Algebra and Applications

Henri Lombardi
Claude Quitté

Commutative Algebra: Constructive Methods

Finite Projective Modules

Springer

Commutative Algebra: Constructive Methods

Algebra and Applications

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Henri Lombardi
University of Franche-Comté
Besançon
France

Claude Quitté
University of Poitiers
Poitiers
France

English translation by Tania K. Roblot

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To James Brewer

Preface to the French Edition

This book is an introductory course to basic commutative algebra with a particular emphasis on finitely generated projective modules, which constitutes the algebraic version of the vector bundles in differential geometry.

We adopt the constructive point of view, with which all existence theorems have an explicit algorithmic content. In particular, when a theorem affirms the existence of an object – the solution of a problem – a construction algorithm of the object can always be extracted from the given proof.

We revisit with a new and often simplifying eye several abstract classical theories. In particular, we review theories which did not have any algorithmic content in their general natural framework, such as Galois theory, the Dedekind rings, the finitely generated projective modules, or the Krull dimension.

Constructive algebra is actually an old discipline, developed among others by Gauss and Kronecker. We are in line with the modern "bible" on the subject, which is the book by Ray Mines, Fred Richman and Wim Ruitenburg, *A Course in Constructive Algebra*, published in 1988. We will cite it in abbreviated form [MRR].

This work corresponds to an MSc graduate level, at least up to Chap. XIV, but only requires as prerequisites the basic notions concerning group theory, linear algebra over fields, determinants, modules over commutative rings, as well as the definition of quotient and localized rings. A familiarity with polynomial rings, the arithmetic properties of $\mathbb{Z}$ and Euclidian rings is also desirable.

Finally, note that we consider the exercises and problems (a little over 320 in total) as an essential part of the book.

We will try to publish the maximum amount of missing solutions, as well as additional exercises on the web page of one of the authors:

http://hlombardi.free.fr/publis/LivresBrochures.html.

Acknowledgments

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August 2011

Henri Lombardi
Claude Quitté

Preface to the English Edition

In this edition, we have corrected the errors that we either found ourselves or that were signalled to us.

We have added some exercise solutions as well as some additional content. Most of that additional content is corrections of exercises, or new exercises or problems.

The additions within the course are the following. A paragraph on the null tensors added at the end of Sect. IV-4. The paragraph on the quotients of flat modules at the end of Sect. VIII-1 has been fleshed out. We have added Sects. 8 and 9 in Chap. XV devoted to the local-global principles.

None of the numbering has changed, except for the local-global principle XII-7.13, which has become XII-7.14.

There are now 297 exercises and 42 problems.

Any useful precisions are on the site:

http://hlombardi.free.fr/publis/LivresBrochures.html.

Acknowledgment

We cannot thank Tania K. Roblot enough for the work achieved translating the book into English.

May 2014

Henri Lombardi
Claude Quitté

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About the Authors

Henri Lombardi Maître de Conférences at the Université de Franche-Comté. His research focuses on constructive mathematics, real algebra and algorithmic complexity. He is one of the founders of the international group M.A.P. (Mathematics, Algorithms, Proofs), created in 2003: see the site http://map.disi.unige.it/. E-mail: henri.lombardi@univ-fcomte.fr. http://hlombardi.free.fr

Claude Quitté Maître de Conférences at the Université de Poitiers. His research focuses on effective commutative algebra and computer algebra. E-mail: claude.quitte@math.univ-poitiers.fr

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