here.

We are grateful to a large number of people for their comments on the material in previous editions. Other colleagues have continued to improve our understanding of the subject of this book in ways too numerous to mention here.

We wish to thank our employers, the University of Auckland and Duke University, for their ongoing support.

Once again we relied on the expertise and hard work of Deborah Frazee for the preparation of our manuscript.

Auckland, New Zealand
Durham, USA

Donald A. Nield
Adrian Bejan

Preface to the Third Edition

Papers on convection in porous media continue to be published at the rate what is now over 200 per year. The indication of the continued importance of the subject, together with the wide acceptance of the first and second editions of this volume, has encouraged us to prepare an expanded third edition. We have retained the basic structure and most of the text of the second edition. We have been somewhat selective in our choice of references, but nevertheless there are over 1,400 new references. Again, we have made an effort to highlight new conceptual developments and engineering applications.

We found that it was possible to fit a lot of the new material under the existing section headings. However, we now have new sections on bidisperse porous media, local thermal nonequilibrium, electrodiffusion, transverse heterogeneity in channels, thermal development of forced convection, effects of temperature-dependent viscosity, constructal multiscale flow structures, optimal spacings for plates separated by porous structures, control of convection using vertical vibration, and bioconvection.

Once again we decided that, except for a brief mention, convection in unsaturated media had to be beyond the scope of this book. Also, we are aware that there are some topics in the area of hydrology that could be regarded as coming under the umbrella of the title of our book but are not treated here.

We are grateful to a large number of people who provided us, prior to publication, with copies of their chapters or books that survey research on various topics. Other colleagues have continued to improve our understanding of the subject of this book in ways too numerous to mention here.

We wish to thank our employers, the University of Auckland and Duke University, for their ongoing support.

Once again we relied on the expertise and hard work of Linda Hayes and Deborah Frazee for the preparation of the electronic version of our manuscript.

Auckland, New Zealand
Durham, USA

Donald A. Nield
Adrian Bejan

Preface to the Second Edition

Papers on convection in porous media continue to be published at the rate of over 100 per year. This indication of the continued importance of the subject, together with the wide acceptance of the first edition, has encouraged us to prepare an expanded second edition. We have retained the basic structure and most of the text of the first edition. With space considerations in mind, we have been selective in our choice of references, but nevertheless there are over 600 new references. We also made an effort to highlight new conceptual developments and engineering applications.

In the introductory material, we judged that Chaps. 2 and 3 needed little alteration (though there is a new Sect. 2.6 on other approaches to the topic), but our improved understanding of the basic modeling of flow through a porous medium has led to a number of changes in Chap. 1, both within the old sections and by the addition of a section on turbulence in porous media and a section on fractured media, deformable media, and complex porous structures.

In Chap. 4, on forced convection, we have added major new sections on compact heat exchangers, on heatlines for visualizing convection, and on constructal tree networks for the geometric minimization of the resistance to volume-to-point flows in heterogeneous porous media.

In Chap. 5 (external natural convection) there is a substantial amount of new material inserted in the existing sections. In Chaps. 6 and 7, on internal natural convection, we now have included descriptions of the effects of a magnetic field and rotation, and there are new sections on periodic heating and on sources in confined or partly confined regions; the latter is a reflection of the current interest in the problem of nuclear waste disposal. In Chap. 8, on mixed convection, there are no new sections, but in a new subsection we have given some prominence to the unified theory that has been developed for boundary layer situations. In Chap. 9, on double-diffusive convection (heat and mass transfer) there is a new section on convection produced by inclined gradients, a topic that has also been given wider coverage in the related section in Chap. 7.

In Chap. 10, which deals with convection with change of phase, we have a new subsection on the solidification of binary alloys, a research area that has blossomed

in the last decade. We also have a new section on spaces filled with fluid and fibers coated with a phase-change material. In the first edition we had little to say about two-phase flow, despite its importance in geothermal and other contexts. We now have included a substantial discussion on this topic, which we have placed at the end of Chap. 11 (geophysical aspects). Once again we decided that, except for a brief mention, convection in unsaturated media had to be beyond the scope of this book.

D.A.N. again enjoyed the hospitality of the Department of Mechanical Engineering and Materials Science at Duke University while on Research and Study Leave from the University of Auckland, and both of those institutions again provided financial support.

We are grateful for comments from Graham Weir and Roger Young on a draft of Sect. 11.9, a topic on which we had much to learn. We also are grateful to a large number of people who provided us with preprints of their papers prior to publication. Other colleagues have improved our understanding of the subject of this book in ways too numerous to mention here.

Once again we relied on the expertise and hard work of Linda Hayes for the preparation of the electronic version of our manuscript, and again the staff at the Engineering Library of Duke University made our search of the literature an enjoyable experience.

Auckland, New Zealand
Durham, USA

Donald A. Nield
Adrian Bejan

Preface to the First Edition

In this book we have tried to provide a user-friendly introduction to the topic of convection in porous media. We have assumed that the reader is conversant with the basic elements of fluid mechanics and heat transfer, but otherwise the book is self-contained. Only routine classic mathematics is employed. We hope that the book will be useful both as a review (for reference) and as a tutorial work (suitable as a textbook in a graduate course or seminar).

This book brings into perspective the voluminous research that has been performed during the last two decades. The field recently has exploded because of worldwide concern with issues such as energy self-sufficiency and pollution of the environment. Areas of application include the insulation of buildings and equipment, energy storage and recovery, geothermal reservoirs, nuclear waste disposal, chemical reactor engineering, and the storage of heat-generating materials such as grain and coal. Geophysical applications range from the flow of groundwater around hot intrusions to the stability of snow against avalanches.

We believe that this book is timely because the subject is now mature in the sense that there is a corpus of material that is unlikely to require major revision in the future. As the reader will find, the relations for heat transfer coefficients and flow parameters for the case of saturated media are now known well enough for engineering design purposes. There is a sound basis of underlying theory that has been validated by experiment. At the same time there are outstanding problems in the cases of unsaturated media and multiphase flow in heterogeneous media, which are relevant to such topics as the drying of porous materials and enhanced oil recovery.

The sheer bulk of the available material has limited the scope of this book. It has forced us to omit a discussion of convection in unsaturated media and also of geothermal reservoir modeling; references to reviews of these topics are given. We also have excluded mention of several hundred additional papers, including some of our own. We have emphasized reports of experimental work, which are in relatively short supply (and in some areas are still lacking). We also have emphasized simple analysis where this illuminates the physics involved. The excluded material includes some good early work, which has now been superseded, and some recent

numerical work involving complex geometry. Also excluded are papers involving the additional effects of rotation or magnetic fields; we know of no reported experimental work or significant applications of these extensions. We regret that our survey could not be exhaustive, but we believe that this book gives a good picture of the current state of research in this field.

The first three chapters provide the background for the rest of the book. Chapters 4 through 8 form the core material on thermal convection. Our original plan, which was to separate foundational material from applications, proved to be impractical, and these chapters are organized according to geometry and the form of heating. Chapter 9 deals with combined heat and mass transfer and Chap. 10 with convection coupled with change of phase. Geophysical themes involve additional physical processes and have given rise to additional theoretical investigations; these are discussed in Chap. 11.

This book was written while D.A.N. was enjoying the hospitality of the Department of Mechanical Engineering and Materials Science at Duke University, while on Research and Study Leave from the University of Auckland. Financial support for this leave was provided by the University of Auckland, Duke University, and the United States—New Zealand Cooperative Science Program. We are particularly grateful to Dean Earl H. Dowell and Prof. Robert M. Hochmuth, both of Duke University, for their help in making this book project possible.

Linda Hayes did all the work of converting our rough handwritten notes into the current high-quality version on computer disk. She did this most efficiently and with tremendous understanding (i.e., patience!) for the many instances in which we changed our minds and modified the manuscript.

At various stages in the preparation of the manuscript and the figures we were assisted by Linda Hayes, Kathy Vickers, Jong S. Lim, Jose L. Lage, and Laurens Howle. Eric Smith and his team at the Engineering Library of Duke University went to great lengths to make our literature search easier. We are very grateful for all the assistance we have received.

Auckland, New Zealand
Durham, USA

Donald A. Nield
Adrian Bejan

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Nomenclature

B	Transition number for electrodiffusion, Eq. (3.95)
Be	Bejan number, Eq. (4.145)
Br	Brinkman number, Sect. 2.2.2
C	Concentration
c	Specific heat
c_a	Acceleration coefficient
c_F	Forchheimer coefficient
c_P	Specific heat at constant pressure
D	Diameter
D	d/dz
D_m	Solute diffusivity
D_{CT}	Thermodiffusion coefficient (Soret coefficient times D_m)
Da	Darcy number
d_p	Particle diameter
Ec	Eckert number, Sect. 2.2.2
g	Gravitational acceleration
Ge	Gebhart number, Sect. 2.2.2
H	Vertical dimension
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors
Ja	Jakob number
K	Permeability
k	Thermal conductivity
k_m	Thermal conductivity of the porous medium
L	Horizontal dimension
Le	Lewis number
N	Buoyancy ratio
Nu	Nusselt number
P	Pressure
Pe	Péclet number
Pr	Prandtl number

q', q'', q'''	Heat transfer rate per unit length, area, volume, respectively
r. e. v	Representative elementary volume
Ra	Thermal Rayleigh (Rayleigh-Darcy) number
Ra_D	Solutal Rayleigh number
Re	Reynolds number
r	Radial coordinate
Sc, Sh	Jakob numbers
Ste	Stefan number
s	Time constant
T	Temperature
t	Time
$\mathbf{V}$	Intrinsic velocity
$\mathbf{v}$	(u, v, w) , seepage velocity
x, y, z	Position coordinates
α	Nondimensional wavenumber
α_{BJ}	Beavers-Joseph coefficient
α_m	Thermal diffusivity of the porous medium
β	Thermal expansion coefficient
β_C	Concentration expansion coefficient
δ	Boundary layer thickness
ζ	Inter-phase momentum transfer coefficient
η	Similarity variable
θ	Angle
θ	Temperature perturbation amplitude
λ	Exponent in power law variation
μ	Dynamic viscosity
$\tilde{\mu}$	Effective viscosity (Brinkman)
ν	Kinematic viscosity
ρ	Density
σ	Heat capacity ratio, $\sigma = \varphi + (1 - \varphi)(\rho c)_s/(\rho c_P)_f$
τ	Nondimensional time
φ	Porosity
φ	Angle
ψ	Streamfunction
ω	Frequency
χ	$c_F K^{1/2}$

Subscripts

b	Basic state
b	Bulk
C	Concentration
c	Critical

D	Parameter based on length D
e	Effective
eff	Effective
f	Fluid
g	Gas
H	Horizontal
L	Parameter based on length L
l	Liquid
m	Porous medium
p	Particle
ref	Reference
s	Solid
V	Vertical
w	Wall
x	Parameter based on length x
0	Reference
∞	Far field

Superscripts

$'$	Perturbation
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Chapter 1

Mechanics of Fluid Flow Through a Porous Medium

1.1 Introduction

By a porous medium, we mean a material consisting of a solid matrix with an interconnected void. We suppose that the solid matrix is either rigid (the usual situation) or it undergoes small deformation. The interconnectedness of the void (the pores) allows the flow of one or more fluids through the material. In the simplest situation (“single-phase flow”), the void is saturated by a single fluid. In “two-phase flow,” a liquid and a gas share the void space.

In a natural porous medium, the distribution of pores with respect to shape and size is irregular. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood, and the human lung (Fig. 1.1 and Table 1.1). Man-made porous media include ceramics, composite materials, and high-porosity metallic foams. On the pore scale (the microscopic scale), the flow quantities (velocity, pressure, etc.) will be clearly irregular. But in typical experiments, the quantities of interest are measured over areas that cross many pores, and such space-averaged (macroscopic) quantities change in a regular manner with respect to space and time, and hence are amenable to theoretical treatment.

How we treat a flow through a porous structure is largely a question of distance—the distance between the problem solver and the actual flow structure (Bejan 2004a, b). When the distance is short, the observer sees only one or two channels, or one or two open or closed cavities. In this case, it is possible to use conventional fluid mechanics and convective heat transfer to describe what happens at every point of the fluid- and solid-filled spaces. When the distance is large so that there are many channels and cavities in the problem solver’s field of vision, the complications of the flow paths rule out the conventional approach. In this limit, volume-averaging and global measurements (e.g., permeability, conductivity) are useful in describing the flow and in simplifying the description. As engineers focus more and more on designed porous media at decreasing pore scales, the problems tend to fall between the extremes noted above. In this intermediate range, the challenge is not only to describe *coarse* porous structures, but also to *optimize*

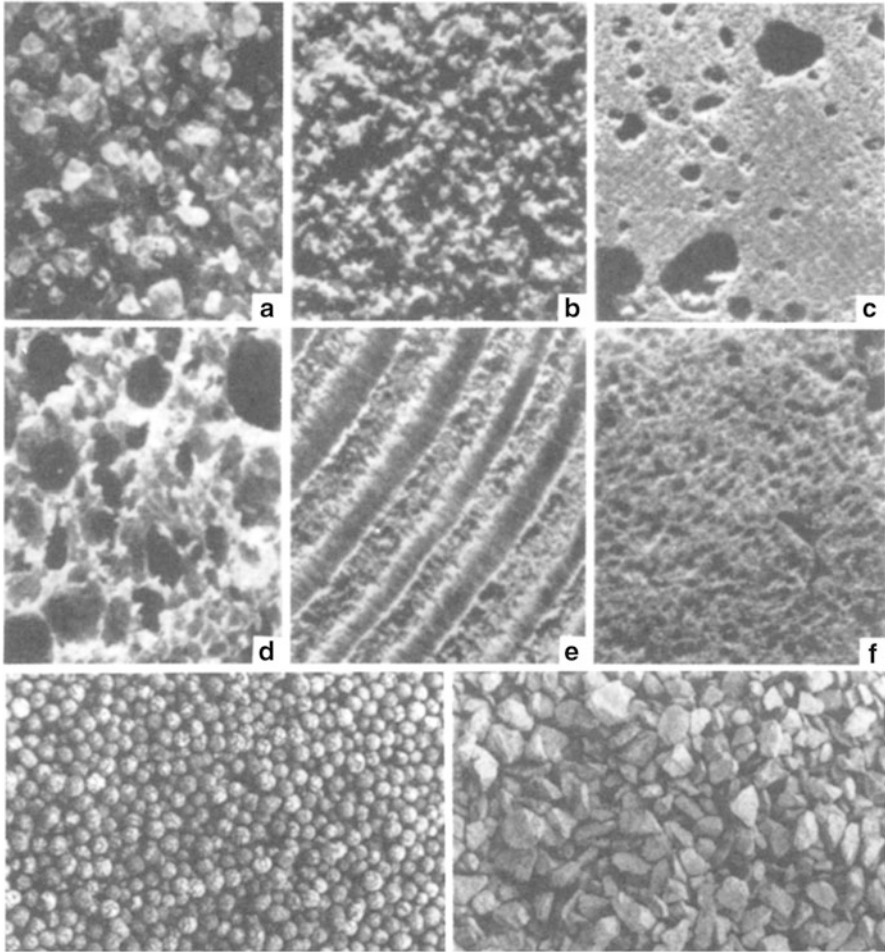


Fig. 1.1 *Top*: Examples of natural porous materials: (a) beach sand, (b) sandstone, (c) limestone, (d) rye bread, (e) wood, and (f) human lung (Collins 1961, with permission from Van Nostrand Reinhold). *Bottom*: Granular porous materials used in the construction industry, 0.5-cm-diameter Liapor® spheres (*left*) and 1-cm-size crushed limestone (*right*) (Bejan 1984)

flow elements, and to *assemble* them. The resulting flow structures are *designed* porous media (see Bejan et al. 2004; Bejan 2004b).

The usual way of deriving the laws governing the macroscopic variables is to begin with the standard equations obeyed by the fluid and to obtain the macroscopic equations by averaging over volumes or areas containing many pores. There are two ways to do the averaging: spatial and statistical. In the spatial approach, a macroscopic variable is defined as an appropriate mean over a sufficiently large *representative elementary volume* (r.e.v.); this operation yields the value of that

Table 1.1 Properties of common porous materials (based on data compiled by Scheidegger 1974; Bejan and Lage 1991)

Material	Porosity φ	Permeability K (cm ²)	Surface per unit volume (cm ⁻¹)
Agar-agar		2×10^{-10} to 4.4×10^{-9}	
Black slate powder	0.57–0.66	4.9×10^{-10} to 1.2×10^{-9}	7×10^3 to 8.9×10^3
Brick	0.12–0.34	4.8×10^{-11} to 2.2×10^{-9}	
Catalyst (Fischer-Tropsch, granules only)	0.45		5.6×10^5
Cigarette		1.1×10^{-5}	
Cigarette filters	0.17–0.49		
Coal	0.02–0.12		
Concrete (ordinary mixes)	0.1		
Concrete (bituminous)		1×10^{-9} to 2.3×10^{-7}	
Copper powder (hot- compacted)	0.09–0.34	3.3×10^{-6} to 1.5×10^{-5}	
Cork board		2.4×10^{-7} to 5.1×10^{-7}	
Fiberglass	0.88–0.93		560–770
Granular crushed rock	0.45		
Hair (on mammals)	0.95–0.99		
Hair felt		8.3×10^{-6} to 1.2×10^{-5}	
Leather	0.56–0.59	9.5×10^{-10} to 1.2×10^{-9}	1.2×10^4 to 1.6×10^4
Limestone (dolomite)	0.04–0.10	2×10^{-11} to 4.5×10^{-10}	
Sand	0.37–0.50	2×10^{-7} to 1.8×10^{-6}	150–220
Sandstone (“oil sand”)	0.08–0.38	5×10^{-12} to 3×10^{-8}	
Silica grains	0.65		
Silica powder	0.37–0.49	1.3×10^{-10} to 5.1×10^{-10}	6.8×10^3 to 8.9×10^3
Soil	0.43–0.54	2.9×10^{-9} to 1.4×10^{-7}	
Spherical packings (well shaken)	0.36–0.43		
Wire crimps	0.68–0.76	3.8×10^{-5} to 1×10^{-4}	29–40

variable at the centroid of the r.e.v. It is assumed that the result is independent of the size of the representative elementary volume. The length scale of the r.e.v. is much larger than the pore scale, but considerably smaller than the length scale of the macroscopic flow domain (Fig. 1.2).

In the statistical approach, the averaging is over an ensemble of possible pore structures that are macroscopically equivalent. A difficulty is that usually the statistical information about the ensemble has to be based on a single sample, and this is possible only if statistical homogeneity (stationarity) is assumed.

If one is concerned only with deriving relationships between the space-averaged quantities and is not concerned about their fluctuation, then the results obtained by using the two approaches are essentially the same. Thus, in this situation, one might as well use the simpler approach, namely, the one based on the r.e.v. An example of its use is given in Sect. 3.5. This approach is discussed at length by Bear and