

Astrophysics and Space Science Library 389

Mikhail Ya Marov
Aleksander V. Kolesnichenko

Turbulence and Self-Organization

Modeling Astrophysical Objects

AS
SL

 Springer

Turbulence and Self-Organization

Astrophysics And Space Science Library

EDITORIAL BOARD

Chairman

W.B. BURTON, *National Radio Astronomy Observatory, Charlottesville, Virginia, U.S.A. (bburton@nrao.edu); University of Leiden, The Netherlands (burton@strw.leidenuniv.nl)*

F. BERTOLA, *University of Padua, Italy*

C.J. CESARSKY, *Commission for Atomic Energy, Saclay, France*

P. EHRENFREUND, *Leiden University, The Netherlands*

O. ENGVOLD, *University of Oslo, Norway*

A. HECK, *Strasbourg Astronomical Observatory, France*

E.P.J. VAN DEN HEUVEL, *University of Amsterdam, The Netherlands*

V.M. KASPI, *McGill University, Montreal, Canada*

J.M.E. KUIJPERS, *University of Nijmegen, The Netherlands*

H. VAN DER LAAN, *University of Utrecht, The Netherlands*

P.G. MURDIN, *Institute of Astronomy, Cambridge, UK*

B.V. SOMOV, *Astronomical Institute, Moscow State University, Russia*

R.A. SUNYAEV, *Space Research Institute, Moscow, Russia*

For further volumes:

<http://www.springer.com/series/5664>

Mikhail Ya Marov • Aleksander V. Kolesnichenko

Turbulence and Self-Organization

Modeling Astrophysical Objects

 Springer

Mikhail Ya Marov
Department of Planetary Sciences
and Cosmochemistry
V.I. Vernadsky Institute of Geochemistry
and Analytical Chemistry
Russian Academy of Sciences
Moscow, Russia

Aleksander V. Kolesnichenko
Department of Planetary Science and Aeronomy
Keldysh Institute of Applied Mathematics
Russian Academy of Sciences
Moscow, Russia

ISSN 0067-0057

ISBN 978-1-4614-5154-9

ISBN 978-1-4614-5155-6 (eBook)

DOI 10.1007/978-1-4614-5155-6

Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2012948282

© Springer Science+Business Media New York 2013

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

*Dedicated to Academician¹ Leonid
Ivanovich Sedov*

¹The title *Academician* denotes a Full Member of an art, literary, or scientific academy. In this case, it refers to the Academy of Sciences of the USSR.

God the Almighty on the throne with a threateningly raised hand:

—Noli turba circulos meos!²

Underneath there are myriads of stars, worlds moving over spheres. And Prometheus stretching his muscles to break the circle of human being. And someone who fell from the circle and is plunging into chaos.

And the threatening, warning Finger of the Almighty:

—Noli turba circulos meos!

From the book by Leonid Andreev S.O.S.

In mechanics, the meaning of the modeling of real bodies and phenomena using objects and processes invented in science is clear to everybody and we all understand well the meaning of our actions in these cases. It is inappropriate to say that “science discovered” an ideal incompressible fluid, or a perfectly rigid body, or a perfectly elastic body. We understand that these objects are absolutely necessary and useful scientific inventions.

***From the book by L.I. Sedov
“Reflections on Science and Scientists”.***

Turbulent motion appears as a very complex motion in open systems emerging from a less ordered motion—“physical chaos”... The transition from a laminar state to a turbulent one may be considered to be an example of self-organization in a nonlinear open system.

***From the book by Yu.L. Klimontovich
“An Introduction to the Physics of Open Systems”.***

²“Don’t disturb my circles”—Archimedes’ dictum.

Preface

There are notable words in the book of the renowned French scientist David Ruelle “Hasard et Chaos” (chance and chaos), “Mechanics was born from the desire to explain the world”. The validity of this utterance, which refers to the history of science, acquires a new deep meaning at the present stage of the development of natural sciences, where not only the symbiosis and close intertwining of various disciplines occur but also the fundamentals of the cognition of natural laws are revealed, whose carrier is traditionally served by mechanics, along with its individual branches and numerous applications. The classical conservation laws and the differential equations of motion, including the motion of celestial bodies, the equations of continuum mechanics, dynamics of rarefied multicomponent gases, and physical kinetics formulated on their basis constitute the foundation for developing various physical, geophysical, and astrophysical models that are designed not only to explain the surrounding world, but also to understand the origins of its birth and evolution. The avalanche-like accumulation of knowledge about the Universe (or in modern notion, Multiverse) based on enormous observational data sets and the continuously improving methods of mathematical modeling, which use high-performance computing systems, open new vast horizons for the cognition of nature.

The rapidly expanding views about the surrounding regions of space near the Earth and far beyond, which is attributable primarily to enormous progress in astronomy and space research, led to a deeper penetration into the physical essence of the processes and phenomena in various natural and cosmic media in diverse states of their constituent matter. This inspired increasingly complicated mathematical models for these media, which were in turn made possible by immense breakthroughs achieved in creating powerful computing systems – their architecture, performance, and software. Truly boundless opportunities open up on this path, because linked to this capacity of the computing systems is the prospect of formulating and solving complex multidimensional nonstationary geo- and astrophysical problems and analyzing evolutionary processes based on large-scale numerical experiments.

Turbulence is at the same time a very common and a complex natural phenomena associated with the emergence and development of an enormous number of vortices on all possible scales (organized vortex structures) under certain regimes of fluid motion in an essentially nonlinear hydrodynamic system. When the stability of a laminar flow defined by the critical Reynolds number is lost, an unsteady fluctuating flow emerges in the hydrodynamic system. A continuous distribution of the velocity fluctuations and other thermohydrodynamic parameters is produced in this flow through the stretching of vortices in a range of smallest wavelengths, determined by dissipative (viscous) forces, to the longest ones, determined by the flow boundaries. The conditions for emerging vorticity and structuring of the developed turbulence are influenced by the physical properties of the medium, such as the molecular transport coefficients to which the energy dissipation processes in a turbulent flow are related, as well as by the conditions at the boundary, where thin boundary vortex layers are observed, whose instability manifests itself in the generation of vortex tubes. Turbulization leads to a rapid mixing of particles in a continuous medium and to an increase in the efficiency of mass, momentum, and heat transfer; in multiphase multicomponent media, it also contributes to the acceleration of phase transitions and chemical reactions. As the knowledge about the various natural objects in which turbulence plays a significant and, in many cases, crucial role is accumulated, modeling this phenomenon and related hydrodynamic effects acquires a key significance. At the same time, a direct numerical simulation of turbulent flows involves great mathematical difficulties, while constructing a general theory of structured turbulence in a compressible fluid is unlikely to be possible, because the mechanisms according to which different-scale vortex structures interact are extremely complex.

This monograph is based on our studies of problems that arise in phenomenological modeling of developed turbulence in multicomponent mixtures of reacting gases and heterogeneous gas-dust media. The first steps of these studies are studying the natural environment and evolution mechanisms of the Earth and planets in the Solar System, protoplanetary gas-dust accretion disks, and turbulent heat and mass transport in the upper atmospheres of planets (the tenuous gaseous envelopes of celestial bodies lying in the boundary regions between the dense atmospheric layers and circumplanetary space). We are concerned here with investigating turbulent flows in gas and gas-dust media with complicated physical-chemical characteristics whose mathematical modeling requires taking into account the flow compressibility, the variability of thermophysical properties, heat and mass transfer, chemical reactions, and radiation, and the influence of gravitational and electromagnetic forces. The additional mechanical effects that emerge under these circumstances generally do not allow one to employ the results obtained in terms of the traditional description of turbulent homogeneous incompressible fluid flows that are used, for example, in meteorology. Therefore, to study natural media of this type, we need to develop new approaches to model turbulence that adequately describe the hydrodynamic motions of compressible mixtures, transport processes, and chemical kinetics in a fluctuating multiphase multicomponent continuum. Since the hydrodynamic and physical-chemical pattern of

turbulent motion is complex, the theoretical approaches to solving this problem must be “semiempirical” in character.

We focus on the thermodynamic construction of continuum models for turbulized natural media in outer space, which are the basis for solving the formation and evolution problems of various astrophysical and geophysical objects. These primarily include turbulent motion models of multicomponent chemically active gases with diffusion, heat transfer, viscosity, and radiation processes; turbulent motion models of gas suspensions and various types of heterogeneous media with phase transitions; and models of structured turbulent flows as well as turbulent flows interacting with an electromagnetic field.

The book is devoted to the topical problem of self-organization in developed turbulent flows, which serves as a reflection of the most general concept of the relationship between order and chaos in natural processes. Submitting our work for the judgment of specialists in various areas of knowledge, we realize that the approach we develop is in some aspects unusual for mechanicians guided by “classical” yardsticks. For this reason, the history of a new understanding of turbulence is of interest. The question about the relative degree of order of laminar and turbulent motions was probably first discussed publicly during the “Synergetics 83” International Conference in the town of Pushchino near Moscow. In his report, Yu. L. Klimontovich formulated a general criterion, the so-called “*S*-theorem” (to which we will return later), which characterizes the relative degree of order of the states for open dissipative systems. This criterion was used by the speaker for a quantitative proof of the assertion that a turbulent motion is more ordered than a laminar one. There were very few supporters of this viewpoint in the hall. I. Prigogine, H. Haken, and B. Ebeling were among them. Curiously, the renowned hydromechanician G.I. Barenblatt was openly indignant during this report, saying: “All mechanicians know excellently well that a turbulent motion is more chaotic”.

Nevertheless, contrary to the traditional viewpoint of mechanicians on turbulence, a large number of various vortex coherent structures (CSs) have been discovered and their topological characteristics have been firmly established in the last decade owing to the progressive development of the methods for a visual observation of turbulized fluid flows. As examples, we can name Taylor vortices, turbulent spots, vortex rings, vortex balls, hairpin-like vortices, burstings, vortex spirals, streaks, Brawn-Thomas structures, mushroom-shaped vortices, etc. The frequency of occurrence of a particular structure depends on the type of flow (a boundary layer, a mixing layer, a jet, etc.), geometry, and the regime of turbulized fluid motion. It is important to ascertain how these coherent structures can emerge. Nonlinear nonequilibrium thermodynamics, whose principles are widely used in the monograph, partly answers this question.

Basically, the distinctive feature of our studies consists of the proposed stochastic-thermodynamic approach to constructing semiempirical turbulence models in reacting multicomponent gases and gas-dust media and structured turbulence in a homogeneous fluid. These studies are oriented mainly toward solving a number of complex present-day problems in astrophysics and geophysics based on the methods of continuum mechanics. The developed approaches have a direct

bearing on modeling the mechanisms that shape the properties of astrophysical and geophysical objects at various stages of their evolution; on investigating the problems of stellar and planetary cosmogony, including the formation of protoplanetary gas-dust accretion disks and the subsequent accumulation of planetary systems; on the early evolutionary stages of planets; on the formation and evolution of planetary and cometary atmospheres; and on the problems of ecology associated with the diffusion of pollutants and natural environmental protection that attract increasingly greater attention.

The book has nine chapters. Some questions are concretized in the appendix. We have attempted to make the chapters of the book independent of one another, where possible, although they are naturally unified by a common conceptual orientation.

In the *first chapter*, which is the introductory one, we briefly consider the properties of turbulent flows, the elements of stochastic nonlinear dynamics, and the relationship between order and chaos, including the synergetic aspects of the formation of ordered structures. The basic views of turbulence as a dynamical system are discussed. Since the emphasis in the monograph is on turbulent flows in natural inhomogeneous media with a variable density, the problem of modeling these media acquires entirely new facets. The classical views of turbulent motions in an incompressible fluid are intertwined with other areas of mechanics that combine a mixture of hydromechanics, thermodynamics, the theory of radiative transfer, and the kinetics of chemical reactions. In addition to velocity, density, and temperature fluctuations, the concentration of individual chemical mixture components acquires a considerable significance. As a result, we face one of the most complex problems in the mechanics of turbulized media, which requires semiempirical modeling of interrelated hydrodynamic, physical-chemical, and radiative processes and phenomena in a turbulent flow.

The possibility of emerging order in a cosmic medium and during the evolution of cosmic objects is discussed from the standpoint of stochastic dynamics and the theory of self-organization in open nonlinear dissipative systems. The questions of dynamical astronomy, the dynamics of the Solar System, the nature of galaxies, stars, planets, and small bodies in the Solar System, the questions of stellar-planetary evolution, including the formation of protoplanetary accretion disks, and some problems of the structure and evolution of the Universe are considered as examples. This discussion goes far beyond the narrower problems of turbulence that the monograph is devoted to. This serves, on the one hand, the purpose of giving examples of order based on present views of the cosmos and the nature of its inhabitants and, on the other hand, the purpose of reflecting the generality of the concept of macromolecular structure formation in cosmic and other natural media.

The *second chapter* is devoted to formulating general mass, momentum, and energy balance laws in a multicomponent chemically active gas mixture. It can be also regarded as auxiliary and is used as the basis for a more detailed consideration of the turbulence problems suggested in the subsequent chapters of the monograph. As is well known, the most complete and rigorous mathematical description of a multicomponent medium in a regular (laminar) flow can be given in terms of the kinetic theory of multicomponent mixtures of polyatomic ionized gases.

The system of generalized integro-differential Boltzmann equations for the distribution functions of particles of each type in the mixture (with the right-hand parts containing the collision and reaction integrals) supplemented by the radiative transfer equation and the Maxwell equations for electromagnetic fields serves as the basic one. This approach was developed in our monograph (Marov and Kolesnichenko 1987), where a generalized Chapman-Enskog method was used to derive the system of differential gas-kinetic equations for a reacting mixture. In addition, from the viewpoint of macroscopic properties, such a multicomponent gas mixture (e.g., the upper atmosphere of a planet) can be considered as a continuous medium and the methods of continuum mechanics for mixtures can be used to describe it adequately. Based on the principles of nonequilibrium thermodynamics, these methods allow one to obtain the system of hydrodynamic equations with all necessary closing relations. This phenomenological approach also allows semiempirical models of turbulent flows in reacting gas media to be developed by using extended irreversible thermodynamics.

The formalism of classical nonequilibrium thermodynamics is used to study the mass, momentum, and energy transfer processes. As is well known, a wide class of nonequilibrium transport processes in gases can be described by means of this formalism fully in accord with the experimental data. In particular, the technique for the thermodynamic derivation of generalized Stefan-Maxwell relations for multicomponent diffusion proposed here allows one to obtain a number of algebraic relations for the transport coefficients that relate, for example, the thermal diffusion ratios to the thermal diffusion and multicomponent diffusion coefficients, the true and partial thermal conductivities, and the multicomponent and binary diffusion coefficients. All these hydrodynamically derived relations agree completely with the results of the gas-kinetic theory for multicomponent mixtures of monatomic gases obtained in the second approximation of the Chapman-Enskog method. However, in contrast to this, the thermodynamic approach is not related to the postulation of a specific microscopic model for the interaction of molecules in the natural medium that is investigated, which is indicative of its universality.

In the *third chapter*, we derive a closed system of averaged hydrodynamic equations for a turbulized multicomponent chemically active gas mixture designed to describe a wide class of turbulent motions and physical-chemical processes in natural media. We analyze the physical meaning of the individual terms in these equations, including the energy transition rates between various energy balance components. Here, we systematically use the weighted-mean Favre averaging, which allows the form and analysis of the averaged equations of motion for chemically active gases with variable thermophysical properties to be simplified considerably, along with the traditional probability-theoretic averaging of fluctuating thermohydrodynamic parameters. Assessing the status of the first-order closure problem on the whole, it should be recognized that no general phenomenological theory of turbulent heat conduction and turbulent diffusion for multicomponent reacting mixtures has existed until now. Therefore, we consider in this chapter a thermodynamic approach to the closure of the averaged hydrodynamic equations for a mixture at the level of first-order turbulence models based on

the methods of extended irreversible thermodynamics. Special attention is paid to deriving closing gradient relations for the Reynolds turbulent stress tensor and the turbulent heat and diffusion fluxes in a multicomponent mixture by thermodynamic methods. The Onsager formalism allows one to obtain the most general structure of these relations, including those in the form of generalized Stefan-Maxwell relations for multicomponent turbulent diffusion. At the closure level under consideration, these relations describe most comprehensively the turbulent heat and mass transport in a multicomponent medium. The turbulent exchange coefficients are determined with classical models dating back to Prandtl, Taylor, and Karman as well as with the more recent second-order closure models that are based on the differential balance equations for the turbulent energy and integral turbulence scale. For the convenience of the reader, all calculations are performed comprehensively and can be traced in all details.

In the *fourth chapter*, we consider the problem of constructing semiempirical second-approximation turbulence models for a multicomponent chemically active gas mixture with a variable density and variable thermophysical properties. We derive closing differential transfer equations for the various one-point (one-time) second-correlation moments of the fluctuating thermohydrodynamic parameters that appear in the averaged hydrodynamic equations of mean motion for a reacting mixture. The closure problem for a chemically active medium is generally highly complicated because one needs to average the nonlinear “source terms” of substance production in chemical reactions with an exponential behavior. Therefore, we propose an original procedure for averaging the rates of chemical reactions of any order and outline a scheme for semiempirical modeling of these additional correlations. Approximating expressions that contain universal empirical coefficients that need not be chosen again for each new flow are used in modeling the third-order correlations in the transfer equations. We emphasize that although these additional equations are semiempirical, the invariant models of fully developed turbulence in chemically active gases based on them are fairly flexible. In particular, they allow taking into account the influence of the mechanisms of convection, diffusion, formation, redistribution, and dissipation of stochastic turbulent characteristics for the field of fluctuating thermohydrodynamic parameters on the spatiotemporal distribution of averaged thermohydrodynamic parameters for the medium. Basically, the approach we developed is widely used in numerical simulations of real reacting turbulized fluid flows with a significant influence of the flow prehistory on the turbulence characteristics at a point. On the other hand, it is used to derive more accurate algebraic relations for the turbulent transport coefficients in multicomponent shear mixture flows (and as applied to the specificity of modeling natural media), which is embodied in this chapter of the book.

The *fifth chapter* describes a phenomenological model for the developed turbulence in a compressible homogeneous medium by taking into account nonlinear cooperative processes. The primary concept is to represent a turbulized fluid motion as a thermodynamic system consisting of two continua, the subsystem of averaged motion and the subsystem of turbulent chaos. This in turn is considered as a conglomerate of vortex structures of various spatiotemporal scales. We develop

the ideas of a stationary nonequilibrium state of the dissipatively active subsystem of turbulent chaos that emerges due to the influx of negentropy from the external medium (the subsystem of averaged motion) and the appearance of relatively stable coherent vortex structures in the system when the flow control parameters are varied. This allows some of the turbulent field rearrangement processes to be considered as self-organization processes in an open system. The methods of the stochastic theory of irreversible processes and extended irreversible thermodynamics are used to derive the defining relations for the turbulent fluxes and forces that close the system of averaged hydrodynamic equations and describe the transport and self-organization processes in the stationary nonequilibrium case with completeness sufficient for practice.

Our original approach to the stochastic-thermohydrodynamic modeling of the subsystem of turbulent chaos is based on introducing a set of random variables into the model—fluctuating internal coordinates (such as the turbulent energy dissipation rates, the intrinsic vorticities of the velocity field fluctuations that refer to mesoscale vortex structures, etc.) that characterize the structure and temporal evolution of the fluctuating field of hydrodynamic flow parameters. This makes it possible to model the Richardson-Kolmogorov cascade process and to derive the kinetic Fokker-Planck-Kolmogorov (FPK) equations, which describe the evolution of the probability density function for small-scale turbulence characteristics with thermodynamic methods. These equations are a basis for analyzing the Markovian diffusion processes of the transition from one stationary-nonequilibrium state to another in the space of internal coordinates through a successive loss of stability (and increase in supercriticality) by the subsystem of turbulent chaos that is far from complete chaos of thermodynamic equilibrium. These transitions can be described as nonequilibrium “second-order phase transitions” in a vortex continuum, causing the internal coordinates at bifurcation points to change abruptly.

We also consider an alternative approach to investigating the mechanisms of such a transition. This is based on stochastic Langevin equations, which are closely related to the derived kinetic FPK equations. We analyze a cardinal problem of the approach being developed—the possibility that asymptotically stable stationary-nonequilibrium states may occur in the subsystem of turbulent chaos. We propose a nonequilibrium thermodynamic potential for the stochastic internal coordinates of turbulent chaos that generalizes the well-known Boltzmann-Planck relation for equilibrium states to stationary nonequilibrium states of the ensemble representing chaos and show that this potential is the Lyapunov function for stationary nonequilibrium states of the ensemble that corresponds to the subsystem of turbulent chaos.

The third section of Chap. 5 is devoted to the thermodynamic derivation of generalized fractional FPK equations that describe based on fractional dynamics how the internal coordinates of the subsystem of turbulent chaos evolve. Introducing fractional time derivatives into the kinetic FPK equation allows one to include the effects of intermittency in time in a unified mathematical formalism. These effects are associated with turbulent bursts against the background of less intense low-frequency background turbulence oscillations.

In the *sixth chapter*, we prove the so-called H -theorem for the Kullback entropy. This postulates that any initial probability distribution for the internal coordinates of the subsystem of turbulent chaos under known assumptions asymptotically approaches a certain stationary state after a sufficiently long time. Here, we demonstrate that self-organization (i.e., the emergence of ordered dissipative structures with a lower symmetry than that of the initial state) is in principle possible in the thermodynamically open subsystem of turbulent chaos in due course of the temporal evolution of the quasi-equilibrium vortex system. This can happen when coherent structures can be generated that are associated with the effect of multiplicative-noise-induced non-equilibrium phase transitions in the subsystem of chaos. We show that if the multiplicative noise of chaos is intense enough, the extrema of the probability density that describe the stationary behavior of a stochastic vortex system differ significantly in both number and position from the stationary states that correspond to a deterministic system. Moreover, multiplicative noise can generate new stationary states, thereby changing the properties (in particular, the bifurcation diagrams) of the local stability of chaos themselves: the transition points can be displaced under the influence of intense noise in a turbulent fluid.

Based on the general concept of generating coherent vortex structures in the thermodynamically open subsystem of turbulent chaos (due to nonequilibrium phase transitions induced by the multiplicative noise of chaos), we also consider in this Chap. one of the specific mechanisms for the formation and evolution of mesoscale vortex structures. These are associated with the phase-frequency synchronization of the self-oscillations of those internal coordinates that refer to the coherent component of chaos. In addition, we study some of the scenarios for the dynamical influence that the incoherent component (fine-grained fluctuating field) of turbulent chaos has on the formation and evolution of vortex structures. These transitions are shown to interrelate with the self-organization of clusters with a lower symmetry than that of the initial state. We conclude that while the growth in size of solid particles during collisions in classical turbulence is hampered, they can coalesce and enlarge within such dissipative ordered structures. In other words, the emergence of vortex clusters facilitates the solution of a key problem of the evolution of accretion disks—the problem of solid particle enlargement through collisions at relatively low velocities. This encounters obvious difficulties in attempting to reproduce similar processes in laboratory experiments.

In the *seventh chapter*, our attention is focused on one of the fundamental problems in astrophysics—the formation of protoplanetary accretion disks around late-type stars, a special case of which is the origin of the Solar System. Particular attention is given to developing a semiempirical approach to modeling heterogeneous turbulence in the accretion disk that surrounded the proto-Sun at the early stage of its existence with the goal of reducing the number of assumptions in the models used. We formulate a complete system of equations of two-phase multi-component mechanics by taking into account the relative motion of the phases, coagulation processes, phase transitions, chemical reactions, and radiation. Basically, it is designed for schematized formulations and numerical solution of specific model problems on mutually consistent modeling of the structure, dynamics,

thermal regime, and chemical composition of the circumsolar disk at various stages of its evolution. The processes in the disk medium in the presence of the developed turbulent motions of a coagulating gas suspension are addressed, which eventually contribute to the formation of a dust subdisk near the equatorial plane of the proto-Sun within the model under consideration. We also consider the emerging hydrodynamic and then gravitational instability in it followed by dust-cluster formation.

For an adequate phenomenological description of turbulent flows in a gas-dust disk, we perform the probability-theoretic Favre averaging of the stochastic equations of heterogeneous mechanics and derive the defining gradient relations for the turbulent interphase diffusion and heat fluxes. We also derive the “relative” and Reynolds stress tensors, which are needed to close the hydrodynamic equations of mean motion. We investigate the influence of the inertial effects of dust particles on the characteristics of turbulence in the disk, particularly on the additional generation of turbulent energy by large particles. We propose a semiempirical modeling method for the turbulent viscosity coefficient in a two-phase disk by taking into account the inverse effects of dispersed phase and heat transport on the development of turbulence with the goal of modeling the vertically inhomogeneous thermohydrodynamic structure of the dust subdisk and the ambient gas.

For a steady motion when solid particles settle down to the central plane of the disk under gravity, we investigate a parametric method of moments for solving the Smoluchowski integro-differential coagulation equation for the particle size distribution function. This states that a priori the sought-for distribution function belongs to a certain parametric class of distributions. We also analyze to which degree fine dust particle saturation can limit the subdisk atmosphere. This is responsible for the intensification of various coagulation mechanisms in a turbulized medium. The results of this chapter open new possibilities for constructing improved (and more realistic) models of stellar-planetary cosmogony, thereby providing a new approach to solving the fundamental problem of the origin and evolution of the Solar System and planetary systems around other stars.

In the *eighth chapter*, the problem of hydrodynamic helicity and, in particular, the influence of helicity on the evolution of disk turbulence is investigated. We show that the relatively long decay of turbulence in the disk is associated with the absence of reflection symmetry (relative to the equatorial plane) in the anisotropic field of turbulent velocities. We formulate the concept of emerging mesoscale coherent vortex structures with a large energy content in the thermodynamically open subsystem of turbulent chaos related to the realization of an inverse cascade of kinetic energy in mirror-asymmetric disk turbulence. We furthermore show that the inverse cascade generates a hierarchical system of clumps with a fractal density distribution that eventually initiate the triggering cluster-formation mechanisms through energy release. In turn, the formation of vortex clusters leads to an intensification of the mechanical and physical-chemical interactions between matter particles. As a result, spontaneous emergence and growth of dust clusters, stimulation of the condensation processes and phase transitions, the processes of heat and mass exchange between various regions of a heterogeneous disk, and significant modification of the spectrum of oscillations (density waves) are possible.

We discuss the influence of helicity on the energy cascade in a rotating disk and negative viscosity (generated by the cascade of helicity in disk turbulence when inverse energy transfer from small vortices to larger ones takes place), which we describe with a phenomenological approach. We also provide the relationships between the shear and rotational viscosities in a turbulent disk.

Finally, in the concluding *ninth chapter*, we address the problem of reconstructing an evolving protoplanetary gas-dust disk including electromagnetic effects, which also applies mainly to the basic concepts of stellar-planetary cosmogony. A closed system of magnetohydrodynamic (MHD) equations of mean motion designed to model shear and convective turbulent flows in electrically conducting media in the presence of a magnetic field is derived in the approximation of single-fluid magnetohydrodynamics. These equations can be used to numerically solve the problems of mutually consistent modeling of powerful turbulent cosmic plasma flows in accretion disks and related coronas. There the magnetic field significantly affects the dynamics of occurring astrophysical processes. We also systematically used weighted Favre averaging in addition to the traditional probability-theoretic averaging of the MHD equations to develop a model of a conducting turbulized medium. Favre averaging considerably simplifies the form of the averaged equations of motion for a compressible electrically conducting fluid and the analysis of the mechanisms for the amplification of macroscopic fields by turbulent flows. For a clear physical interpretation of the individual components of the plasma-field energy balance, we derive various energy equations that allow us to trace the possible transitions of energy from one form to another and, in particular, to understand the transfer mechanisms of the gravitational and kinetic energies of mean motion to magnetic energy.

In this context we focus on deriving the closing relations for the total (including the magnetic field) kinetic turbulent stress tensor in an electrically conducting fluid and the turbulent electromotive force (or the so-called magnetic Reynolds tensor) within the framework of extended irreversible thermodynamics. This also allows us to analyze the constraints imposed by the entropy growth condition on the turbulent transport coefficients. We propose a technique for modeling the turbulent transport coefficients, in particular, the kinematic turbulent viscosity, which makes it possible to take into account the influence of a magnetic field and the inverse effect of heat transfer on developing turbulence in a differentially rotating electrically conducting accretion disk.

It obviously follows from this brief discussion of the content that the monograph is basically oriented toward solving problems that are traditionally related to astrophysics and geophysics by the methods of mechanics. Leaving aside the debatable question regarding the very artificial breakdown of science into divisions and branches, we will only note that the merits and advantages of our approach are justified if they ensure the most comprehensive modeling of natural phenomena. This equally applies to cosmic objects that are inaccessible to direct study or to an attempt to reconstruct the events responsible for their set-up. The appearance and evolution of turbulized gas-dust accretion disks that result in the formation of planetary systems is just an example of the modeling

approach developed in the monograph. At the same time, the monograph content in general may be considered as a theoretical basis for numerical simulations of a wide class of phenomena in which the mechanics of inhomogeneous (multi-component, multiphase) and structured turbulent media widespread in nature plays a crucial role. An important constituent of these studies is the fundamentally new stochastic-thermodynamic approach to modeling developed turbulent flows and structured turbulence in astro- and geophysical systems considered from the standpoint of stochastic dynamics of open dissipative systems, which we have developed for many years. This is why we describe turbulent flows in natural phenomena as a typical example of nonequilibrium nonlinear dynamics, and this is the pivot around which the exposition in the monograph is built.

The book is the joint work of two authors. We gratefully point out that the discussions of several key problems with L.I. Sedov had a profound influence on the basic concepts underlying the content as well as on our own works on the mechanics of cosmic and natural media. In this monograph, we have tried to reflect many of L.I. Sedov's ideas on model construction of continuous media with complicated physical-chemical and thermal properties, in particular, on modeling multiphase multicomponent hydromechanics and multicomponent turbulent media.

The materials of Chaps. 2–9 and partially of Chap. 1 are based entirely on our original studies. The results of these studies are used in the numerical models that we are developing in collaboration with our colleagues and students. Some of the results have been published previously in the papers presented in the list of references and in our monographs “An Introduction to Planetary Aeronomy”, Nauka, Moscow, 1987; “Turbulence of Multicomponent Media”, Nauka-Interperiodika, Moscow, 1998; “Mechanics of Turbulence of Multicomponent Gases”, ASSL series, vol. 269, Kluwer Academic Publishers, Dordrecht/Boston/London, 2001; and “Astrophysical Discs” (Ed. by A.M. Fridman, M.Ya. Marov, and I.G. Kovalenko), ASSL series, vol. 337, Springer, 2006.

We are well aware that we have not managed to cover equally completely all questions in the extensive subject matter that the monograph is concerned with. This is primarily because, despite certain progress achieved in recent decades in studying such a complex phenomenon as turbulence, especially the turbulence of inhomogeneous media and structured turbulence, much still remains unclear and the emerging mathematical difficulties often seem insurmountable. Another not exhaustively studied area are the physical-chemical characteristics of a cosmic medium, to which the observed peculiarity of accretion disks is directly related, especially their structure-forming mechanisms and evolution, including the evolution of planetary systems and the formation of peculiar phenomena on the planets and their satellites. In particular, developing the theory of structured turbulence will play a key role in understanding the natural self-organization mechanisms. This is an appeal to find answers to these challenging questions, or at least to place more rigorous constraints on the developed models.

We are also aware that it is a desideratum to develop new original approaches that would allow one to efficiently model turbulent dynamics when describing complex natural phenomena, in particular, cosmic media. This is our goal in our studies.

A. Buckingham from the Livermore Laboratory apparently completely agrees with this concept. In his review of our book “Mechanics of Turbulence of Multicomponent Gases” mentioned above, he noted that at this juncture, it is more useful to develop “. . . a model that would allow one to calculate how turbulence affects the accompanying physical processes than to concentrate on a deeper understanding of the essence of turbulence – the goal that, while being academically attractive, is fraught with potential disappointments ” (Appl. Mech. Rev, vol. 56, no.1. 2003).

We hope that our new book will be received with interest by a wide circle of specialists in the fields of astrophysics, geophysics, mechanics, plasma physics, and space research. Astrophysicists, geophysicists, and planetologists will find in it a fairly deep justification of the theoretical approaches and methods for mathematical modeling of turbulence used in describing various natural and cosmic media, in particular, disk structures widespread in the cosmos, from the standpoint of mechanics. In turn, mechanicians will find a new rapidly progressing area of knowledge and the possibility of applying this fundamental science to the fascinating prospects of penetrating into the deepest mysteries of nature.

Numerous discussions of the questions that are touched upon here with our colleagues from various organizations contributed to writing the monograph and to improving its content. We would like to express our gratitude primarily to A.M. Fridman, E.M. Galimov, A.B. Makalkin, V.I. Maron, and I.S. Veselovskii. We are also grateful to K.K. Manuilov and O.A. Devina for their technical assistance in preparing the electronic version of the monograph and V. Astakhov for translating it into English. The publication of the book in Russian became possible owing to financial support by the Russian Foundation for Basic Research (project no. 08-01-07033), for which we express our gratitude.

We are grateful to Springer for offering to publish the book in English. It is worth to note that no specific advancement of the problems concerned was made since it was published in Russian, except for some new factual data about the inner planets of the Solar System and exoplanets, which do not impact, however, on our basic knowledge about planetary systems described in Chap. 1. Most importantly, we underline that the theoretical ground and principal concepts we deal with correspond to the contemporary views on the problems of self-organization in turbulent media and their application to accretion disk evolution.

Any critical remarks on the content of the book are welcome.

Moscow, Russia

Mikhail Ya Marov
Aleksander V. Kolesnichenko

Contents

1	Turbulent Chaos and Self-Organization in Cosmic Natural Media	1
1.1	Turbulent Fluid Motion. General Principles	2
1.1.1	Physical Nature of Turbulence and Scenarios for Its Generation	3
1.1.2	Developed Turbulence. Kolmogorov's Theory	9
1.1.3	On the Spectrum of Developed Turbulence	13
1.1.4	Turbulent Diffusion	16
1.1.5	Geophysical Turbulence	18
1.1.6	Turbulence Modeling Methods	21
1.2	Chaos and Self-Organization in Dynamical Systems	23
1.2.1	Elements of Stochastic Dynamics	24
1.2.2	Relationships between Order and Turbulent Chaos	32
1.2.3	The Emergence of Order in Turbulent Flows. A Stochastic-Thermodynamic Model	37
1.3	Cosmic Objects and Self-Organization	39
1.3.1	Dynamical Astronomy. General Principles	39
1.3.2	The Solar System: Dynamical Properties	42
1.3.3	The Solar System: Nature of the Planets and Satellites	46
1.3.4	Atmospheres of the Inner and Outer Planets	91
1.3.5	Nature and Dynamics of Small Bodies	107
1.3.6	Protoplanetary Accretion Disks	116
1.3.7	Evolution of Cosmic Objects in the Universe	130
2	Foundations of Mathematical Modeling of Reacting Gas Mixtures	145
2.1	Basic Conservation Laws and Balance Equations for Regular Motions of a Gas Mixture	146
2.1.1	On the Models of Continuous Media and Mathematical Modeling	146

2.1.2	The General Balance Equation	149
2.1.3	The Mass Balance Equations for a Reacting Mixture of Gases	151
2.1.4	The Equation of Motion for a Multicomponent Gas Mixture	153
2.1.5	The Energy Balance Equations	154
2.1.6	The Internal Energy Balance Equation for a Medium	157
2.1.7	The Thermal Equation of State	159
2.2	The Second Law of Thermodynamics	160
2.2.1	The Emergence of Entropy in Viscous Heat-Conducting Gas Mixtures	160
2.2.2	The Onsager Principle	161
2.2.3	The Balance Equation for Entropy and Entropy Production in Reacting Gas Mixtures	166
2.3	Defining Relations for the Diffusion and Heat Fluxes and the Viscous Stress Tensor	169
2.3.1	Linear Kinematic Constitutive Equations	169
2.3.2	Viscous Flow of an Isotropic Fluid	170
2.3.3	Heat Conduction, Diffusion, and Cross Effects	171
2.3.4	The Stefan–Maxwell Relations for Multicomponent Diffusion	176
2.3.5	Formulas for Defining Multicomponent Diffusion Coefficients via Binary Coefficients	186
3	Closed System of Hydrodynamic Equations to Describe Turbulent Motions of Multicomponent Media	189
3.1	Basic Concepts and Equations of Mechanics of Turbulence for a Mixture of Reacting Gases	190
3.1.1	Choosing the Averaging Operator	191
3.1.2	Mass and Momentum Conservation Laws for Averaged Motion	196
3.1.3	The Energetics of a Turbulent Flow	201
3.1.4	Equation of State for a Turbulized Mixture as a Whole	211
3.1.5	The Closure Problem of the Averaged Equations for a Mixture	212
3.2	Rheological Relations for the Turbulent Diffusion and Heat Fluxes and the Reynolds Stress Tensor	215
3.2.1	Balance Equation for the Weighted-Mean Entropy of a Mixture	216
3.2.2	Entropy Balance Equations and Entropy Production for the Subsystem of Turbulent Chaos	220
3.2.3	Balance Equation for the Total Entropy of a Turbulized Continuum	223

3.2.4	Linear Closing Relations for a Turbulized Multicomponent Mixture of Gases	226
3.2.5	Formulas to Determine the Correlations Including Density Fluctuations	232
3.2.6	Rheological Relations for the Turbulent Diffusion and Heat Fluxes in the Case of Strongly Developed Turbulence	235
3.3	Modeling the Turbulent Transport Coefficients	238
3.3.1	The Turbulence Scale	238
3.3.2	Gradient Hypothesis	239
3.3.3	First Approximation Modeling for the Turbulent Transport Coefficients	244
3.3.4	Differential Kolmogorov–Prandtl Model [b – L Model]	248
3.3.5	Equations for the Turbulence Scale: A Model with Two Transfer Equations	252
4	Differential Models for the Closure of Turbulently Averaged Hydrodynamic Equations for a Chemically Active Continuous Medium	255
4.1	Nonequilibrium Arrhenius Kinetics in a Turbulized Flow	257
4.1.1	Elements of Nonequilibrium Arrhenius Kinetics	258
4.1.2	Averaging the Nonequilibrium Chemical Reaction Rates	261
4.1.3	Formula for the Correlation Moments Including the Fluctuations in Substance Source Through Chemical Reactions	266
4.2	Model Transfer Equations for the Second Moments for a Multicomponent Gas Mixture	268
4.2.1	General Form of the Transfer Equation for the One-Point Second Moments for a Turbulized Mixture	269
4.2.2	Transfer Equations for the Turbulent Stress Tensor for a Multicomponent Medium with a Variable Density	272
4.2.3	Transfer Equations for the Turbulent Diffusion and Heat Fluxes for a Multicomponent Medium with a Variable Density	281
4.2.4	Transfer Equations and Dissipation of the Scalar Second Moments for a Multicomponent Medium with a Variable Density	285
4.3	Algebraic Closure Models for a Multicomponent Chemically Active Medium	290

4.3.1	Local Equilibrium Approximation (the <i>K</i> -Theory of Turbulence for a Chemically Reacting Gas Mixture)	290
4.3.2	Quasi-Equilibrium Approximation	293
5	Stochastic-Thermodynamic Modeling of Developed Structured Turbulence	295
5.1	Synergetic Approach to Describing Stationary-Nonequilibrium Turbulence	299
5.1.1	System of Hydrodynamic Equations of Mean Motion for a Single-Component Compressible Fluid	301
5.1.2	Thermodynamics of Structured Turbulence: The Internal Fluctuating Coordinates of the Subsystem of Turbulent Chaos	303
5.1.3	Balance Equation for the Total Entropy of the Subsystems of Averaged Motion and Structured Turbulent Chaos	317
5.1.4	Stationary-Nonequilibrium State of the Turbulent Field: Defining Relations for Structured Turbulence	319
5.1.5	Prigogine's Principle: Thermodynamic Derivation of the Fokker–Planck–Kolmogorov Equations	322
5.1.6	Examples of the Fokker–Planck–Kolmogorov Equations Describing the Evolution of the Fluctuating Characteristics of Turbulent Chaos	324
5.2	Investigation of the Self-Organization of Turbulent Chaos Based on Stochastic Langevin Equations	333
5.2.1	Stochastic Approach to Studying the Evolution of Turbulent Chaos: A Gaussian Process	335
5.2.2	Stochastic Langevin Equations in the Space of Internal Coordinates	341
5.2.3	Nonequilibrium Stationary States of Turbulent Chaos	343
5.2.4	Thermodynamic Stability of Stationary States and Critical Stationary States	349
5.2.5	Intermittency Effects	356
5.3	Fractional-Order FPK Equation to Describe Turbulent Chaos with Memory	360
5.3.1	Causality Principle for Non-Markovian Processes in the Subsystem of Turbulent Chaos	362
5.3.2	Fractional Integral and Fractional Derivative (Introductory Information)	367
5.3.3	FFPK Equation to Describe Evolutionary Processes in Fractal Time	370

6	Self-Organization of Developed Turbulence and Formation Mechanisms of Coherent Structures	373
6.1	Role of Nonequilibrium Phase Transitions in Structuring Hydrodynamic Turbulence	375
6.1.1	Basic Mathematical Apparatus	378
6.1.2	<i>H</i> -Theorem for Stationary States	384
6.1.3	Phenomenology of Small-Scale Turbulence	388
6.1.4	Model Stochastic Differential Equations and the Fokker–Planck–Kolmogorov Equation for the Turbulent Energy Dissipation Rate	393
6.1.5	Phase Transitions Induced by the Multiplicative Noise of Turbulent Chaos	395
6.1.6	Analysis of Verhulst’s Mathematical Model for the Dissipation of Turbulent Energy	399
6.2	Generation of Structured Turbulence Through Phase Synchronization	402
6.2.1	Stable Limit Cycles and the Related Synchronization of Periodic Self-Oscillations (Phase Dynamics Approximation)	406
6.2.2	Mechanism for the Formation of Mesoscale Coherent Structures (Clusters) in the Subsystem of Turbulent Chaos	411
6.2.3	Phase Dynamics Equations	414
6.2.4	Solution of the Stochastic Equations for the Phase Difference Between the Oscillations of a Synchronized Cluster in a Stationary State	418
7	Foundations of Mechanics of Heterogeneous Media for Accretion Disks	425
7.1	Theoretical Prerequisites to Modeling the Evolution of Turbulized Accretion Disks	426
7.1.1	Basic Concepts of Planetary Cosmogony	426
7.1.2	Basic Assumptions of the Model	434
7.2	Equations of Mechanics of Heterogeneous Media to Describe a Protoplanetary Gas–Dust Cloud	436
7.2.1	Mass Conservation: A Monodisperse Gas–Dust Medium	437
7.2.2	Interphase Diffusion: The Aerodynamic Drag Coefficient for Disk Dust Particles	441
7.2.3	Allowance for the Multifractional Composition of Dust: The Kinetic Coagulation Equation	447
7.2.4	Momentum Conservation Equation for Gas–Dust Matter and Radiation	451
7.2.5	Heat Influx Equation for a Heterogeneous Gas–Dust Medium and Radiation in a Disk	454

7.2.6	Thermodynamic Equation of State for Disk Matter	459
7.2.7	Radiative Transfer Equation for a Gas–Dust Disk: Optical Properties of Dust Grains	460
7.2.8	Basic System of Laminar Equations of Motion for a Gas–Dust Disk	465
7.3	Averaged Equations of Two-Phase Mechanics to Describe a Turbulized Gas–Dust Disk	466
7.3.1	Averaged Mass Balance Equations for Gas–Dust Matter: The Turbulent Transport Coefficient	469
7.3.2	Averaged Smoluchowski Coagulation Equation	476
7.3.3	Averaged Equation of Motion for a Gas–Dust Disk Medium	478
7.3.4	Balance Equation for the Averaged Internal Energy of a Gas Suspension	481
7.3.5	Energy Balance Equations for Disk Matter	486
7.3.6	Modeling the Turbulent Viscosity Coefficient in a Dust Subdisk	494
7.4	Steady Motions in a Turbulized Gas–Dust Subdisk	503
7.4.1	Axisymmetric Motion in a Gas–Dust Disk	504
7.4.2	Turbulent Viscosity Coefficient in a Gas–Dust Disk	512
7.4.3	Limiting Saturation of a Rotating Gas–Dust Disk with Fine Dust Particles	515
7.4.4	Analytical Solution of the Coagulation Kinetics Equation by the Method of Moments	519
8	Influence of Hydrodynamic Helicity on the Evolution of Turbulence in an Accretion Disk	525
8.1	Theoretical Prerequisites to Modeling Hydrodynamic Helicity	525
8.2	Energy Cascade in Isotropic Turbulence with Reflection Symmetry	530
8.2.1	Equations of Turbulent Chaos in the Presence of a Mean Flow	531
8.2.2	Conservation Laws in Locally Isotropic Turbulence	533
8.2.3	Vorticity Dynamics and Energy Cascade	534
8.2.4	Two-Dimensional Turbulence	536
8.3	On the Energy and Helicity Cascades in Disk Reflection-Invariant Turbulence	537
8.3.1	Mirror Symmetry Breaking in a Protoplanetary Disk	537
8.3.2	Influence of Helicity on the Energy Cascade	539
8.3.3	Generation of Hydrodynamic Helicity in a Rotating Disk	542

- 8.4 Negative Viscosity in Rotating Disk Turbulence
as a Manifestation of the Helicity Cascade 545
 - 8.4.1 Difficulties of the Momentum Transfer Theory 545
 - 8.4.2 Negative Viscosity (Thermodynamic Approach) 547
 - 8.4.3 Rotational Viscosity 549
- 9 Thermodynamic Model of MHD Turbulence and Some
of Its Applications to Accretion Disks 555**
 - 9.1 Basic Equations of Magnetohydrodynamics
for Modeling the Disk Structure and Corona 556
 - 9.1.1 Magnetic Induction Equation 556
 - 9.1.2 Mass and Momentum Conservation Equations 559
 - 9.1.3 Various Forms of the Energy and Heat Influx
Equations for an Electrically Conducting Medium 562
 - 9.1.4 Equations of State 566
 - 9.2 Equations of Turbulent Motion for a Conducting Medium
in the Presence of a Magnetic Field 567
 - 9.2.1 Averaged Continuity Equation 569
 - 9.2.2 Magnetic Induction Equation for Mean Fields 569
 - 9.2.3 Averaged Equation of Motion 571
 - 9.2.4 Energy Equations of Mean Motion for Electrically
Conducting Matter 572
 - 9.2.5 Equations for the Magnetic Energy of a Turbulized
Plasma 577
 - 9.3 Derivation of the Defining Relations or Turbulent Flows
of an Electrically Conducting Medium in the Presence
of a Magnetic Field 582
 - 9.3.1 The Balance Equation for the Averaged Entropy
of a Conducting Medium 583
 - 9.3.2 Entropy Balance Equations and Entropy Production
for the Subsystem of Turbulent Chaos in a
Conducting Medium 584
 - 9.3.3 Balance Equation for the Total Entropy 586
 - 9.3.4 Stationary-Nonequilibrium Regime
of the Subsystem of Turbulent Chaos. Derivation
of the Defining Relations 588
 - 9.3.5 Derivation of the Correction Function to the
Turbulent Viscosity Coefficient for a Conducting
Medium with a Variable Density 594
 - 9.4 Modeling the Turbulent Transport Coefficients
in a Thin Accretion Disk 598
 - 9.4.1 Viscosity Law in Thin Keplerian Disks 600
 - 9.4.2 Modeling the Turbulent Viscosity Coefficient
in a Protoplanetary Disk of Finite Thickness 603

Appendix: Elements of Tensor Calculus	607
Conclusion	611
References	613
Index	653

Chapter 1

Turbulent Chaos and Self-Organization in Cosmic Natural Media

Natural objects evolve from initial chaotic motions to order through a fascinating internal self-organization, which is embedded in their structure. The dynamics of this process is the focus of this work. They can be subjected to temporal and spatial variations or retain their stability for a long time. Ordered structures surround us ubiquitously on Earth; numerous examples of self-organization are observed in space. Turbulent flows characterized by a great variety of dynamical processes are widespread in the surrounding world. We mainly focus on the problems of macroscopic modeling these natural flows.

Turbulence is a widespread and extremely complex physical phenomenon associated with the fluctuating motion of a fluid that is present in various engineering systems and natural media. The atmospheres of planets in the Solar System, including the outer gaseous envelopes of these celestial bodies that lie in the atmosphere-space boundary regions, are typical examples of turbulized cosmic natural media. Developed turbulence also plays an important role in forming the structure and properties of astrophysical objects – galaxies and stars at various evolutionary stages as well as protoplanetary clouds and accretion disks that serve as a basis for cosmogonic models. Numerous present-day experimental studies of the peculiar features of turbulent motions of liquid and gaseous media and the key mechanisms determining their nature create the necessary prerequisites for developing theoretical approaches aimed at constructing appropriate mathematical models of this phenomenon.

Although the theory of hydrodynamic turbulence has been developed for more than a century now, it is still far from complete. New advanced statistical and phenomenological approaches continue to appear. There are increasingly more various mathematical models developed for a better understanding of the formation and evolution of turbulent motions of a homogeneous fluid (the classical theory of turbulence) and fluids with complicated physical-chemical and thermal properties. Numerous semiempirical (engineering) turbulence models designed to solve practical problems based on large-scale numerical simulations are developed based on the concepts of turbulent exchange coefficients for various transferable substances. Our objective here is to familiarize the reader with some important ideas that inspire