

A portrait of Alfred Tarski, a man with short dark hair, wearing a suit and tie, looking directly at the camera. The portrait is overlaid with a semi-transparent teal color. A vertical red bar is visible on the left side of the cover.

Andrew McFarland  
Joanna McFarland  
James T. Smith Editors

# Alfred Tarski

Early Work in Poland –  
Geometry and Teaching

 Birkhäuser



*Alfred Tarski*

*This book is dedicated to Helen Marie Smith, in gratitude for her advice and support, and to Maria Anna McFarland, as she enters a world of new experiences.*

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Andrew McFarland • Joanna McFarland  
James T. Smith  
Editors

# Alfred Tarski

Early Work in Poland—Geometry  
and Teaching

with a Bibliographic Supplement

Foreword by Ivor Grattan-Guinness

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# Foreword

by Ivor Grattan-Guinness

## 1. Context

It is common in the history of ideas to find that the reception and development of some theories of a historical figure proceeded much more rapidly than others, both during his lifetime and afterward. As a result a historian can enjoy expounding the “neglected” contributions of the figure, and search for causes of this oversight. However, it is rather rare for a body of his work to disappear entirely from sight, especially when it was published at the time of its creation and the figure in question has been subject to considerable attention both by contemporary co-workers and by historians.

Yet this situation pertains to some of the early work of the great mathematician and logician Alfred Tarski (1901–1983). It is well known that he worked as a schoolteacher in Poland from the early 1920s and that he published papers in logic and mathematics regularly, mostly in Polish or Austrian journals; there is even an edition of English translations of versions of many of these papers (Tarski [1956] 1983). However, during this period he also published a substantial number of papers, comments, and problems in Polish journals oriented around mathematics education; but they are not handled in the fine biography, Feferman and Feferman 2004. Further, the journals in question escaped the attention of the German mathematics abstracting journal *Jahrbuch über die Fortschritte der Mathematik*. In addition, Tarski coauthored an elementary textbook on geometry in 1935 that has generally been overlooked. The purpose of the present volume is to bring this material to light in English translation, and also to amplify the historical context in which the teacher Tarski produced it in the first place.

There are two trends in the development of logic and the foundations of mathematics that became of major significance during this period, so that their bearing upon Tarski’s work are worth seeking. One was the development of nonclassical logics to complement the classical two-valued logic and, in the excessively polemical case of the intuitionism of L. E. J. Brouwer, to replace it (Mancosu 1998). Tarski does not seem to have been concerned with other logics in the papers here. The other was the growing recognition of the need to work in both logic and mathematics with *hierarchies* of theories, distinguishing mathematics from metamathematics (David Hilbert’s program) and logic from metalogic (a central feature of Kurt Gödel’s famous paper of 1931 on the incompleteness of first-order arithmetic, which stimulated his friend Rudolf Carnap to coin “metalogic” that year as a technical term. Tarski would soon contribute notably to this trend by coining the word “metalanguage” in connection with his semantic theory of truth of the 1930s.

## 2. Cleft

One would have thought that since logic was among other things a study of the theory of deduction and since mathematics deployed deduction in proofs of theorems and definitions of concepts, the two disciplines would happily live side by side. However, this has never been the case (as is already evident in, for example, Euclid's *Elements*); in particular, the rise in interest in foundational subjects from the late nineteenth century onward did not eliminate it at all.

A very important source of maintaining the cleft during the nineteenth century is the founding from the late 1810s onward of the “mathematical analysis” of real variables, grounded upon an articulated theory of limits, by the French mathematician Augustin Louis Cauchy. He and his followers extolled rigor, in particular careful nominal definitions of major concepts and detailed proofs of theorems. From the 1850s onward this aim was enriched by the German mathematician Karl Weierstrass and his many followers—they brought in, for example, multiple-limit theory, definitions of irrational numbers, and an increasing use of symbols—and from the early 1870s, Georg Cantor and his set theory. However, none of these developments explicitly drew upon any kind of logic.

This silence continued among the many set theorists who helped to develop measure theory, functional analysis, and integral equations (Jahnke 2003). Even the extensive dispute over the axiom of choice (a Tarski favorite) from 1904 onward focused mostly on its legitimacy as an assumption in set theory and mathematics and on the use of higher-order quantification (Moore 1982): its need to state an infinitude of independent choices within finitary logic was a trouble for logicians.

The creators of symbolic logics were exceptional among mathematicians in attending to logic, but they made little impact on their colleagues. The algebraic tradition with George Boole, Charles Sanders Peirce, Ernst Schröder, and others from the mid-nineteenth century was just a curiosity to most of their contemporaries. Similarly, when mathematical logic developed from the late 1870s, especially with Giuseppe Peano's “logistic” program at Turin from around 1890, it gained many followers there (Luciano and Roero 2010) but few elsewhere. However, followers in the 1900s included the Britons Bertrand Russell and Alfred North Whitehead, who adopted logistic (including Cantor's set theory) and converted it into their “logicistic” thesis that all the “objects” of mathematics could be obtained from it; G. H. Hardy but not many other mathematicians responded (Grattan-Guinness 2000, chapters 8 and 9). From 1903 onward Russell had also publicized the mathematical logic and arithmetic logicism put forward from the late 1870s onward by Gottlob Frege, which had gained little attention hitherto even from students of foundations and did not gain much more in the following decades. Hilbert's program of metamathematics attracted several followers at the University of Göttingen and a few elsewhere; however, its impact among mathematicians was limited even in Germany.

The situation in Poland is quite poignant; for soon after the re-creation of the country after the Great War, major schools of mathematicians and logicians emerged. Moreover, many of the mathematicians worked in set theory and/or its applications, so that links with logic could be close. In 1920 the Poles launched in Warsaw a journal, *Fundamenta Mathematicae*, to cover both disciplines (Lebesgue 1922). The two coeditors for logic were Jan Łukasiewicz and Stanisław Leśniewski, who had obtained chairs at the University of Warsaw in 1915 and 1919 respectively after taking doctorates at Lwów. They and their associates formed the largest community in the world working on logic and related topics (McCall 1967). I am told that Polish has many properties relevant to logic and set theory, which may have helped stimulate the interest in the first place. However, not many logic papers appeared in the journal (Tarski being the most frequent author), and the two logicians resigned from its board in 1928, with little regret from the mathematicians (Kuratowski 1980, 33–34).

By contrast, a most praiseworthy feature of Tarski's work is that he researched in both logic and branches of mathematics (especially geometry) at the same time, thereby consciously ignoring the cleft. In particular, the material newly come to light treats not only logic and set theory but also elementary geometry and common algebra. But he wrote it mostly in Polish, which will have reduced its market, although some of its contents appeared elsewhere in papers in German or French. Let us now encounter it in English guise.





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# Preface

Alfred Tarski is regarded as one of the four greatest logicians of all time—the others being Aristotle, Gottlob Frege, and Kurt Gödel.<sup>1</sup> Most notably, Tarski was largely responsible for designing the infrastructure on which most logical research has been based since 1950. Using that structure, he achieved major technical results in logic, foundations of geometry, and abstract algebra. Outside that framework, Tarski discovered major theorems in set theory and set-theoretic aspects of geometry, and completed some works about teaching geometry.

Tarski was born in Warsaw in 1901, and grew up there in a time of turmoil. He completed doctoral studies in 1924, just after a major expansion of the Polish system of universities. The economic climate was adverse, as was the growing antisemitism. Tarski obtained full-time employment as a secondary-school teacher, and worked part-time as a university assistant and researcher. During the next fifteen years, he gained world renown in the fields of set theory and logic. On a 1939 lecture tour, he was stranded, fortunately, in the United States, when the Germans and Soviets invaded Poland. After some trying years, he secured a professorship at the University of California, Berkeley. There, according to the eminent Polish philosopher Jan Woleński,

Tarski ... created the great Californian School of logic. He ... had a dominant influence upon the development of logic after World War II.<sup>2</sup>

After four decades of service at Berkeley, Tarski died there in 1983.

Much of Tarski's scientific work has been accessible and has become rather well known. Three years after his death, his colleagues Steven R. Givant and Ralph McKenzie published the remarkably complete *Collected Papers* volumes and a detailed bibliography. An excellent biography by Anita B. and Solomon Feferman is available in English and Polish.<sup>3</sup> Tarski took meticulous care to document his Berkeley research program, and that material is readily available for historical study. But a few of Tarski's early works have been difficult to access, and for some of them, hardly any background was even detectible. This was particularly true for some of his early work about geometry. He had completed that in Warsaw, which was largely destroyed during World War II. There is almost nothing of his left there, and Tarski's archive in Berkeley starts in 1939.

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<sup>1</sup> Corcoran 1991.

<sup>2</sup> Woleński 1989, 20.

<sup>3</sup> Tarski 1986a; Givant 1986; Feferman and Feferman 2004 and 2009. For descriptions of these works see section 16.2 and chapter 17.

The present book has three main goals:

- (1) To publish translations as necessary so that
  - Alfred Tarski's works will all be accessible in English, French, or German; and
  - his geometric works will all be accessible in English.
- (2) To provide scientific and cultural background information about the works translated here: their origin, context, structure, and impact.
- (3) To update Givant's 1986 bibliography of Tarski's publications, and include an annotated list of major studies of Tarski's life and work.

By including ample background material in this book, the editors have heeded an opinion expressed succinctly by a leading mathematics historian, David E. Rowe:

The type of knowledge mathematicians have produced has depended heavily on cultural, political, and institutional factors that shaped the various environments in which they have worked.<sup>4</sup>

The book stresses the connection between Tarski's work as a teacher and the subjects of some of his research. It does not itself pursue other connections between environment and research, but does aim to provide scholars interested in that kind of inquiry a glimpse of the background and routes to deeper and broader study. It does not supplant the Fefermans' 2004 biography, but fills some gaps in their coverage. In some cases, the present editors have included significant background that was apparently not familiar to others who have written about Tarski. This is especially true for the year, 1920, of the Polish–Soviet War, and for Tarski's work as a secondary-school teacher and teacher-trainer.

The present editors expect that further background material specifically about Tarski will come to light only through historians' studies of other students, teachers, and scientists associated with him. This is particularly the case for Tarski's disrupted first university year, 1918–1919. Since articles about mathematics or philosophy are usually edited to exclude material not directly related to their theoretical content, the social context of Tarski's work will probably be better revealed by investigating his activity in teaching and teacher-training. Researchers should adopt a maxim: if Tarski is one of the top four logicians of all time, he must be one of the top  $n$  thinkers, and what he thought about almost anything should be interesting!

This book is organized into four parts:

Part One . . . . <i>Debut</i>	Part Three . . . <i>Teaching</i>
Part Two . . . . <i>Geometry</i>	Part Four . . . . <i>Supplement</i>

The supplement contains some translations not directly related to geometry research or teaching, an update of the 1986 bibliography, and annotated lists of major studies of Tarski and his work.

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<sup>4</sup> Rowe 2003, 114.

Readers will find general background material in chapters 1, 3; 4, 8; and 9, 14: the first and last chapters of its three main parts. Background specific to particular translations is located in the chapters devoted to them.

Part One of the book, *Debut*, contains only one translation: Tarski's first published paper, *A Contribution to the Axiomatics of Well-Ordered Sets*, written in 1921 while he was still a student. Two background chapters describe his life through the completion of his doctoral study in 1924. They complement the Fefermans' biography, adding material about the turbulent years 1918–1920 in Warsaw. There remain gaps in the story: for example, it is still not clear why Tarski changed his major interest from biology to logic and mathematics.

Part Two, *Geometry*, is devoted to Tarski's work on equidecomposability. The background chapter 4 summarizes the elementary theory of area and volume, covered to some extent in secondary schools. It then considers the measure-theoretic approach that led Stefan Banach and Tarski to show in 1924 that set-theoretic decomposition alone will not yield a theory in accordance with intuition: a marble and the earth can each be dissected into the same finite number of disjoint sets of points, which are congruent in pairs. Their famous paper is translated in this part, along with Tarski's 1924 paper that showed that this counterintuitive result has no analogue in the plane. The latter paper was published in a journal aimed at secondary-school teachers. Attention is then turned to Tarski's work on the more elementary concept of decomposition of plane polygons into subpolygons whose interiors are disjoint but whose boundaries may overlap. Three elegant little papers are translated, which appeared during 1931–1932 in journals aimed at secondary-school students and their teachers. Two of those are by Tarski; the third, by another schoolteacher, Henryk Moese. Inadequate translations of the three were published obscurely half a century ago, but the translations in the present book are new. The subsequent impact of all five papers is traced in chapter 8, which concludes Part Two.

Part Three of the book, *Teaching*, presents a variety of material. Its first chapter describes Tarski's family situation and his teaching in a secondary school and in university lectures and research seminars. It complements the Fefermans' treatment of these aspects of his life. Tarski's 1929 report to teachers about an important research conference and his 1932 suggestions on teaching about circles are translated. During 1930–1932 Tarski published fourteen exercises to challenge teachers and talented students; they are translated and analyzed here. So are representative sections of his [1935] 1946 coauthored secondary-school text on geometry. The main portion of the present book concludes with chapter 14, which describes some of Tarski's activity during the 1930s that is background for several translations in the supplement, and leads to his 1939 voyage to the New World.

Part Four, the supplement, begins with chapter 15, which consists of translations of the eleven remaining works of Tarski that until now were accessible only in Polish. They are about various subjects not closely related to the earlier chapters, and are all very short. Each is accompanied by a brief discussion that places it in context and renders it intelligible. The most significant one is the report of Tarski's 1930 presentation in Lwów:

the first appearance in print of his celebrated theory of truth. The book concludes with chapters 16–18, which contain annotated lists that update the 1986 bibliography of Tarski's publications and identify major studies of his life and his work.

The material gathered in this book will increase the accessibility of Tarski's early work and explain some of its relationships to the intellectual, political, and social milieu of Poland between the world wars. The present editors hope that it will spur broader investigation into the connections between mathematics and its cultural setting during that era. This would be particularly welcome for the connection between mathematical research and mathematics education, as displayed by Tarski's research on geometry and his practice both in teaching secondary-school students and training secondary-school teachers. This hope is a major reason for including works of such contrasting mathematical sophistication in a single volume of selected translations.

Rough maps on pages xxii and xxiii depict Poland and the surrounding area, with the international borders of 1914 and 1924—before and after the First World War. Cities of importance to this book have been identified. For place names in this region, English versions are used when available. Otherwise, for places that have been in more than one country, the names used in this book are the official names used in 1924.

In 1936 Poland officially adopted some changes in spelling that affect words quoted in this book, particularly involving the letters *i*, *j*, and *y*. The editors have tried to adhere to the spellings in the original texts.

The translations are meant to be as faithful as possible to the originals.<sup>5</sup> Bibliographic references and personal names have been adjusted to conform with conventions of the present book. Some uses of alternative type styles for emphasis, enunciations, and personal names have been modified. The only intentional modernizations are punctuation and occasional changes in symbols, where Tarski's conflict with others used throughout this book. Those are discussed in the introductions to the individual translations. As an aspect of adjusting punctuation, the editors modified the use of white space to enhance visual organization. All [square] brackets in the translations enclose editorial comments. These are inserted, usually as footnotes, to document changes in technical terms, to note or suggest corrections for occasional errors in the originals, to clarify possibly troublesome translation details, and to explain a few passages that seem opaque.

Polish surnames often have gender-specific suffixes. Thus, the wife or unmarried daughter of Tarski is surnamed Tarska; the wife and the unmarried daughter of Łukasiewicz are surnamed Łukasiewiczowa and Łukasiewiczówna, respectively. Married women often use these names hyphenated with those of their husbands. The order varies; in this book, the husbands' surnames come second.

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<sup>5</sup> The introduction by Magda Stroińska and David Hitchcock to Tarski [1935] 2002 was helpful in planning the Polish translation process.

In this book, the term *gimnazjum* refers only to Polish secondary schools that prepared pupils for university studies during 1900–1939.

Spaces and diacritical and punctuation marks have been ignored during all alphabetizations, particularly for the bibliography and index. Warning: this resulted in alphabetization different from the Polish standard! Throughout the book, capitalization of Polish titles reflects the conventions of the Chicago 1993 manual, which disagree with those now taught in Polish schools.

The huge bibliography lists all and only works referred to in the book. Each entry indicates where citations occur. The author–date system is employed for citations: for example, *Tarski 1986a* is a citation for a work published under Tarski’s name in 1986. The present book mentions more than one author named Tarski; citations that include this surname alone are references to Alfred Tarski. Sometimes an author is to be inferred from the context, so that a date alone may also serve as a citation of a work.

Biographical information about more than sixty individuals involved with Tarski is presented in boxes or notes located in various chapters and cross-referenced, as appropriate, in others. The emphasis is on the years before 1945.<sup>6</sup> Sources for the data are identified by footnotes inside the boxes. The book’s index lists both subjects and persons. The latter entries include personal dates when known.

The project culminating in the present book started with brief conversations, years apart, between James T. Smith and Steven R. Givant at Berkeley logic colloquia. Smith recalled that during 1965–1970, he had heard Tarski speak eloquently to general audiences about the degree of equidecomposability of polygons. Although that material had been published, it was nevertheless virtually inaccessible. Smith mentioned this at the fall 2007 meeting of the State of Jefferson Mathematics Congress. Joanna and Andrew McFarland were also attending. Joanna is a teacher of Polish from Płock, Poland. Andrew, for whom Polish was also a first language, was on the mathematics faculty at Sonoma State University. The three discussed the possibility of working together to republish the equidecomposability papers. The critical resources for collaboration had converged: interest in the project and facility with logic, mathematics, English, and Polish. Later, Smith and Givant mused that a few other works from Tarski’s early years had suffered the same neglect, and concluded that publishing a volume of translations from Polish might be feasible. The McFarlands, Smith, and Springer editor Ann Kostant discussed this further. It was decided to include translations of representative sections of the virtually forgotten [1935] 1946 secondary-school text *Geometrja* by Tarski and two coauthors. With these translations all of Tarski’s geometric work would be accessible in English except the famous 1924 paper, in French, by Banach and Tarski on set-theoretic equidecomposability of geometric figures. This book’s selection of translations was rounded out by including that paper as well. Investigation of the background and impact of these works suggested the utility of including an update of the 1986 Tarski

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<sup>6</sup> Many of these individuals participated in the Polish underground or clandestine educational system during World War II. The famous 1944 book by Jan Karski is a gripping first-hand description of those activities.

bibliography. Springer Science+Business Media (Birkhäuser) agreed to this plan in October 2009.

For the present book, translations from Polish were drafted by the McFarlands, and edited jointly with Smith. Translations from other languages were done by Smith. The McFarlands carried out library and archival research in Poland, particularly on Tarski's teaching. Smith did that for material accessible in libraries in the United States. Smith assembled the background information into the present organization, and the result was edited jointly. Smith was responsible for the design and composition of the book.

The editors wish to acknowledge professors Edith Mendez and Elena A. Marchisotto for inspiration to undertake historical studies, and Helen M. Smith for her patience, generosity, insight, and ingenuity. Alfred Tarski's son, Prof. Jan Tarski, is especially recognized for his assistance to the present editors, and for his editorial work on other publications listed in chapters 16 and 17. On matters of content, the editors are grateful for the advice and assistance of Sheldon Axler, John Corcoran, Stanisław Domoradzki, Steven R. Givant, Jacek Juliusz Jadacki, Anna Jaroszyńska-Kirchmann, Andrzej Jerzmanowski, Anna Kozłowska, Witold Kozłowski, Renato Lewin, Paulo Mancosu, Antony Polonsky, V. Frederick Rickey, Janusz Rudziński, Andrzej Schinzel, James R. Shilleto, and Jan Zygmunt. For help with translation, we are indebted to Grażyna Ula Furman, Arek Goetz, Sergei Ovchinnikov, and Michael Thaler. We are immensely thankful for the library services provided by the Archiwum Akt Nowych, Archiwum Polskiej Akademii Nauk, Archiwum Państwowe Miasta Stołecznego Warszawy, Archiwum Państwowe w Lublinie, Archiwum Uniwersytetu Warszawskiego, Biblioteka Narodowa, Centralna Biblioteka Matematyczna Polskiej Akademii Nauk, Biblioteka im. Zielińskich Towarzystwa Naukowego Płockiego, Książnica Kopernikańska, Narodowe Archiwum Cyfrowe, the United States Holocaust Memorial Museum, the University of California libraries in Berkeley and Richmond, and for the splendid interlibrary loan services of San Francisco State University. This book was made possible by the retirement system of that university.

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# Contents

Foreword, by Ivor Grattan-Guinness .....	v
Preface .....	ix
Illustrations .....	xix
Maps of Central Europe .....	xxii

## Part One: Debut

Introduction .....	1
<b>1 School, University, Strife</b> .....	3
1.1 Coming of Age in Warsaw .....	3
1.2 Well-Ordered Sets .....	15
<b>2 A Contribution to the Axiomatics of Well-Ordered Sets (1921)</b> .....	19
<b>3 Doctoral Research</b> .....	31

## Part Two: Geometry

Introduction .....	43
<b>4 Area, Volume, Measure</b> .....	45
4.1 Area and Volume .....	45
4.2 The Measure Problem .....	51
4.3 Equidecomposability in the Plane .....	58
4.4 The Banach–Tarski Theorems .....	64
<b>5 On the Equivalence of Polygons (1924)</b> .....	77
5.1 §1 .....	80
5.2 §2 .....	83
5.3 §3 .....	84
5.4 Summary .....	90



<b>6</b>	<b><i>On Decomposition of Point Sets into Respectively Congruent Parts (1924)</i></b> . . . . .	93
6.1	General Properties of Equivalence by Finite or Denumerable Decomposition . . . . .	96
6.2	The Fundamental Theorems about Equivalence by Finite Decomposition . . . . .	106
	A. Euclidean Space with One or Two Dimensions . . . . .	106
	B. Euclidean Space of Three (and More) Dimensions . . . . .	109
	C. The Surface of a Sphere . . . . .	112
6.3	The Fundamental Theorems about Equivalence by Denumerable Decomposition . . . . .	115
<b>7</b>	<b>Degree of Equivalence of Polygons</b> . . . . .	125
7.1	Introduction . . . . .	126
7.2	<i>The Degree of Equivalence of Polygons (1931)</i> . . . . .	135
7.3	Moese's Contribution (1932) . . . . .	144
7.4	<i>Remarks on the Degree of Equivalence of Polygons (1932)</i> . . . . .	151
<b>8</b>	<b>Research Threads</b> . . . . .	159
8.1	Two Threads in Logic . . . . .	160
8.2	Thread: Well-Ordering and Finiteness . . . . .	161
8.3	Thread: Cardinal Arithmetic and the Axiom of Choice . . . . .	161
8.4	Thread: Application of Set Theory to Geometry . . . . .	162
8.5	Equidecomposability in Elementary Geometry . . . . .	162
8.6	Equidecomposability in Set Theory . . . . .	163
8.7	The Measure Problem . . . . .	165
8.8	Generalizing Cantor–Bernstein . . . . .	167

## Part Three: Teaching

<b>Introduction</b> . . . . .	169	
<b>9 Career and Family</b> . . . . .	171	
9.1	Employment and Marriage . . . . .	171
9.2	Teaching Geometry . . . . .	179
9.3	Teaching Teachers . . . . .	184
9.4	Research Seminars . . . . .	194
9.5	Warsaw/Poznań Congress, 1929 . . . . .	200
9.6	Organizations and Journals for Teachers . . . . .	203
9.7	<i>Parametr</i> and <i>Młody Matematyk</i> . . . . .	204
9.8	Tarski's Contributions to <i>Parametr</i> and <i>Młody Matematyk</i> . . . . .	211
9.9	Tarski's Coauthored Textbook <i>Geometrja</i> . . . . .	214

<b>10 Congress of Mathematicians of Slavic Countries (1929)</b> .....	225
<b>11 Circumference of a Circle (1932)</b> .....	229
<b>12 Exercises Posed by Tarski</b> .....	243
12.1 Diluting Wine .....	249
12.2 Iteration of the Absolute Value Symbol .....	250
12.3 Decomposition into Factors .....	251
12.4 Interesting Identity .....	253
12.5 Exercise 118 .....	255
12.6 Equidistant .....	256
12.7 System of Inequalities .....	257
12.8 Another System of Inequalities .....	259
12.9 Cutting a Rectangle Out of a Square .....	261
12.10 The Degree of Equivalence of Polygons .....	263
12.11 Postulate about Parallels .....	264
12.12 Analytic Geometry of Space .....	268
12.13 Stereometry .....	268
12.14 Arranging Two Segments in a Plane .....	271
<b>13 Geometry for the Third Gimnazjum Class (1935)</b> .....	273
13.1 §1 Circle and Disk .....	277
13.2 §2 Construction Problems .....	279
13.3 §3 Relative Positions of a Circle and a Line Lying in a Plane .....	281
13.4 §4 Construction Problems .....	285
13.5 Summary of §5–§24 .....	291
13.6 §25 Incommensurable Segments—Irrational Numbers .....	292
13.7 §26 Measuring Incommensurable Segments with a Unit .....	296
13.8 §27 Rational Approximations of Irrational Numbers —Operations on Irrational Numbers .....	297
13.9 Summary of §28–§32 .....	301
13.10 §33 Equivalent Polygons .....	302
13.11 §34 On Measuring Areas of Polygons .....	304
13.12 §35 Areas of Similar Polygons .....	316
<b>14 Teaching and Logic—to the New World</b> .....	319
14.1 Teaching .....	319
14.2 Logic .....	321
14.3 To the New World .....	330
14.4 Epilogue .....	336

## Part Four: Supplement

<b>Introduction</b> .....	337
<b>15 Assorted Contributions</b> .....	339
15.1 Discussion of Łukasiewicz 1925, on Understanding Deduction .....	340
15.2 Discussion of Greniewski [1927] 1928, on Action .....	341
15.3 Discussion of Czeżowski [1927] 1928, on Causality .....	343
15.4 Łukasiewicz 1928–1929, on Definitions .....	346
15.5 Tarski 1929, on Polish Pensions .....	349
15.6 Tarski 1930–1931, on the Concept of Truth .....	356
15.7 Tarski 1932, on Banach’s Measure .....	363
15.8 Discussion of Ajdukiewicz 1936, on Idealism .....	365
15.9 Discussion of Wilkosz 1936, on the Significance of Logic .....	367
15.10 Discussion of Zawirski 1936b, on Synthesis .....	370
15.11 Discussion of Kokoszyńska 1936a, on Relative Truth .....	374
15.12 Two Letters to Sierpiński, 1946–1947 .....	376
<b>16 Posthumous Publications</b> .....	385
16.1 Papers .....	385
16.2 Monographs .....	390
16.3 Letters .....	392
<b>17 Biographical Studies</b> .....	399
<b>18 Research Surveys</b> .....	411
18.1 1986, 1988 JSL Surveys .....	411
18.2 Other surveys .....	413
<b>Bibliography</b> .....	421
<b>Permissions</b> .....	473
<b>Index of Persons</b> .....	477
<b>Index of Subjects</b> .....	485

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# Illustrations

## Portraits

section, page

<i>Alfred Tarski</i> . . . . .	frontispiece
<i>Stefan Mazurkiewicz</i> . . . . .	1.1, 6
<i>Zygmunt Janiszewski</i> . . . . .	1.1, 6
<i>Tadeusz Kotarbiński</i> . . . . .	1.1, 6
<i>Leon Trotsky as the Devil</i> . . . . .	1.1, 11
<i>Alfred Teitelbaum (Tarski)</i> . . . . .	1.1, 13
<i>Stanisław Leśniewski</i> . . . . .	1.1, 13
<i>Kazimierz Twardowski</i> . . . . .	1.2, 18
<i>Kazimierz Pasenkiewicz</i> . . . . .	ch. 3, 32
<i>Aleksander Jabłoński</i> . . . . .	ch. 3, 34
<i>Mordchaj Wajsberg</i> . . . . .	ch. 3, 34
<i>Kazimierz Ajdukiewicz</i> . . . . .	ch. 3, 37
<i>Wacław Sierpiński</i> . . . . .	4.1, 48
<i>Stefan Banach</i> . . . . .	4.2, 56
<i>Felix Hausdorff</i> . . . . .	4.4, 68
<i>Kazimierz Kuratowski</i> . . . . .	4.4, 72
<i>Henryk Moese</i> . . . . .	7.1, 131
<i>Maria Tarska and Others in the 1919–1920 Polish–Soviet War</i> . . . . .	9.1, 175
<i>Ugo Amaldi</i> . . . . .	9.2, 182
<i>Federigo Enriques</i> . . . . .	9.2, 182
<i>David Hilbert</i> . . . . .	9.3, 185
<i>Mario Pieri</i> . . . . .	9.3, 187
<i>Jan Łukasiewicz</i> . . . . .	9.4, 196
<i>Mojżesz Presburger</i> . . . . .	9.4, 196
<i>Andrzej Mostowski</i> . . . . .	9.4, 198
<i>Wanda Szmielew</i> . . . . .	9.4, 198
<i>Antoni Marian Rusiecki</i> . . . . .	9.7, 209
<i>Stefan Straszewicz</i> . . . . .	9.7, 209
<i>Zygmunt Chwiałkowski and Others</i> . . . . .	9.9, 218
<i>Wacław Schayer</i> . . . . .	9.9, 220
<i>Maria Kokoszyńska and Alfred Tarski</i> . . . . .	14.2, 327
<i>Leon Chwistek</i> . . . . .	14.2, 329
<i>Janina Hosiassonówna</i> . . . . .	14.3, 335
<i>Adolph Lindenbaum</i> . . . . .	14.3, 335
<i>Tadeusz Czeżowski</i> . . . . .	15.3, 345
<i>Witold Wilkosz</i> . . . . .	15.9, 369
<i>Zygmunt Zawirski</i> . . . . .	15.10, 373
<i>Alfred Tarski</i> . . . . .	ch.16, 384

## Figures

Many figures are not titled, but numbered as in the original versions of translated material. For example, figure 1 in chapter 2 of this book is adapted from figure 1 in the original version of Tarski [1921] 2014.

	<i>section, page</i>
<i>Alfred Tarski Signature</i> .....	frontispiece
<i>Central Europe in 1914</i> .....	xxii
<i>Central Europe in 1924</i> .....	xxiii
<i>Alfred's Autumn 1919 –Winter 1920 Enrollment Record</i> .....	1.1, 6
<i>1920 Propaganda (3 figures)</i> .....	1.1, 11
<i>Przegląd Filozoficzny Cover</i> .....	ch. 2, 20
<i>Figure 1</i> .....	ch. 2, 28
<i>Figures 2–3</i> .....	ch. 2, 29
<i>To the Bursar, Autumn 1923</i> .....	ch. 3, 35
<i>Three Signatures</i> .....	ch. 3, 38
<i>Przegląd Matematyczno-Fizyczny, Masthead</i> .....	4.3, 58
<i>Hausdorff's Classification of Rotations in <math>G</math></i> .....	4.4, 68
<i>Przegląd Matematyczno-Fizyczny, Cover</i> .....	ch. 5, 78
<i>A Square and an Isosceles Right Triangle with the Same Area</i> .....	5.3, 85
<i>Fundamenta Mathematicae 6 (1924), Cover and Table of Contents</i> .....	ch. 6, 94
<i>Parallelograms with the Same Base and Altitude</i> .....	7.1, 128
<i>Adaptation of Wojtowicz's Argument</i> .....	7.1, 128
<i>Gimnazjum in Kepno, Where Henryk Moese Taught</i> .....	7.1, 131
<i>Dividing a Square into Two Polygonal Regions</i> .....	7.1, 134
<i>Figures 1 to 4</i> .....	7.2, 136
<i>Figures 5 to 7</i> .....	7.2, 141
<i>Figure 1</i> .....	7.3, 145
<i>Figure 2</i> .....	7.3, 146
<i>Figure 3</i> .....	7.3, 148
<i>Figures 4 to 6</i> .....	7.3, 149
<i>Figure 7</i> .....	7.3, 150
<i>Warsaw, 1925: 140–150 Marszałkowska Street</i> .....	9.1, 173
<i>51 Koszykowa Street</i> .....	9.1, 178
<i>4 Sutkowski Street</i> .....	9.1, 178
<i>Hilbert's Foundations of Geometry: Title Page</i> .....	9.3, 185
<i>Pieri's Point and Sphere Memoir: Polish Translation</i> .....	9.3, 187
<i>Tarski's Teaching Load at the University of Warsaw (table)</i> .....	9.3, 191
<i>First Congress of Mathematicians of Slavic Countries (group portrait)</i> ....	9.5, 201
<i>General Polish Exposition, Poznań, May–September 1929</i> .....	9.5, 202
<i>Cover and Contents, Młody Matematyk 2 (1931, issue 3)</i> .....	9.7, 206
<i>Masthead and Contents, Parameter 2 (1931, issues 8–10)</i> .....	9.7, 206

	<i>section, page</i>
<i>Advertisement for Geometrja</i> .....	9.9, 215
<i>Figure 1: Graph of Function <math>g \rightarrow \log_g n</math></i> .....	12.4, 254
<i>Figure 2: Equidistant Curve</i> .....	12.6, 256
<i>Figure 3</i> .....	12.7, 258
<i>Figures 4 and 5</i> .....	12.9, 262
<i>Figures 6 and 7</i> .....	12.11, 266
<i>Figure 8: Tarski's Parallel Postulate</i> .....	12.11, 266
<i>Figure 9</i> .....	12.13, 270
<i>Geometrja Table of Contents</i> .....	ch.13, 275
<i>Geometrja Cover</i> .....	ch.13, 276
<i>Figure 1, for §1*</i> .....	13.1, 277
<i>Figure 2, for §2</i> .....	13.2, 279
<i>Figure 3, for §2</i> .....	13.2, 280
<i>Figures 4 and 5, for §3</i> .....	13.3, 281
<i>Figure 6, for §3</i> .....	13.3, 282
<i>Figures 7 and 8, for §3</i> .....	13.3, 284
<i>Figures 9 and 10, for §4</i> .....	13.4, 286
<i>Figure 11, for §4</i> .....	13.4, 287
<i>Figure 12, for §4</i> .....	13.4, 288
<i>Figure 13, for §4</i> .....	13.4, 289
<i>Figure 75, for §25</i> .....	13.6, 292
<i>Figure 76, for §25</i> .....	13.6, 294
<i>Figure 77, for §27</i> .....	13.8, 298
<i>Figure 78, for §27</i> .....	13.8, 299
<i>Figures 106 to 110, for §33</i> .....	13.10, 303
<i>Figures 111 and 112, for §34</i> .....	13.11, 305
<i>Figures 113 to 116, for §34</i> .....	13.11, 306
<i>Figures 117 and 118, for §34</i> .....	13.11, 308
<i>Figures 119 and 120, for §34</i> .....	13.11, 309
<i>Figure 121, for §34</i> .....	13.11, 310
<i>Figures 122 to 124, for §34</i> .....	13.11, 311
<i>Figure 125, for §34</i> .....	13.11, 312
<i>Figure 126, for §34</i> .....	13.11, 315
<i>Figures 127 to 130, for §35</i> .....	13.12, 317
<i>Fencing</i> .....	14.2, 329
<i>Headlines from the Völkischer Beobachter</i> .....	14.3, 330
<i>Gdynia–America Poster</i> .....	14.3, 332
<i>M/S Piłsudski Menu</i> .....	14.3, 332
<i>Arrival in New York</i> .....	14.3, 332

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\* This and the following § numbers refer to sections of *Geometrja*.



*Central Europe in 1914*



*Central Europe in 1924*



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# Part One

# Debut

These first three chapters describe Alfred Tarski's childhood, schooling, and university studies during a time of political chaos and threats of war. Chapter 1 includes background for his first paper, *A Contribution to the Axiomatics of Well-Ordered Sets*, published while he was still a student. The paper is translated in chapter 2. Chapter 3 provides background for Tarski's doctoral research, supervised by Stanisław Leśniewski.

Tarski's life in Poland unfolded amid a chaotic vortex of political, social, economic, and scientific developments. Accounts of events, people, and ideas were merged from several dimensions to form the linear sequence of pages of this book. That is reflected in the background chapters 1 and 3 by the use of boxes interspersed in the main narrative. They contain biographical sketches of some persons associated with Tarski, and informational essays about several other topics. Each box can be read independently: readers are not expected to visit them in sequence. Cross-references refer to them from the main narrative.

# School, University, Strife

The first section of this chapter describes Alfred Tarski's childhood, schooling, and initial university studies in Warsaw, the city that in 1918 became the capital of the new independent Polish republic. The second section provides background for his first publication, a paper on set theory and logic that he completed while still a student. That paper is translated in the next chapter.

## 1.1 Coming of Age in Warsaw

Alfred Tarski was born Alfred Teitelbaum in January 1901 in Warsaw, Poland, an oppressed part of the Russian Empire.<sup>1</sup> His father, Ignacy, or Izaak, stemmed from Warsaw; his mother, Róża Prussak, from Łódź. Their families were engaged in the lumber and textile businesses. The Teitelbaums had two children: Alfred's brother, Waclaw, was two years younger. The family lived at 51 Koszykowa Street, apartment 14, on the second floor of a five-story building that they owned.<sup>2</sup> It was located in the center of the city, about three kilometers from the University of Warsaw. From September 1910 to summer 1915, Alfred attended the State Gimnazjum 4 in Warsaw.<sup>3</sup> The family was Jewish, but secular in outlook. The language at home was Polish; in school, Russian. Alfred was precocious, particularly in languages: he studied French, German, Latin, and Greek at school, and after school went to synagogue to learn Hebrew. Alfred translated a German story into Polish at age twelve as an anniversary gift to his parents. Even earlier, he had shown interest in politics and social justice.

---

<sup>1</sup> For biographical information supporting this section, unless another source is cited, consult the biographies by Anita B. and Solomon Feferman (2004) and by Jacek Juliusz Jadacki (2003a). Also note the descriptions of those works in chapter 17. The Fefermans emphasized personal-interview sources; Jadacki, published records. For historical and sociological information, consult the works by Norman Davies (1982, volume 2, chapters 18–19), Celia S. Heller (1994), and Richard M. Watt (1979).

<sup>2</sup> The spellings of the names are from Tarski [1918] 2014, translated in section 16.4. Jadacki (2003a, 143) verified ownership from 1930 data. Jadacki and the Fefermans (2004, 8) claimed that 15 Koszykowa had been destroyed in the 1940s. But it still stands as 51a Koszykowa: see Golińska, Porębska, and Srebrny 2009a and 2009b. It is pictured on page 176 of the present book.

<sup>3</sup> Tarski [1918] 2014: *rzqd gimnazjum*. A Polish secondary school that prepared students for eventual university study was called a *gimnazjum* or *gimnazjum*. The latter term is used in this book to avoid confusion with various types of secondary schools of other times and places.

For more than a century, Poland had been partitioned between Russia on the east, Germany to the west, and Austria to the south. Thoughts of Polish unification and independence had long spawned agitation and intrigue. World War I broke out in August 1914 for other reasons. Its conclusion four years later resulted in a unified Polish republic, but almost by accident. Many Polish military organizations were formed in 1914, and within two years, nearly two million Polish men were involved. Poles had enjoyed greater freedom in the Austrian partition, and the Germans made promises, so most of the Polish armies served, under the leadership of Józef Piłsudski, with Austria and Germany against Russia. Russia's allies Great Britain and France regarded the question of Polish independence as an internal Russian matter. Piłsudski played each side against the other. International posturing and intrigue increased in intensity. Amid this clamor, Alfred became a Polish nationalist.

In September 1915 Alfred transferred to the small, elite Mazowieckie Gimnazjum.<sup>4</sup> Its faculty were highly educated scholars, including two with doctorates in philosophy from the University of Lwów: Stefan Frycz and Bogdan Nawroczyński. Alfred's favorite subject was biology; his teacher, Stanisław Przylecki, had recently earned a medical degree in Zurich. Alfred's brilliance impressed both his teachers and his fellow students. That summer, the Germans entered Warsaw, which they would occupy until the end of the war. They instituted a number of reforms immediately, including permission for Alfred's school to switch its instruction from Russian to Polish.

Since 1870 the University of Warsaw had functioned as a Russian institution, serving the Imperial establishment. Most Polish students had to attend university abroad. After student boycotts and the outbreak of the war, the Russians closed the university altogether in 1914 and moved its faculty back to the homeland. The German occupation supported its autumn 1915 reopening as a Polish university. The new Polish faculty were assembled from various institutes in Warsaw, from universities in the other partitions of Poland, and from exile abroad. The university expanded rapidly during the war years, from one thousand to more than four thousand students. Most belonged to the urban middle or upper class, from central Poland; about 75% had to work to offset expenses. About 65% were male; and about 25%, Jewish. During that time the philosophical faculty grew to about forty, of all ranks.<sup>5</sup> According to the mathematician Kazimierz Kuratowski, who was a student in Warsaw at that time,

... the restored institutions of higher education were ... a fulfilment of the dreams of many generations, the attainment of the goal of a persistent struggle for Polish education. Therefore, beside young students in classes one could see adult representatives of the Warsaw intelligentsia, for whom a direct contact with the restored Polish universities and colleges was a deep emotional experience.

... the atmosphere in which the institutions of higher education in Warsaw began their work ... released a great creative potential ... which produced a surprising development in many branches of science, including Polish mathematics.

---

<sup>4</sup> Tarski [1918] 2014: *Szkola Ziemi Mazowieckiej*.

<sup>5</sup> Garlicki 1982, 49, 53, 314–315, 343. See also Manteuffel 1936, 156–175.

Two mathematicians, the topologists Zygmunt Janiszewski and Stefan Mazurkiewicz, gathered several others, including Samuel Dickstein, Stefan Kwietniewski, and the logician Jan Łukasiewicz, to start building the mathematics faculty.<sup>6</sup>

At the Mazowieckie Gimnazjum, Alfred prepared to enter the university. He was graduated in June 1918 with “excellent, 5” marks in all subjects covered that year:

Polish	German	History	Logic	Mathematics	Physics
Latin	French	Civics	Hygiene	Cosmography	

His mathematics courses had included analytic geometry but not calculus. Two of his nine classmates were too ill to complete the year. Typhus was raging, due to the huge influx of refugees into Warsaw, overcrowding, and poor sanitation. For example, 775 new cases were reported in the city during the week ending 23 February. Nearly three-fourths of these were among the Jewish population; the fatalities, about 9% of the victims, were most common among those over forty, and twice as prevalent among Christians.<sup>7</sup>

Alfred’s graduation picture, on page 13, shows his intensity. In October 1918, he enrolled in a broad university curriculum, intending to concentrate in biology.<sup>8</sup> The war ended officially in November. The Germans released Piłsudski, whom they had imprisoned for a year when he stopped cooperating. Immediately, he assumed command of the Polish government in Warsaw. But armed conflict continued in the German part of Poland to the west. The university suspended classes for the academic year 1918–1919 and urged all its students to join the Polish army; about half did so. Alfred was declared unfit for service, but his studies were interrupted by the continuing Polish struggle for independence. The present editors do not know what occupied Alfred during the rest of 1918–1919.<sup>9</sup>

In 1918, Janiszewski published a plan for realizing in the discipline of mathematics the great creative potential that had stemmed from the fulfilment of dreams for restored institutions. According to Kuratowski,

One of the principal means suggested... for attaining that end was the concentration... in a relatively narrow field of mathematics... one in which Polish mathematicians had common interests and... achievements which counted on a world scale. This field comprised set theory together with topology, and the foundations of mathematics together with mathematical logic.

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<sup>6</sup> Kuratowski 1980, 20, 25–26, 28. Łukasiewicz served in the university administration (Łukasiewicz[1953] 1994, 133). For portraits and biographical sketches of Janiszewski and Mazurkiewicz, see pages 6 and 14; for Łukasiewicz, see section 9.4; a biographical sketch of Kwietniewski is in section 9.3.

<sup>7</sup> Szkoła Ziemi Mazowieckiej [1918] 1927. Jadacki 2003a, 141. The typhus report Goodall 1920 presents a vivid account of conditions in Warsaw. Tuberculosis was a comparable threat (Wynot 1983, 340).

<sup>8</sup> Tarski [1918] 2014, translated in section 16.4. Tarski 1924f.

<sup>9</sup> Garlicki 1982, 341; Manteuffel 1936, 26–28. The archive document Warsaw 1918–1919 has top and bottom parts. The top, from the university secretariat, 8 November 1918, certified that Alfred was student number 2909 of the Philosophical Faculty (*Wydział*) and said that he should give it to the military and return it when he was released from service. The bottom, from a military doctor on 5 February 1919, said “By reason of §1D and age, the requirement not applying, [Alfred] was declared unfit for military service.”

Semestr *zimowy*

*Biblioteka Korzystania* №  
Roku akad. 1919/20

Imię i Nazwisko Wykładającego	Tytuł Wykładu	Liczba wykładów	CZESNE	POWIADZENIE KWESTURY	Własnoręcznie poświadczanie Wykładających			
					Podwizdzenie o osobistym zapoznaniu ze studenta	Data	Powizdzenie rozpoczęcia	Data
St. Mazurkiewicz	Rechnunek różniczkowy i całkowy	4	16		St. Mazurkiewicz	4/10	St. Mazurkiewicz	4/10
Z. Janiszewski	Geometria analityczna i cząstek	4	16		Z. Janiszewski	5/10	Z. Janiszewski	5/10
W. Sierpolski	Teoria mnogości i liczb	4	16		W. Sierpolski	5/10	W. Sierpolski	9/10
Z. Sierpolski	Wymiarzalniki i równania różnicowe	1	4		Z. Sierpolski	5/10	Z. Sierpolski	9/10
St. Lemnarski	Chemia z sokami podłoża matematyki	2	-		St. Lemnarski	15/10	St. Lemnarski	15/10
S. Pienkowski	Fizyka do trójkątów	4	16	PAROWOZ Cesne 12 Opłata za korzystanie z pracowni 12	S. Pienkowski	6/10	S. Pienkowski	10/10
Kotarbiński	Kurs logiki elementarnej	2	8	Razem 72 Oltre 12 wskazanie 12	Kotarbiński	11/10	Kotarbiński	11/10
Petracycki	Sociologia	6	-	KWESTURA	Petracycki	13/10	Petracycki	13/10

Dziekan Z. Hoyerdy 13. 11. 1920.

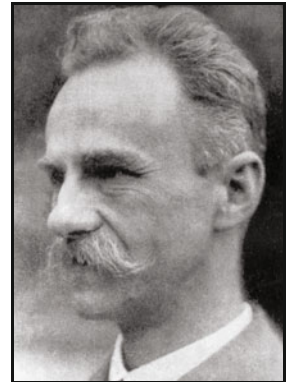
Alfred's Autumn 1919–Winter 1920 Enrollment Record



Stefan Mazurkiewicz around 1930



Zygmunt Janiszewski around 1915



Tadeusz Kotarbiński in 1933

Janiszewski, Łukasiewicz, and Mazurkiewicz would undertake that development. They were joined by the Polish mathematician of greatest world note, who had come to Warsaw that year: Waclaw Sierpiński, a specialist in set theory.<sup>10</sup>

Alfred must have caught a glimpse of this new direction during the disrupted year 1918–1919. Reenrolling for the academic year 1919–1920, he would continue some of his general scientific studies. But his new emphasis was mathematics and logic.<sup>11</sup> The atmosphere was electric:

... students could be met more often at political rallies and meetings than in university lecture rooms or laboratories.<sup>12</sup>

Withstanding the distraction, Alfred signed up for thirty-one hours of classes per week. The corresponding entries of his enrollment record are displayed on the facing page.<sup>13</sup> They show that Alfred attended

lecture/exercises courses by

- Mazurkiewicz on differential calculus
- Janiszewski on analytic geometry, and
- Sierpiński on set theory;

lectures by

- Sierpiński on determinants and linear equations;

exercises with

- Stanisław Leśniewski on foundations of mathematics; and

lectures by

- Stefan Pieńkowski on experimental physics,
- Tadeusz Kotarbiński on elementary logic, and
- Leon Petrażycki on sociology.

The stamp near the bottom of the left-hand page is the bursar's receipt for payment of 76 Polish marks. Alfred was charged no laboratory fees, and only for the nineteen hours of lectures checked in the third column. (Footnotes will lead to biographical sketches and portraits of many of Alfred's teachers.)<sup>14</sup>

<sup>10</sup> Janiszewski's proposal ([1918] 1968) was published by the Mianowski Fund, which supported many academic activities described in this book. For more information about the fund, consult a box in section 9.3. Kuratowski 1980, 29–31. For a portrait and biographical sketch of Sierpiński, see section 4.1.

<sup>11</sup> Givant (1991, 28) mentioned this change but presented no background for its occurrence in 1918–1919.

<sup>12</sup> Garlicki 1982, 341.

<sup>13</sup> Tarski 1924f. The legend at the top reads "Semestr *zimowy*. Roku akad. 1919/20." When Alfred's enrollment booklet was issued in 1918, academic years consisted of winter and summer semesters. In 1919–1920, the university converted to three trimesters: autumn, winter, summer (*jesień, zima, letni*). Its documentation placed data for the first two trimesters in the space for the former winter semester. The headings identify columns for lecturers' names, lecture titles, hours, tuition, bursar's certification, and lecturers' signatures and dates to certify enrollment and attendance. Summer data are on the following pages. Jadacki (2003a, 142) reported some of this information, but inaccurately.

<sup>14</sup> Biographical sketches of Kotarbiński and Leśniewski are on page 9; portraits, on pages 6 and 13.