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Andreas Eichler · Timo Leuders *Editors*

Views and Beliefs in Mathematics Education

Results of the 19th MAVI Conference

IMBF

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Editorial

Ralf Erens und Carola Bernack-Schüler

Looking at venues of the various MAVI conferences, the 19th international conference on Mathematical Beliefs (MAVI) moved southwards into the heart of the Black Forest in southwestern Germany after it had been held in Talinn, Bochum and Helsinki in the last three years respectively. The 19th MAVI conference was organized by the University of Education in Freiburg from September 25th to September 28th, 2013. The current proceedings is/are published in the Freiburg Springer series of empirical research in mathematics education. We are grateful for the financial support of the University of Education Freiburg which enabled the publication of this conference issue

The founding fathers of these conferences, Erkki Pekhonen (Helsinki) und Günter Törner (Duisburg), initiated the MAVI group as a bilateral Finnish-German cooperation in 1995 which soon grew into an international community of researchers who met at yearly meetings e.g. in Pisa and Genova (Italy), Vienna and St. Wolfgang (Austria), Kristianstad and Gävle (Sweden) and Nikosia (Cyprus). The remarkable body of MAVI volumes of each meeting displays a variety of research papers on beliefs, attitudes and emotions in mathematics education.

A noteworthy characteristic of MAVI conferences is the lack of formal organization. Each participant enjoys an equal status and all accepted contributions are given the same time for presentation and discussion. According to the MAVI tradition, papers were submitted in advance and pass through a peer review procedure. For the 19th MAVI altogether 21 papers were presented and actively discussed by the participants.

The first section of this volume consists of six papers looking into teachers' beliefs. Their working contexts reach from pre-school (Sumpter, p.63) across out-of-field mathematics teaching (Bosse & Törner, p.10) to secondary school. The papers focus on mathematics in general but also on specific contents as curriculum reform (Berg et al., p.75) or beliefs about geometry (Girnat, p.xy). But also the belief change over 25 years in the Finnish teacher population was subject of research (Oksanen & Pehkonen, p.35) as well as conceptions about mathematics in different countries (Salo i Nevado et al., p.51).

The following four papers focus on belief research during teacher training and during the experiences as novice teachers (teacher trainees). The professional development of pre-service teachers is described either based on a longitudinal case study of a novice teacher (Palmér, p.131), taking into account external influences outside university (Ebbelind, p.119) or based on belief changes through a teacher training course (Bernack-Schüler et al., p.91). Furthermore the novice teachers' belief system concerning the teaching and learning of arithmetics was subject of research (Bräunling & Eichler, p.105).

The next three contributions deal with mathematics beliefs in the domain of technology. One paper is looking into pre-service teachers' beliefs concerning the teaching of mathematics with technology using metaphors (Portaankorva-Koivisto, p.135) but also the beliefs about the implementation and use of technology for the specific domain of calculus is described (Erens & Eichler, p.143). Moreover, Sundberg (p.169) glance at teachers' technological pedagogical content knowledge.

Another two papers look into beliefs related to problem solving and posing (Kontorovich, p.181; Papadopoulos, p.193). Beyond one theoretical framework from the history of mathematics is presented to investigate beliefs (Haapasalo & Zimmermann, p.207) and epistemological judgments about the certainty of mathematics knowledge were subject to research using a special approach in terms of interviews (Rott et al., p.235). Liljedahl & Andra (p.223) get a deep insight in students' interactions.

The different contributions address different topics and groups of students, teachers etc. The 19th MAVI conference added a variety or research perspectives to the international discussions of mathematics related beliefs and affect. With the feedback of the reviewers and the discussion in the conference the authors of this volume produced the existing contribution of a rich selection of research methods and may further enhance the discussion of MAVI topics in the future.

The editors

Carola Bernack-Schüler

Ralf Erens

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Teacher Identity as a Theoretical Framework for Researching Out-of-Field Teaching Mathematics Teachers

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Abstract

This theoretical essay deals with teacher identity as an approach for studying out-of-field teaching mathematics teachers, i.e. teachers without formal education for teaching mathematics. It thinks about teacher identity as a unifying concept that is able to connect cognitive and affective-motivational perspectives. The connecting framework helps us to explain how out-of-field teaching in mathematics works: teachers' shortcomings in cognitive domains are indirectly compensated due to the prioritization-function of affective-motivational realms. Furthermore, other advantages of the identity approach are stated by referring to characteristics of identity concepts that can be found in literature. Finally, our conclusion leads to recommendations for designing in-service training programs for these teachers.

1 Introduction and Motivation

Some teachers have neither been formally educated as mathematics teachers at university nor in pre-service courses but actually teach mathematics in school. Following the terminology of Ingersoll (2001), we want to call this group of teachers *out-of-field teaching mathematics teachers*. In Germany – for example – we can observe this out-of-field teaching phenomenon both in primary schools and in secondary schools.

Only recently, the German *Institut zur Qualitätsentwicklung im Bildungswesen (IQB)* found out that in some federal states the percentage of out-of-field teaching mathematics teachers is about 50 % in grade 4 (Richter, D., Kuhl, P., Reimers, H., & Pant, H. A., 2012) and about 37 % in grade 9 (Richter, D., Kuhl, P., Haag, N., & Pant, H. A., 2013). Further, the authors of the quoted studies claim to show that there are significant differences in mathematics-related student achievement between students who were taught out-of-field and those who were not. Especially low-performing students achieved even worse, when educated by out-of-field teaching teachers.

Not only in Germany (Bosse & Törner, 2012; Törner & Törner, 2012) but also in other countries out-of-field teaching poses a challenge for teachers, teacher educators and politics (Crisan & Rodd, 2011; Dee & Cohodes, 2008; Hobbs, 2012; Ingersoll, 2001; Ingersoll & Curran, 2004; McConney & Price, 2009a; McConney & Price, 2009b; Rodd, 2012; Vale, 2010).

Having in mind that out-of-field teaching has negative influence on student achievement, it seems reasonable to cope with the phenomenon. Professional development programs – like those provided by the German National Centre for Mathematics Teacher Education (DZLM) – deal with this topic and offer in-service teacher education in order to support teachers in teaching mathematics. The crucial questions in this context are: What support do out-of-field teachers need? How can professional development programs help these teachers? And what has to be considered when designing in-service training courses? In order to answer these questions we have to understand the practical experiences made by those teachers. We have to get insight into their needs, into their self-conception, into their cognitive and affective problems and challenges, and – of course – into their relation to mathematics and to mathematics education.

Therefore, we need a theoretical framework in which we can analyze not only the phenomenon per se but the individuals who teach out-of-field. In the following paragraphs we will show that the theory of *teacher identity* is a useful approach to answer the crucial questions. We will proceed as follows:

First, we will explain the rather general term *identity* by referring to current research literature and by highlighting its benefits for our research purposes.

Second, we will connect the concept of *identity* to an established theoretical approach in mathematics teacher education: we will show how the identity concept can profitably enhance models for describing and analyzing teachers' professional competencies. The aim of this step is to develop a theoretical framework we want to call *subject-related teacher identity*. This framework helps both analyzing out-of-field teaching mathematics teachers and understanding how to and where to start when support shall be offered.

Third, we will critical reflect on the framework of *subject-related teacher identity*. Therefore, we will consider practical and methodological challenges that occur when researching out-of-field teaching mathematics teachers.

Fourth, we will make some concluding remarks about further steps in a possible research project. It has to be pondered whether, and if so, how the framework actually helps with designing professional development activities.

2 Teacher Identity as a Research Approach

2.1 A Rough Survey of Existing Research

Researching *Identity* is a broad field that is dealt with in many different educational and non-educational disciplines. In the last decades, the concept of identity has often changed depending on the theoretical perspective and depending on problems researchers want to work on. Grootenboer, Lowrie and Smith (2006) suggest that one can have three different views on the concept of identity: a psychological-developmental perspective, a social-cultural perspective and a poststructural perspective. Recently, researches in (mathematics) education have preferred social-cultural approaches (e.g. Boaler, 2002) like the theory of Communities of Practice by Wenger (2007). Apart from that, one can find studies in which teacher identity is seen in a poststructural framework (e.g. Walshaw, 2004). Other authors differ from the named trisection of perspectives. In this case, identity is seen as a narrative or at least in narrative contexts (Alsup, 2006; Sfard & Prusak, 2005).

On the one hand, each of these four perspectives implicates another research approach and another understanding of the identity concept. On the other hand, multiple perspectives (or facets of several single perspectives) are chosen and arranged for a specific research purpose. According to Grootenboer et al. (2006), the way the term identity is used in research contexts depends on the persuasions of the researcher.

Thus, it is difficult to provide an exact definition of the term *identity.* Some authors (e.g. Kelchtermans, 2009) prefer a quite narrow definition of identity by fixing static identity components, by framing their scope and by determining the meaning of the identity concept with a new term. Kelchtermans (2009) for example speaks about *self-understanding* instead of *identity*, because he wants to emphasize the teachers' conceptions of themselves as teachers.

Having in mind that out-of-field teaching of mathematics is an almost unexplored research field, we prefer a broader definition of *identity*. Therefore, we want to refer to Grootenboer et al. (2006) and Grootenboer and Zevenbergen (2008). They provide a "unifying" and "connecting" concept that can bring together multiple and interrelated elements which are relevant for out-of-field teaching mathematics teachers' professional activities, too. More precisely, the authors suggest using the term *identity* by framing beliefs, attitudes, emotions, cognitive capacity and life history. In this way, the identity concept lets us explore many different facets of out-of-field teaching mathematics teachers' activities, challenges and needs. Further, we cannot only have a look at cognitive but also on affective questions. With the help of this identity concept, the different identity facets are not detached anymore. They are brought together and allow us to gain a holistic view of our research objects.

2.2 Complex-Systemic Aspects of Teacher Identity

As a consequence of the broad definition, teacher identity becomes more complex. In fact, complexity appears in four ways:

First, the unifying concept of identity allows us to draw links to other relevant domains. Beauchamp and Thomas (2009) give an overview of such links and show that teacher identity is at least connected to activities of self-reflection, to the role of emotions, to agency, to stories and discourses, and to context. These domains are partly connected to each other. Thus, we can speak about identity as a systemic concept.

Second, we have to respect the situational character of identity (Beauchamp & Thomas, 2009; Grootenboer & Zevenbergen, 2007; Wenger, 2007). According to that, identity is something highly dynamic. It cannot be treated like a static entity you can have a look at without considering physical, social, institutional and affective contexts. If identity is context-dependent, we have to assume that there is something like a mathematical identity and a specific identity of out-of-field teaching mathematics teachers. Having said this, we should have in mind that teaching out-of-field is another context than teaching in-field and that *crossing* from one context to the other might have implications on many different levels (Hobbs, 2012).

Third, the dependence on context implies that identity comprises the existence of something like *sub-identities*. The identity of a *german out-of-field teaching primary mathematics teacher* belongs somehow or other to the identity of the *german primary teacher* that in turn belongs to the identity of the *german teacher* and so on. Such vertical identity hierarchies as well as horizontal identity overlapping – for example the identities of a mathematics teacher who also teaches geography – have to be considered.

Fourth, every teacher has two sub-identities that come along with special implications and interdependences: the professional and the personal identity (Alsup, 2006). In the context of our research purposes, this fact plays an important role in two ways: On the one hand, the out-of-field teaching teachers are professionals in teaching one (or more) subject(s) apart from mathematics. One can assume that the knowledge and the skills that are related to that (or these) subject(s) influence the personal identity. On the other hand, we can assume that a strong and developed professional identity in teaching mathematics is missing. However, these teachers encountered mathematics and teaching mathematics in their personal, non-professional life when they were students themselves. Thus, the personal identity might play a significant role when talking about the professional one.

2.3 Historic-Process-Oriented Aspects of Teacher Identity

In the identity concept, there is not only a dynamic momentum due to contextuality but also due to variability. Beauchamp and Thomas (2009) assemble different terms that are used in literature and that explain the process of identity shifting and reshaping (e.g. development, construction, formation, making, creation, building, architecture and so forth). Every term describes that identity is something that has to be built. The authors underline that identity development is not a process that eventually ends. Moreover, they say that identities are developed and re-developed constantly – which is obvious as the individual enters new contexts and makes new experiences constantly.

Also Sachs (2001) and Kelchtermans (2009) emphasize the importance of the dynamic nature of identity. In Kelchtermans' opinion, identity itself is an "ongoing process of making sense of one's experiences and their impact on the 'self'" (ibid., p. 261). Regarding the broad definition, talking about an individual's identity also means talking about life history (Grootenboer et al., 2006) and about the individual's biographic experiences (e.g. Alsup, 2006).

When using identity as an approach, the individual's past, present and future is considered. Bernstein (2000) distinguishes between the retroperspective and the properspective identity, having in mind that identity has developed from experiences in the past and will be developed from expectations and goals in the future. Also Sfard and Prusak (2005) consider the time-related variability of identity. They differentiate between the actual and the designated identity by explaining that there is a difference between actual experiences and those that are expected in the future.

2.4 Narrative Aspects of Teacher Identity

Stories and discourses are not only a way to express identity (Beauchamp & Thomas, 2009). Moreover, they are seen as a means of identity-making (ibid.; Alsup, 2006) or even as the identity itself (Sfard & Prusak, 2005). For the analysis of out-of-field teaching mathematics teachers it is mainly important, that narrative activities and communicational practice can support these teachers in developing a stronger mathematical identity (e.g. when consulting colleagues or attending an in-service course). In relation to this, Alsup (2006) ascribes narratives the power of transformation in thinking. Narratives become also relevant for methodological considerations, as they are able to open windows through which one can have a look at a facet of an individual's identity.

3 From Teacher Identity to Subject-Related Teacher Identity

3.1 Professional Competence of Mathematics Teachers

One option to analyze and to describe mathematics teachers' knowledge, skills and affective-motivational dispositions is the use of competence models. Especially empirical research deals with measuring teachers' competencies in order to model these with *competence profiles* (Blömeke, Kaiser, & Lehmann, 2010; Blömeke, Suhl, & Döhrmann, 2012). For example, the model that is used in the TEDS-M study (ibid.) contains both cognitive (PCK, CK, PK) and affective-motivational (beliefs, motivations, self-regulation) domains. Of course, we could list other competence models for modeling mathematics teachers' competencies.

3.2 Connecting Cognitive and Affective-Motivational Components

Having a look at each of these models, the question is left open how these two domains are connected to each other. The authors of such models try to describe teachers' cognitive and affective competencies as precisely as possible by dividing them into sub-competencies that are sometimes divided into sub-sub-competencies again. Once a research field is defined and a corresponding (sub-)-competence is located in the model, other competencies are faded out (above all affective-motivational competencies when someone has chosen a cognitive (sub)competence). At the best, a study is extensive enough to research many different competencies and sub-competencies. That has to be the necessary condition if someone wants to get a holistic picture of a teacher, his or her skills, needs, and shortcomings.

The unifying concept of *identity* as described above gives us the chance to build a theoretical framework in which cognitive and affective-motivational components are already connected to each other. The framework does not claim to be able to define fixed competence fields but can, however, provide a holistic picture. Moreover, the theory makes it possible to explore the same research objects based on both cognitive *and* affective perspectives.

When researching out-of-field teaching mathematics teachers, it stands to reason that the cognitive focus is on mathematics and teaching mathematics. Using the terminology of the established competence models, the theory of identity is able to relate affective-motivational competencies to any item dedicated to PCK and CK. Having this in mind, we do not have to care about asking ourselves if we want to analyze a sub-competence of PCK, if we want to explore a CK related question exclusively or if we are just interested in a perspective shedding light on a teacher's beliefs.

Let us have a look at an example. If we want to analyze how out-of-field teaching mathematics teachers cope with the Theorem of Pythagoras, how they implement the theorem in teaching contexts and how a professional development program can support these teachers in doing that, we can examine this against the background of identity very well.

One can observe how they encountered the theorem in professional and/or personal domains and ask whether, and if so, how this plays a role in present teaching (*life history* aspect of identity). Further, one can ask whether, and if so, how they have actually learned to give a fruitful introduction to this topic in lessons. Supporting colleagues might play an important role when answering this question (*contextuality* and *variability* of identity). Apart from that, one can research which proof of the theorem the teacher uses in teaching and why he or she favors it (connections to facets of *self-regulation* and *self-understanding*). It is also an interesting opportunity to ask about likes and dislikes around the theorem and its possible proofs (*emotional* aspects) and about personal and professional experiences, expectations and views towards it (aspects of *beliefs*, *attitudes* and *biography*). Of course, one could find many other practical approaches.

Every identity-related item of this example is somehow or other linked to the subject (CK and/or PCK). Therefore, we want to call the theory *subject-related teacher identity.*

3.3 Affective-Motivational Components as Shortcomings-Compensating Factors

When dealing with out-of-field teaching mathematics teachers, we have to assume that they have shortcomings in PCK and CK, as they were not educated in these domains (Bosse & Törner, 2012). Schoenfeld (2011) suggests that the elements of the affective-motivational domain ("orientations") have a function of prioritizing cognitive and other resources. If cognitive resources are missing, then other resources are consulted (e.g. colleagues, textbooks, own experiences in life history). What other resources are used for teaching and how the process of choosing resources looks like, is – according to Schoenfeld – initiated by the affective-motivational domain (cf. ibid, p. 30). From this, one can conclude that out-of-field teaching teachers' shortcomings in the cognitive domain are indirectly compensated by affective-motivational components.

In this respect, the theory of *subject-related teacher identity* – as combining both components – helps to clarify basics and mechanisms of compensating. Therefore, it helps to understand how teachers teach out-of-field and how teachers can be supported.

4 Critical Reflection

4.1 Challenges and Advantages of the Identity Approach

Argument of ethics and appreciation: In our opinion an analysis that investigates shortcomings of out-of-field teaching mathematics teachers exclusively is not an option. Some of these teachers are highly motivated and enthusiastic to teach mathematics out-of-field. Thus, their work has to be appreciated instead of highlighting the deficits. A study that concentrates on shortcomings would ignore that these teachers already compensate drawbacks by their own methods and strategies. The identity framework makes these approaches visible and considers both shortcomings and the individual ways to cope with them.

Argument of dissolving uncertainty: It seems to us that analyzing competence profiles carries a certain risk. Similar to Heisenberg's uncertainty principle, investigating a competence object to close – for example in the context of a sub-sub-competence field – leads to loosing information about the object itself. The identity approach is a more holistic way that allows us to research many different facets of out-of-field teaching mathematics teachers without the need to be perfectly precise on subatomic-like levels. Nevertheless, one gets a broad picture of teaching and possible starting points for supporting these teachers.

Argument of cognitive-empowerment: The cognitive and the affective domain are related to each other. The identity approach respects this due to its unifying character. Separating cognitive and affective components, as it is often done in competence-oriented approaches, limits the opportunities to understand interdependencies that are especially relevant for out-of-field teaching teachers. That has to be underlined since affective components are able to empower cognitive ones (see a multitude of findings in the wake of emotion, motivation and belief research).

Argument of model learning: Not only teachers but also students own a mathematical identity (Boaler, 2002). In order to avoid that students acquire negative attitudes towards mathematics, teachers should develop a fruitful mathematical identity themselves. Bandura (1977) showed that students learn from role models. If an out-of-field teaching mathematics teacher is not able to develop an appropriate identity, his or her students will not do either (see Boaler, 2002; Grootenboer & Zevenbergen, 2008).

Argument of practice: The starting points for the study as well as the practical implementation of the findings are related to questions of teaching in practice. The theoretical framework considers this: Since identity and practical experiences can be made accessible due to narratives, the approach provides ways for involving practical relevance.

4.2 Methodological Considerations towards Further Research Activities

While out-of-field teaching mathematics teachers often refuse to participate in our studies due to affective reasons (fear, shame, embarrassment), it is difficult to undertake a broad quantitative study. Further, it is often unknown who actually teaches out-of-field, as there are almost no statistics about the phenomenon. The identity concept provides a theoretical framework that can also be used for doing reasonable qualitative research on a small group of teachers.

Despite of the group size, the identity approach lets us achieve findings that provide a holistic picture. A qualitative analysis of different and manifold facets is possible – and not only an investigation of a specific competence-field. The identity approach helps us to research *beliefs*, contexts of *self-image* and *sense of self, motivations* and *emotions*. Every facet can be projected into mathematics-related realms, since the identity approach is a unifying concept.

Besides, the identity concept provides narrative approaches that are especially appropriate for qualitative interviews and narrative data collection. To this effect, a useful research method is – for example – asking out-of-field teaching mathematics teachers to write short essays about the topic “mathematics and me”.

5 Conclusions and Recommendations for Professional Development Programs

Sachs (2005) claims, that teachers’ “professional identity […] stands at the core of the teaching profession. It provides a framework for teachers to construct their own ideas of ‘how to be’, ‘how to act’ and ‘how to understand’ their work” (p. 15). If we project this into mathematics and mathematics education as explained above, we are able to comprehend how to support out-of-field teaching mathematics teachers. Subject-related teacher identity as a theoretical framework provides starting-points for both research and intervention due to professional development. Further, it allows us to focus not only on shortcomings in cognitive knowledge but to consider affective-motivational dispositions. As affective-motivational components are responsible for prioritizing knowledge and other resources (cf. Schoenfeld, 2011), we can explain how out-of-field teaching works in spite of shortcomings in CK and PCK.

A professional development program for out-of-field teaching teachers should not only spend time on fostering subject-related cognitive competencies. Of course, this is necessary and important; but in addition to that, in-service training courses should have an eye on the teachers' subject-related identity. We are convinced that being able to explain out-of-field teaching in such a holistic way leads to a better understanding for designing effective and successful professional development.

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Teachers' Geometrical Paradigms as Central Curricular Beliefs in the Context of Mathematical Worldviews and Goals of Education

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Abstract

This article presents some results of a qualitative study on secondary teachers' beliefs, reconstructed as so-called individual curricula, a concept to represent a teacher's argumentative connections between his choice of content, methods, and goals of education. Within these individual curricula, two archetypes are figured out that are supposed to be oppositional in three dimensions: in the use of Geometrical Working Spaces in classroom teaching, in the general mathematical worldview, and in the choice of goals of education a teacher intends to achieve by teaching elementary geometry. The first archetype is characterised by deductive standards, a static view on mathematics, and expert-oriented goals of education; the second one is more empirical, dynamic, and guided by pragmatic goals of education.

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1 Interest of Research and Background of the Study

This article presents some results of a qualitative interview study concerning secondary school teachers' individual curricula on teaching elementary geometry. The core framework is based on the concept of *individual curricula* (Eichler, 2007) which are used to describe the part of a teacher's *beliefs system* (cf. Philipp, 2007) that contains argumentative connections between content, methods, and goals of education and has a similar function as a written curriculum (cf. Stein, Remillard & Smith, 2007), especially the task to justify the choice of contents and teaching methods against to the background of a teacher's individual goals of education.

After reconstructing nine individual curricula out of in-depth interviews, the study was faced to the problem to compare and to categorise the findings. Since an individual curriculum – even restricted to teaching elementary geometry – is a "holistic" conception, it is not sufficient to use just one framework for a categorisation, e. g. just a purely geometrical one; rather it is advisable to use discriminations on three typical levels of a curriculum: the level of goals of educations, the geometrical level, and the geometrical aspect seen in a broader context of general beliefs of the "nature" of mathematics. To do so, three background theories were combined, namely the *theory of Geometrical Working Spaces* (Kuzniak, 2006), a classification of *goals of education* (Graumann, 1993) and a framework to analyse general understandings of mathematics, called the theory of *mathematical worldviews* (Grigutsch, Raatz, & Törner, 1998). Insofar, the central research question of this study is as follows: How can individual curricula on teaching elementary geometry be classified based on these three levels and are there any systematic connections between them? It will be argued that the answer is positive and that it is possible to identify two archetypes of systematic connections between these levels and that each of the nine teachers can be attached to one of the two archetypes.

2 Theoretical Background

Before we can start to describe the study and its method, it is necessary to make some remarks on the three theoretical backgrounds used for the classification.

2.1 Geometrical Working Spaces

The framework of Geometrical Working Spaces (GWS) is based on the idea that three *geometrical paradigms* are relevant to the history and philosophy of elementary geometry which are fundamentally different in ontological, epistemo-