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Lesson Play in Mathematics Education

A Tool for Research and Professional Development

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ISBN 978-1-4614-3548-8 ISBN 978-1-4614-3549-5 (eBook)
DOI 10.1007/978-1-4614-3549-5
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2012951136

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Teacher education is currently receiving extensive attention in mathematics education research, as evidenced by the amount of research articles, books, as well as series of books devoted to this theme. Anna Sfard, in her plenary address at the International Congress of Mathematics Education in Copenhagen in 2004, noted that she was "pleased to find out that the last few years have been the *era of the teacher* as the almost uncontested focus of researchers' attention" (Sfard, 2004, p. 90). She also described the last two decades of the twentieth century as "almost exclusively *the era of the learner*", and the several decades prior to that as the "*era of the curriculum*" (ibid.). This research focus has been accompanied by a growing interest in the education of prospective teachers.

Among a large variety of studies devoted to the education of future teachers of mathematics, several directions are being pursued: a focus on teachers' knowledge and/or knowledge-in-use, a focus on teachers' interpretation of student thinking and classroom situations, and an examination of the tools that assist the development of teachers' mathematical knowledge and pedagogical sensitivities. Our research for this book fits within this latter focus: it introduces a novel tool—lesson play—and discusses various examples of its implementation.

Lesson play is a novel construct in research on teachers' professional development in mathematics education. Lesson play refers to a lesson or part of a lesson presented, written—and sometimes performed!—in a script form, featuring imagined interactions between a teacher and her students. We have been using and refining our use of this tool for a number of years in a variety of situations involving mathematics thinking and learning. We have asked prospective teachers to write lesson plays on a variety of themes and following a variety of prompts. The goal of this book is to offer a comprehensive survey of the affordances of the tool, the results of our studies—particularly in the area of pre-service teacher education—and the reasons for which the tool offers such productive possibilities for both researchers and teacher educators.

Although we claim that lesson play is a novel method, its roots can be traced to Socratic dialogue, a genre of prose in which a 'wise man' leads a discussion, often pointing to flaws in the thinking of his interlocutor. Jumping to modern times, we are further influenced by the work of Sfard (2010) that focuses on communication and, in particular, that describes thinking as communication. The task of writing a lesson play allows an individual to re-embody different selves—that of a teacher-character and of different student-characters. Moreover, elaborating on the theatrical interpretation of the word 'play' in reference to a script to be potentially performed on stage, we are influenced by research that focuses on improvisation and on the importance of role playing in education. Indeed, we consider teaching as an act of improvisation and we note, metaphorically, that every skillful jazz improviser spent his or her youth practicing scales and chords. As such, creating a script for a play can be considered as role playing in one's thinking. It is practiced in a safe environment of one's cubicle, without the need to "think on your feet". We see this role playing as a valuable part in preparing for "real teaching".

In teacher education we are constantly seeking methods that improve our practice and consequently the practice of teachers that are enrolled in the courses we teach. We are not alone in this endeavor. Mason, Watson, and Zaslavsky devoted a special issue of the *Journal of Mathematics Teacher Education* (2007, volume 10) to the nature and role of tasks in teacher education. Following up on this initiative, three edited books were published in these series: Tasks in Primary Mathematics Teacher Education (2009), Teaching with Tasks for Effective Mathematics Learning (2012), and Constructing Knowledge for Teaching Secondary Mathematics (2011). Although the latter book does not have "tasks" in its title, its focus is on illustrative tasks for use in teacher education at the secondary level.

The lesson play task is a contribution to this endeavor. Although it focuses on one particular kind of task, it is flexible and can be adjusted to different populations and different mathematical topics. As we demonstrate, it can be used in both pre-service and in-service teacher education. It can also be used at any level of mathematical curriculum, though our focus in this book is on the elementary school grades. We further believe that the task can be extended beyond mathematics; we thus invite colleagues in teacher educators more broadly (in the sciences and humanities) to adapt it to their contexts.

In Part I—Chaps. 1 to 3—we introduce the lesson play, describe our gradual development of this tool, and contrast it with other ways of planning for instruction. Part II—Chaps. 4 to 9—is devoted to the analysis of the plays that are based on particular prompts. In Part III—Chaps. 10 to 13—we present a cross analysis of previous chapters and also discuss various uses of this tool in our work with teachers.

Overall, we present a compelling argument for lesson play as a valuable tool for teachers preparing their lessons, for instructors/teacher educators who work with teachers in various professional development settings, and for mathematics education researchers who study teachers' knowledge and development.

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Part III

Part I

In Part I we introduce the lesson play, describe our gradual development of this tool and contrast it with other ways of planning for instruction. In Chap. 1 we examine different modes of planning for instruction that have been used and developed over the past century. We point to some of the assumptions underlying these modes of planning, concerning ourselves in particular with the ways in which they might get in the way of the kind of rich, interactive, problem-solving based teaching and learning that standards and curricula around the world are promoting. Our central conceit is that the kind of planning that is needed for this kind of mathematics classroom should look radically different than the type of planning that was developed for the more traditional, static and one-way image of mathematics teaching.

In Chap. 2 we provide an example of an actual lesson play. We highlight the affordances of this tool by drawing attention to the particular decisions made by the playwright, in ascribing utterances to both the teacher and student characters. We point to more general teacher moves that are evident in the play, as well as to specific language choices in the pedagogical and mathematical interactions with students. Then in Chap. 3 we share with the reader our gradual development of the lesson play task, which we now use as an integral part of our methods courses. We demonstrate how the task of writing a script for a play evolved from the general assignment to a request to respond to a particular prompt that introduces a common mistake.

Chapter 1
Planning for Instruction

The "lesson plan" has been a staple of pre-service teacher education for many decades. In fact, almost everyone who has undergone a formal teacher education program has had to devise a lesson plan according to some prescribed format. Indeed, it is hard to imagine what teachers did before they used lesson plans! In this chapter, we describe the emergence of the lesson plan as we know it today and the educational assumptions it carries with it. We then provide an exemplar lesson plan in order to highlight both its strengths and weaknesses as a mode of planning to teach a mathematics lesson. As we will show in the next chapters, the "lesson play" offers a mode of planning that addresses these weaknesses of the lesson plan.

Legacy of the Tylerian Lesson Plan

The roots of the traditional instructional planning in general, and lesson planning in particular, can be traced to the work of Tyler (1949). His framework is based on four components: specifying objectives, selecting learning experiences for attaining objectives, organizing learning experiences, and evaluating the effectiveness of learning experiences. Tyler considered the specification of objectives "the most critical criteria for guiding all the other activities of the curriculum-maker" (p. 62). Elaboration of Tyler's ideas resulted in a variety of instructional design models, whose common components are the identification of: goals and objectives, a teacher's and students' activities (teaching and learning strategies), materials to be used in a lesson, feedback and guidance for students, and assessment/evaluation procedures determining whether the identified objectives have been met (Freiberg and Driscoll 2000).

The practical implementation of these models resulted in the creation of a variety of forms or templates, as can be easily found by searching for "lesson plan" on the Internet. Quite often, these templates do not explicitly embody the

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DOI: 10.1007/978-1-4614-3549-5_1,

ideals and theories that justify their existence. As such, when a prospective teacher is handed a template, she is not receiving the full benefit of the work that went into creating it, but rather an empty shell that stands in the place of grounded theories of teaching practice. Indeed, these templates have been criticized in the scholarly literature (see John 2006; Maroney and Searcey 1996) for oversimplifying what it means to teach, as well as for failing to consider how teachers actually plan. Of course, they are worth criticizing if and only if they are used as proxies for preparation, which *can* be how they appear to future teachers. Future teachers can easily assume that the clear identification and organization of content outcomes will result in the acquisition of this same content by the students.

We know now that the articulation of objectives, although necessary, is far from sufficient when planning for teaching. Research from the 1970s and 1980s showed that specifying objectives is not a central part of teachers' planning (Peterson et al. 1978; Zahorik 1970). Yinger (1980) found that when using Tyler's model "no provision was made for planning based on behavioral objectives or previously stated instructional goals" (p. 124). More recently, John (2006) conducted a comprehensive analysis and critique of the dominant Tylerian model and its extensions. He argued that the emphasis on "outcome-based education" has "led to teaching based on a restricted set of aims, which can in turn misrepresent the richer expectations that might emerge from constructive and creative curriculum documents" (p. 484), and that the approach does not acknowledge elements of teaching "that are not endorsed by the assessment structure" (p. 485). However, as Maroney and Searcy (1996) point out, the results of these studies have also had little influence on current practice: "teacher educators are not assisting teachers or their students by continuing to teach only traditional comprehensive lesson planning models, knowing that the majority of teachers will not use those models" (p. 200).

Why, despite the ongoing criticism and acknowledgment that "real teachers do not plan that way" has the traditional rational model sustained its popularity? John (2006) suggests several interrelated reasons. He believes that "much of the attraction of this approach to planning lies in its elegant simplicity" (p. 485). Related to this, the model reinforces a sense of control based on prediction and prescription. Like a grocery list, it ascertains that no ingredients will be forgotten while also ensuring that certain meals will be made. But there are other reasons too. One is the belief that prospective teachers need to know how to plan in a rational-traditional framework before they can attend to the complexities of particular curricular elements. Another is that many official curriculum documents prescribe the model for teachers to follow. In addition, the model is seen to offer a continuity between school practice and teacher education institutions; indeed, many teachers are required—by their principals or for a substitute teacher—to prepare lesson plans in advance and keep them as documentation of classroom activity. And of course, by virtue of being written down, as a prescription for *one* class, the lesson plan can easily become a recipe for *any* class—independent of the teacher, the students, the school, and even the country.

In the mathematics education literature, teacher-researchers such as Lampert (2001) have shown how expert planning and preparation for teaching a lesson involves extensive work in connecting particular mathematics to particular students, moving back and forth between mathematics, and the structure of tasks appropriate for particular learners. Thus, Lampert begins her planning by first designing a mathematical task, but then the implementation of these tasks shifts in accordance with students' responses. Yinger might describe this type of by-the-seat-of-your-pants teaching as improvisation. In order to improvise well, one must be able to deal with unexpected situations, handle new questions, propose alternate problems, and be able to interpret and evaluate unfamiliar forms of reasoning. What kind of planning can help to support this style of teaching? The notion of improvisation evokes images of jazz musicians, whose ability to improvise depends on extensive practicing of chords, scales and melodies, on creative variation of chord progressions, and on an ability to respond to fellow players. The traditional lesson plan is an ill-suited way of practicing the kinds of moves that would be needed to respond spontaneously and creatively to the rhythm of the problem-solving classroom.

In this book we introduce the "lesson play", which we propose might provide a novel juxtaposition to the traditional planning framework as a method of preparing—and even practicing—to teach a lesson. We recognize that these two methods structure the act of preparation in two fundamentally different ways, each with its own affordances. However, they draw on fundamentally different metaphors for what it means to prepare for teaching. Behind the form of the lesson play lies an image of teaching that is closer to rehearsed in-the-moment choices and decisions than it is to predetermined plots and outcomes. However, as we show in the next section, there are also positive aspects of the traditional lesson plan.

Lesson Plan: An Example

Let's consider a sample lesson plan, as shown in Fig. 1. Following a possible variation of the Tylerian model, this plan clearly identifies learning objectives, sets procedures for attaining these objectives, and specifies the procedures for evaluation. While adopting a half-century old mode of planning, this lesson plan also incorporates many aspects of reform-based mathematics teaching. Indeed, we note the following aspects of the planned lesson that meets contemporary criteria for active, participatory, and conceptually-driven mathematics instruction:

- Students are engaged in an activity of producing rectangular arrays. This occurs after the teacher has provided clear directions and illustrated using 6 as an example.
- Students are using manipulatives to construct the array.
- The teacher attempts to mediate between the students' work with concrete objects and the mathematical ideas of prime and composite numbers.

LESSON PLAN

Objectives
SWAT
Model prime and composite numbers
Recognize prime and composite numbers
Define prime and composite numbers (explain which numbers are prime and which are not)

Materials
5-6 sets of 30 counters (pennies, cubes, chips)

TEACHER'S ACTIVITY	STUDENTS' ACTIVITY
▪ Teacher provides instructions and exemplifies activity: *Our goal today is to make rectangular arrays from a given number of counters. We would like to make as many rectangular arrays as possible for any number. We will do this for every number from 2 to 30. For example, if we take 6 counters, they can be arranged in 1 row, in 1 column, in 2 rows and 3 columns or in 3 rows and 2 columns. So altogether we have 4 possible arrangements.*	▪ Students working in groups of 3 build rectangular arrays. They record the information on the provided worksheet.
▪ Teacher asks students to consider the table they made and list what they notice.	▪ Students take notes.
▪ Teacher asks students to share their notes.	▪ Students share observations about the table.
▪ Teacher focuses on or explicitly provokes a specific observation: *which numbers can be built only in one row or in one column?*	▪ Students list these numbers: 2,3,5,7,11,13,17,19, 23,29
▪ Teacher asks why this is so.	▪ Students make suggestions.
▪ Teacher introduces the term "prime number" and describes what numbers are prime.	▪ Students connect the notion of "prime number" to the table they created.

Evaluation:
Students are given a list of numbers between 5 and 100 and are asked to determine which of the numbers are prime.

Challenge:
Students are asked to find a prime number larger that 100 and explain why they think the number is prime

Fig. 1 Example of a 'good' lesson plan

- The teacher asks students to make observations based on a completed table. This represents a thoughtful attempt to build on students' ideas rather than simply provide information.
- Students have an opportunity to share their ideas and observations regarding the patterns they see.
- The lesson is organized so that the main concept—prime numbers—can be built out of reflection on the activity.
- Evaluation procedures are set to check the degree to which the concepts of prime and composite numbers have been built.
- There is an opportunity for students who complete their work before their classmates to extend/challenge their understanding by exploring numbers greater than 100.

We submit this as a 'good' plan in the sense that it appears to present a constructivist student-centered approach, in which concepts are built through reflection on an activity. Like an abstract, or a book review, it is descriptive—and thus summarizes what a good lesson would look like. However, as John (2006) points out, "the model does not take into account contingencies of teaching" (p. 487). Indeed, like most lesson plans, this one presents a "powerful generic idea", however "it tells us very little about the substance of the particular activity we apply it to" (*ibid*). While economic, and perhaps even iconic, this particular lesson plan ignores the following aspects of the lesson that would provide the substance to which John refers:

- what definition for a prime number the teacher might use in relation to the manipulatives and the students' prior experiences;
- what observations might emerge from considering the table;
- how students' observations emerging from the table, which are not related to prime numbers, might be treated;
- what student difficulties are expected and how those might be addressed;
- what questions the teacher might use to assess or expand student understanding;
- what mathematical language might be introduced or supported.

These lacunae, we argue, are not shortcomings of the specific lesson plan, but the artifacts of the planning structure, which is necessarily prescriptive and summative. The standard format for planning does not encourage, and at times does not leave room for, anticipation of faulty extensions, misconceptions, difficulties, and possibilities for alternative explanations or examples, or consideration of interactions that takes place in a lesson. Indeed, this is a *lesson plan* and not a *teaching plan.* The Tylerian planning framework, as well as variances of this framework, is explicitly designed to focus on predetermining outcomes; it is prescriptive.

Alternative Models

Given the limitation of the traditional model, several alternatives to lesson planning have been suggested. For example, Egan (1988, 2005), who has also critiqued the Tylerian model, suggests creating frameworks that focus less on content delivery and more on the deployment of developmentally appropriate cognitive tools that foster the imaginative engagement of learners. However, while the role of the imagination in teaching and learning is masterfully outlined, the planning for instruction is reduced, yet again, to filling out templates of a pre-determined rubric.

As an alternative to traditional lesson planning, John advocates for a model that gradually adds layers to the Tylerian one. This model places the objective outcomes in the center, and through a circular approach adds to this kernel additional consideration, or so-called satellite components, without suggesting a fixed order. These components include, but are not limited to, key questions, students' learning, professional values, resource availability, classroom control, and degree of difficulty of material. The image of this lesson plan, layered and de-linearized as it is, seems more diagrammatic, inviting perhaps the kinds of links and connections that the medium of the Internet has familiarized us with. Though John's model has the potential to capture many valuable aspects of teaching, it draws more on what experienced teachers do than on what novice teachers should learn to do. In other words, experienced teachers rely on their practice-based knowledge of students and of material in order to add layers to their plans, while novice teachers do not have sufficient resources to draw from. Moreover, while considering expert practice is important for novice teachers, this multi-layered model does not provide an instrument in which planning across the multiple layers can be captured and shared.

Other approaches to lesson planning, such as the one offered in the very popular *Elementary and Middle School Mathematics: Teaching Developmentally* by Van de Walle and Folk (2008), attempt to move away from the template approach to lesson planning altogether and, instead, offer a 10-step approach to planning in a problem-based classroom. Within this process, the lesson plan itself constitutes the three following steps: Plan the introductory activities, Plan the developmental activities, and Plan the follow-up discussion. Instead of having the objective outcome (the learning of a given concept) as the focal point of radiating components, this model privileges the process of learning through initiation, development, and discussion. As we will show, our "lesson play" complements well this approach and focuses on the crucial task of planning what *might* happen during these three segments of the lesson.

Before turning to the "lesson play", we would like to point to one other model of planning that has gained much popularity in mathematics education circles and that envisages planning as a much more public and shared endeavor—thus moving away from the image of lesson planning as an individual, private ritual. Indeed, the Japanese "lesson study" presents a unique approach to planning that involves a number of educators in a process of investigation, anticipation, implementation, reflection, and revision. Given the social nature of the planning, there remains at least a verbal trace of the decisions that were made in creating the final product so that the ensuing plan functions less as a starting point to prescribe action and more as a record of interrogation and reflection (at least for those who participated in the lesson study).

The applicability of lesson study in pre-service teacher education has limitations, however. The process is very time-intensive, requiring many hours of meetings spread over a long period. The process is also heavily dependent on teachers' experience to more effectively anticipate students' reactions to specific activities. Indeed, researchers working within the context of lesson study have shown that anticipating student responses to questions and tasks stands out as one of the most challenging aspects of lesson study, especially for beginning teachers (Stigler and Hiebert 1999). In fact, one of our motivations for designing the lesson play has been to engage prospective teachers in honing their ability to predict and reflect on students' reactions through an interpretive exploration of possibilities.

Conclusion

Based on Davis and Simmt's (2006) distinction between planned (or prescribed) and emergent (or proscribed) events, we see the act of preparing to teach as one that is interpretive in nature, and that shifts focus "from what *must* or *should* happen toward what *might* or *could* happen" (p. 147). In this chapter, we have seen how traditional lesson planning does little to encourage interpretive planning. Our goal is thus to offer a mode of planning through which the attention of prospective teachers is drawn to considering the different possibilities occasioned by a question or task, the different responses a student might offer, the different conceptions a student might build, and the different effects a certain response by the teacher might produce.

Chapter 2
Introducing Lesson Play

In Chap. 1, we offered an example of a lesson plan that satisfied many of the goals of reform-based teaching. Of course, as we know, there can be an enormous distance between planned lessons and implemented lessons. Indeed, when working with prospective teachers, we noticed that they were able to produce impressive lesson plans but, when we observed them teaching mathematics, the careful attention to the use of manipulatives, to problem-based learning and to group work was almost swept away by their actual interactions with students. In these interactions, we saw the same kind of moves that have been reported in the literature such as:

- An emphasis on procedural thinking (Crespo et al. 2010)
- A tendency to ask fact-based questions rather than questions that invite mathematical reasoning (Vacc 1993)
- The use of misleading or erroneous mathematical explanations
- A tendency to position the textbook or the teacher as the mathematical authority in the classroom (Herbel-Eisenmann and Wagner 2007)

These observations led us to believe that prospective teachers needed help in developing more strategies needed to achieve their global goal of reform-based teaching. They needed to think about and pay attention to the way in which they asked questions, responded to students, and provided direction. In the next section, we provide a brief overview of how we came to develop the idea of lesson play that is used in this book. We then provide an example of a lesson play and point to the particular opportunities it offers for helping teachers develop the kinds of moves they need to respond to the complex environment of the reform-based mathematics classroom.

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DOI: 10.1007/978-1-4614-3549-5_2,

Developing the "Lesson Play"

As mentioned above, the idea of lesson play grew out of our frustration with 'good' lesson plans that did not attend, or had no place to attend, to what we consider important features in planning for instruction. Over the past 7 years it evolved from a general instruction to "write a play as an imagined interaction" to an explicit request to attend to a presented problematic, the way it could have emerged and the way it could be resolved. This alternative attends to John's (2006) suggestion that "the lesson plan should not be viewed as a blueprint for action, but should also be a record of interaction" (p. 495). In Chap. 3 we outline the evolution of the lesson play task from infancy to the stage of its current implementation. However, in the next section we invite the reader to consider several potential in-class interactions and an example of a lesson play.

Potential Interactions

Imagine the following interaction, in which a teacher is asking students to identify whether different numbers are prime.

Teacher: Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita: 91 is prime.

Although the student is an imaginary one, her statement is not uncommon, as evidenced in the literature (Zazkis and Campbell 1996a). How might you respond to this student? You are unlikely to follow-up in this manner:

Teacher: Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita: 91 is prime
Teacher: You are wrong. 91 is 7 times 13.

Instead, you will probably want to let Rita engage in some mathematical reasoning. We challenge you to take 5 minutes and actually write down the next five or six exchanges. Perhaps you want to incorporate the voices of other students in the class. We think you will find that actually selecting the words that you use to respond to the student takes some thought, and you will probably find yourself editing your first attempt. You will certainly notice that there are many options available, perhaps more than you had first considered. For example, consider the two options offered below.

Prime Follow-up A		Prime Follow-up B	
Teacher:	Everyone finished? Good. Let's check the rest of the numbers. How about 91?	Teacher:	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita:	91 is prime.	Rita:	91 is prime.
Teacher:	What is a prime number?	Teacher:	I'm going to ask you to add one more column of your 12 by 12 multiplication table.

In Prime Follow-up A, the teacher's question carries with it the assumption that Rita does not understand what it means to be a prime number. The teacher's imagined trajectory looks like this: first, establish a correct definition of prime number; then, when Rita uses this definition for 91, she will find that it is not prime. Presumably, Rita has already encountered the definition for prime number, but the teacher might assume she does not remember it. In Prime Follow-up B, the teacher assumes that Rita thinks that numbers not in the multiplication table are prime. The teacher's imagined trajectory is thus to extend the multiplication table, which will enable Rita to see the number 91 appear, which will lead her to recognize that 91 is not prime. We note that both options communicate the fact that Rita is wrong, without saying so explicitly. But each option will play out very differently in the classroom and affect the way Rita will think of prime numbers and, even, the way she thinks of mathematics—in Prime Follow-up A, mathematics is framed as an activity based on definitions while in Prime Follow-up B, it is an activity involving computation.

While the lesson plan makes quite clear the content in focus (identifying prime numbers), the lesson play and the dialogue between the teacher and the students draws much more attention to the process through which that content will be communicated in the classroom. At a mathematical level, the imagined verbal exchanges necessarily bring into focus both the actual use of mathematical language in communicating and the forms in which ideas are explained or justified. At the pedagogical level, the imagined exchange articulates assumptions about how students are thinking and how their thinking might be changed; it also articulates possible teaching trajectories. And, as shown in the two options above, the lesson play suggests something about the very nature of learning without falling into any pre-fixed pedagogical "ism".

In our work with prospective teachers, we ask them to continue the exchange far beyond the follow-ups exemplified above. Not only do they have to imagine what they would say, as teachers, but also how students might respond. We also invite them to imagine what might have happened *before* a given prompt. So, just as we provided, in Chap. 1, a model lesson plan, we offer here a model lesson play based on the prompt offered at the beginning of the section (which appears at the beginning of Scene 2, in this play). As you read, we invite you to think about the different assumptions the teacher made about the students and to try to identify the general teaching trajectory that the playwright had in mind. What do you notice about the way the teacher asks questions or responds to the students? What choices has the teacher made about her use of mathematical language?

A Sample Lesson Play

Scene 1

1		*(Students were given a list of numbers and asked to determine which ones are prime and which ones are composite, and to explain their decisions. After about* 5 *min of silent individual work, some students are half way through the task, while others are hesitating. The teacher decides to check some of the work to assure students are on the right track.)*
2	Teacher	So, class, let's check what we have come up with so far. Please pay attention, I know you have not finished, you can continue later. Let's start with the first number on our list—23. Is it prime or composite? Yes, Susan.
3	Susan	Prime.
4	Teacher	Okay, and why do you say this?
5	Susan	Because nothing goes into it.
6	Teacher	Goes into?
7	Susan	I mean nothing divides it.
8	Teacher	Nothing? Nothing at all?
9	Maria	She means no numbers other than 23 and 1. You can write it as 23 times 1, but no other options.
10	Teacher	Good. So rather than "nothing", we say 23 has exactly 2 divisors, 23 and 1.
11	Susan	And also when we worked with chips we could only put them in one long line, and you could not make another rectangle without leftovers.
12	Teacher	Indeed, excellent. Let's move on. How about 34, is it prime or composite? Yes, Jamie.
13	Jamie	Composite.
14	Teacher	And you say this because …
15	Jamie	Because it is even.
16	Teacher	So? Please explain.
17	Jamie	We know it is even, right, and if it is even it has 2 in it.
18	Teacher	Has 2 in it? Hmm, I see 34, I see a 3 and a 4. Where is the 2?
19	Maria	What he means is 2 is a factor. Even numbers have 2 as a factor, so it cannot be prime.
20	Teacher	So you are saying that an even number cannot be prime?
21	Maria	Sure. All even numbers are 2 times something, so they are not prime. Primes are odd.
22	Teacher	And what about the number 2?
23	Jamie	2 is prime, and 2 is even.
24	Teacher	So I am confused here. Can you help?
25	Maria	Sure. No need for confusion. What I mean to say is 2 is an exception. It is the only even prime because it is in the very beginning. The other primes are odd. 2 is the only exception.

(continued)

(continued)

26	Teacher	Okay, good. We figured this out. Let us proceed—68?
27	Marty	Composite of course. We just said that even numbers, not 2, but bigger even numbers cannot be prime. So no need to go over even numbers on the list, they are all composite.
28	Teacher	Does everyone agree? Great, so this makes our work easier, of course. Let's go over odd numbers only. The next on our list is 19, Kevin?
29	Kevin	It is composite because … it almost looks like prime but then I remembered in my times tables it is 7 times 7. And the same is with the next one, 63, it is 7 times 9.
30	Teacher	Very good. Your multiplication tables helped you decide. Okay. Now let us take a few more minutes and complete the work. If you have already decided whether each number is prime or composite, please turn to problem 7 on page 106.

Scene 2

31		*(Students continue to work on their own. Some are just finishing up with the list of numbers provided while others have moved onto working on the problem in the textbook.)*
32	Teacher	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
33	Rita	91 is prime.
34	Teacher	And you say so because?
35	Rita	It is not anywhere on the times tables.
36	Teacher	Interesting. So are you saying that only composite numbers are on our multiplication tables?
37	Rita	*(hesitating)* That's what Kevin said and you said "Okay."
38	Teacher	What exactly did Kevin say?
39	Rita	That 49 is 7 times 7 and 63 is 7 times 9 on the times tables. And he is right, and you said "Okay", and 91 is not there.
40	Teacher	I see. When do we say that a number is prime?
41	Students	2 factors only, no factors other than itself and 1.
42	Teacher	So if 63 is 7 times 9, what do we know about its factors?
43	Tina	We know it has 7 and 9 as its factors.
44	Teacher	Exactly, that is why it cannot be prime. But is it possible that 91 has factors that are not on our multiplication table?
45	Rita	*(hesitating)* No, I think, because it is smaller than 100.
46	Teacher	Let's look at 34. Can you find it on the table *(pointing to a* 12 by 12 *multiplication table mounted on the wall).*
47	Tina	It is not there, but it is even. So for even numbers no need to look at the table. We KNOW they are not prime. Like 38 is also not on the tables but it is not prime.
48	Teacher	So we cannot find 34 and 38 on the tables, but they are not prime. Isn't this strange?
49	Rita	Yeah, because they are even, but 91 is not even.
50	Teacher	I see. Let's look at… look at *(thinking)* an odd number … 39.
51	Tina	It is not on the tables.
52	Teacher	So what are you saying?
53	Rita	I say it is 3 times 13, so I say it is composite.
54	Teacher	Isn't it interesting! Can we find another ODD number that is NOT on the tables, but is composite?

(continued)

(continued)

55	Kevin	51?
56	Mary	65 and 75 and 85 and 95!
57	Teacher	Anything else?
58	Mark	57.
59	Teacher	Good. Let's gather all these numbers you found, that are not on the tables and are odd and composite, and write them as products, show them in multiplication. So we have 39, 51, 57, 65, 75, 85, and 95.
60	Mark	Mary's are easy, because they all are 5 times something.
61	Teacher	Nice observation, but let's work out all of them.
62	Students	*(pause)* $39 = 3 \times 13$, $51 = 3 \times 17$, $57 = 3 \times 19$, $65 = 5 \times 13$, $75 = 5 \times 15$, $85 = 5 \times 17$, $95 = 5 \times 19$.
63	Teacher	Very nice. Now, I look carefully at all these COMPOSITE numbers, and I wonder, why are they not on our multiplication table?
64	Rita	Because there are big numbers you are timesing by, and the table does not go that far.
65	Teacher	So where does this bring us with respect to 91?
66	Rita	That what we said, it is not on the times tables, was wrong. I mean it is right that it is not there, but it does not mean it is prime. So this was wrong. It is 7×13. It is not prime, it is composite. Actually, all the people at my table said it was prime, but now we figured it out. It is not prime because it is 7×13, so it has these factors.
67	Teacher	Excellent, Rita. Is it clear to everyone what she said?
68	Mark	She said that we cannot use the times tables to decide what is prime.
69	Teacher	*(smiles)* Yes, that's basically it. Right. So NOW I have a challenge for the class. Let us find ALL the composite numbers that are ODD and that DO NOT appear anywhere on the multiplication table.

Of course, the lesson plan that led to this particular interaction could also have led to millions of others. Thus, what we are interested in here are the particular goals, choices, and assumptions that can be seen within the imagined interactions. We focus first on the mathematical features of the interaction, and then turn our attention to the pedagogical ones. Our intention is not to separate the mathematical from the pedagogical, but to use these two lenses as ways of analyzing the lesson play.

In terms of the mathematical features then, we elaborate on two main points. First, the lesson play deals explicitly with the use of mathematical language. The teacher is constantly attending to the students' language. For example, the teacher repeats Susan's use of the vague phrase "goes into" [5, 6] in an effort to prompt more precise mathematical language. Later, the same thing happens with Jamie's use of "has 2 in it" [17, 18]. Both Jamie and Susan may see the teacher's words as simple synonyms for their own, but in the lesson play, the teacher offers the more precise vocabulary that will be needed for effective communication about prime numbers, not just for Jamie and Susan, but for their classmates as well. The teacher's responses not only offer alternative ways of talking about composite numbers, but also show how nonmathematical language such as "has 2 in it" can be communicatively misleading (since 34 clearly has no 2 in it). This close attention to language, and to the need for precision in communication cannot be separated from the content in question, but it is specific to the way in which the content is worked on in the classroom. Broadly, we might say that the teacher works to bridge the students’ everyday language to formal mathematical language (see Herbel-Eisenmann 2002). While such a goal might be included in a lesson plan, the lesson play offers the specific details of how and when this happens.

In addition to the language focus, the lesson play also makes explicit the various forms of mathematical reasoning that might emerge in the classroom. For instance, when Maria makes the argument that “all even numbers are 2 times something, so they are not prime” [21], the teacher evaluates the argument and proposes a counter-example [22]. This occurs again with respect to Rita’s claim about composite numbers appearing on the times table [35, 36, 50]. In both cases, the students have made quite a reasonable inference, perhaps even a necessary one given their current experiences, and the teacher must recognize them and then devise ways in which the students can come to more appropriate inferences. The actual counter-examples used by the teacher (2 for Maria and 39 for Rita) are highly specific in their responsiveness, and emerge directly from the dialogue.

In the lesson play, we can also identify specific “pedagogical moves” that the teacher makes in order to sustain the interaction. We have already noted the attention to language, but the teacher’s way of working with language involves some “re-voicing” of students’ statements. This move enables the teacher to acknowledge the student’s statement while also offering a mathematically preferable rendition. So, for example, the teacher re-voices Maria’s statement about prime numbers by saying “Good. So rather than ‘nothing,’ we say 23 has exactly 2 divisors, 23 and 1” [10]. Another example of re-voicing comes later on, when the teacher re-voices Rita’s response as a conjecture (that numbers not on the times table are prime [36]) that Rita can then investigate.

In addition to instances of re-voicing, we can also attend to the kinds of questions that the teacher asks. We know from research that teachers tend to ask fact-based questions that require little reasoning (Vacc 1993). For example, after Rita says that 91 is a prime, the teacher might ask fact-based questions such as “Is 91 on the times table?” or “What is 91 divided by 13?” The first requires the student to scan her times table and the second requires her to undertake a calculation. Neither necessarily involves reasoning. In this lesson play, the teacher chooses to ask the question “And you say so because?” [34]. By asking this question, the teacher is able to elicit the student’s reasoning and use it to help Rita see how this reasoning leads to a contradiction. Unlike in Prime Follow-up B, the teacher does not *assume* that Rita’s error involves the multiplication table. Further, unlike Prime Follow-up B, the teacher does not immediately engage Rita in calculation but, instead, re-voices Rita’s response as a conjecture.

Re-voicing and probing student thinking (through reasoning-based questions) are two of the “talk moves” that Chapin et al. (2009) identify as promoting classroom discussion. In many mathematics classrooms, the interaction follows what is known as the IRE format (initiation-response-evaluation), which leads students through a predetermined set of information and does little to encourage students to express their thinking (Cazden 2001; Nystrand 1997). In promoting "talk moves", Chapin et al. seek alternative interactions that engage students and foster reasoning. So, while we can focus on re-voicing or reasoning-based questioning as talk moves, it is important also to zoom out somewhat and consider the kind of interaction that follows from these moves.

In her work on mathematics discussions in the classroom, Wood (1998) identifies two forms of classroom interaction: focusing and funneling. Similar to IRE, funneling occurs when the teacher asks a series of questions that guide the students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the question to arrive at the solution, often without seeing the connection among the questions. Consider how the following lesson play differs from the one offered above.

Teacher	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita	91 is prime.
Teacher	I am going to ask you to add one more column of your 12 by 12 multiplication table.
Rita	Okay. I will add the column for 13.
Teacher	And what do you notice?
Rita	I see that 91 is there.
Teacher	What are its factors?
Rita	13 and 7.
Teacher	So is it prime?
Rita	No.

In this example, although the teacher asks some open questions (such as "What do you notice"), the teacher is focused on getting Rita to find the factors of 91. In this scene of a lesson play, the teacher does not find out why Rita thinks 91 is prime. Nor does the teacher enable Rita to make sense of her generalization from the previous class (drawing on the interaction with Kevin). Indeed, in examining the lesson plays written by prospective teachers, based on a prompt in which a student mistakenly identified 91 as prime (see Chap. 6) we have found that the vast majority of them lead students through a process of extending the multiplication table. This is not, of course, an incorrect method, but it leads to a funneled discussion in which the interaction is necessarily pre-determined—which does not make for a very interesting discussion!

In contrast to funneling, focusing requires the teacher to listen to the students' responses and to guide them based on what the students are thinking rather than how the teacher would solve the problem. Achieving this kind of focusing interaction can be very challenging, and requires the use of moves that go beyond simple initiation and feedback. Indeed, in the model lesson play we offered, the teacher needs to deal with Kevin's generalization, with multiples of 2 and 5 that are not on the times table, as well as with counter-examples involving composite numbers that are odd. Instead of having a fixed endpoint to the discussion, the teacher must remain responsive to the student and open to the possibility that the student pursues a method of solving the problem that is initially unknown. This does not mean that the teacher does not have a goal. Indeed, we can see in the model lesson play that the teacher wants to help Rita see that there are many numbers that are composite—and that Rita knows are composite—that are not on the times table. The teaching trajectory is thus to help Rita refute the implicit conjecture about the times table by considering the numbers that are not on it and thus revisiting the idea of what it means to be prime.

In terms of the pedagogical features of the lesson play, we wish also to draw attention to some aspects of its format. The structure of the lesson play—as a dialogue occurring over time with possibilities for different points of view—allows for the portrayal of the messy, sometimes repetitive interactions of an inquiry-based classroom. This structure stands in stark contrast to a necessarily ordered and simplified list of actions such as: take up homework, state definition, provide examples, give problems, and evaluate solutions. In this lesson play, we see the teacher revisiting definitions of "prime" and "composite" that were used in Scene 1 with the help of new ideas that emerge in Scene 2, such as the multiplication table. The lesson play communicates the fact that the meanings of definitions change for students as they encounter new examples or problems. It also probes the way in which student interpretations can lead to unexpected consequences.

For example, at the beginning of Scene 2, we see Rita defending her claim that 91 is prime because it is not on the multiplication table: "That's what Kevin said and you said 'Okay'. " [37]. Here the teacher has the option of proposing a counter-example, returning to the definition of prime, or arguing about the context of her response to Kevin. The lesson play tests out these different options by 'running' them like a script and seeing how Rita (and other students) might respond. Being interpretations, these different options can now be critiqued, so that decisions can be evaluated. In contrast to a lesson plan, which may be "good" or "bad", the lesson play, as an interpretation, invites questioning about the different ways in which teachers might respond to students, and the different conditions under which students might build understandings.

This leads to a final point about the lesson play that relates to its 'playfulness'. By its very nature, the lesson play requires a focus on specific and particular imagined interactions. In a lesson plan, one can include directives such as "call on different students to answer questions". In a lesson play, those students must be named, individually, and the playwright has to decide quite explicitly whether, for example, Tina or Rita will answer a teacher's question. The playwright is forced to consider whether it is more important to make Tina follow through or to give Rita a chance to participate. This may, at one level, sound trivial, but we see it as part of the imaginative work that teachers must do to prepare and practice for the classroom—much the same way children practice routines of communication in their self talk.

By being forced to make a choice, one must follow through with the consequences of each option, and one might even find it necessary to evaluate the outcomes of different choices. Further, the playwright must do this imaginative work not only for the teacher (the role she will eventually play), but also for the students—the playwright must try to think or talk like a student. We conjecture