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Norma B. Goethe
Philip Beeley
David Rabouin *Editors*

G.W. Leibniz, Interrelations between Mathematics and Philosophy

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David Rabouin
Editors

G.W. Leibniz, Interrelations Between Mathematics and Philosophy

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Editors
Norma B. Goethe
School of Philosophy
National University of Cordoba
Córdoba
Argentina

David Rabouin
CNRS
University Paris Diderot
Paris Cedex 13
France

Philip Beeley
Faculty of History
University of Oxford
Oxford
United Kingdom

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Preface

The papers in this collection focus on the study of Leibniz's mathematical and philosophical thought and the interrelations between the two. They take advantage of the fact that we are today in the privileged position of being able to take a fresh look at material which has long been available in conjunction with those letters and papers recently published thanks to the remarkable efforts of the editors of the Academy Edition. With the benefit of a considerably extended textual basis, compared even to twenty years ago, we seek to examine Leibniz's mathematical practice with philosophical eyes exploring his goals and the underlying values and ideas that guided so many of his investigations.

The present volume traces its origin to a memorable workshop on the interrelationships between mathematics and philosophy in G. W. Leibniz which was organized by Mic Detlefsen and David Rabouin, and which took place at Université Paris Diderot (Laboratoire SPHERE, CNRS, UMR 7219) and at the École Normale Supérieure in Paris, 8–10 March 2010. The workshop was conceived within the framework of the "Ideals of proof" project under the direction of Detlefsen and funded by the Agence Nationale de la Recherche. Besides providing the ideal setting for discussion, that event revealed a common sentiment amongst all participants that a more in-depth study of the interrelations between these two fundamental aspects in Leibniz's thought was not only highly desirable, but also most timely on account of growing interest in the philosophy and history of mathematical practice.

Initial plans for this volume were drawn up immediately after the workshop by Norma Goethe during long hours of lively discussions over coffee and with three other participants, Richard Arthur, Philip Beeley, and David Rabouin, in the wonderful old Café Gay Lussac at the corner of Rue d'Ulm and Rue Claude Bernard. In fact, Norma Goethe, a fellow at the Lichtenberg-Kolleg (University of Göttingen) for the academic year 2009–2010, came from Göttingen with the undisclosed aim of persuading Philip and David of the timeliness of the project and invited them to join the editorial team. She should like to thank the Lichtenberg-Kolleg (University of Göttingen) and the German Research Foundation (DFG) for providing support for her participation in the workshop and the ongoing work towards the present volume which took her to Oxford and Nancy for exchanges with Philip and David. All of the editors should like to express their sincere gratitude to Jed Buchwald, editor of the

Archimedes series, for his interest in the project and also to Lucy Fleet and Mireille van Kan for their patience in the face of considerable delays in submission.

Some of the essays commissioned for this volume have grown out of papers presented in Paris, while others have been conceived and written since that time specifically for publication in this volume. All contributions have in no small measure benefitted from those three days of intense intellectual exchange and debate first in the Rue d'Ulm and then on the banks of the Seine in the Rue Thomas Mann.

The editors should like to thank all the participants of the workshop for the insights on Leibniz's mathematics which they shared and for the fruitful exchanges that were thereby made possible. Their thanks go especially to Mic Detlefsen who understood the significance of organizing such a scholarly gathering at that time and for the intellectually stimulating way in which he conducted the workshop. Particularly remembered is how his enthusiasm engendered lively interaction between all participants and how discussion continued through coffee breaks and well into the evenings.

In addition to thanking the authors who contributed to this volume, the editors should also like to thank all of the invited referees for the way in which they brought to bear their dedication to high scholarly standards. Besides those listed, we should also like to thank Marco Panza for the sound academic advice he gave. Special thanks go to Siegmund Probst for his unlimited generosity in providing all kinds of assistance to our book project. Finally, we should like to express our gratitude to Kirsti Andersen and Henk Bos for insightful exchanges and comments, wonderful conversations, and a most enjoyable time spent on the Rive Gauche after the conference was over.

Norma B. Goethe
Philip Beeley
David Rabouin

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Contributors

Richard T. W. Arthur Department of Philosophy, McMaster University, Hamilton, ON, Canada

Philip Beeley Faculty of History, University of Oxford, Oxford, UK

Norma B. Goethe School of Philosophy, National University of Cordoba, Cordoba, Argentina

Emily R. Grosholz Department of Philosophy, Pennsylvania State University, University Park, PA, USA

Laboratoire SPHERE, UMR 7219, CNRS—Université Paris Diderot, Paris, France

Douglas M. Jesseph Department of Philosophy, South Florida University, Tampa, FL, USA

Eberhard Knobloch Technische Universität Berlin, Berlin, Germany

Samuel Levey Philosophy Department, Dartmouth College, Hanover, NH, USA

Siegmund Probst Leibniz Archives, Hanover, Germany

David Rabouin Laboratoire SPHERE, UMR 7219, CNRS—Université Paris Diderot, Paris, France

Invited Referees

O. Bradley Bassler University of Georgia, USA

Daniel Garber Princeton University, USA

Emily R. Grosholz Pennsylvania State University & REHSEIS / SPHERE, University Paris Diderot, Paris, France

Niccolò Guicciardini University of Bergamo, Italy

Samuel Levey Dartmouth College, USA

Jeanne Peiffer (CNRS), Centre Alexandre Koyré, Paris, France

Siegmund Probst Leibniz Archives, Hanover, Germany

Staffan Rodhe University of Uppsala, Sweden

Donald Rutherford University of California at San Diego, USA

Part I
Mathematics and Philosophy

The Interrelations Between Mathematics and Philosophy in Leibniz's Thought

Norma B. Goethe, Philip Beeley and David Rabouin

Les Mathematiciens ont autant besoin d'estre philosophes, que les philosophes d'estre Mathematiciens.

Leibniz to Malebranche, 13/23 March 1699 (A II, 3, 539)

1 A Mixture of Philosophical and Mathematical Reflections and Deliberations

The aim of this collection is to explore the ways in which mathematics and philosophy (metaphysics and broader philosophical questions) are interrelated in the letters and papers of Gottfried Wilhelm Leibniz. Taking up one of his most notable expressions, the essays collected in this volume are all in some way concerned with "a curious mixture of philosophical and mathematical thought" which characterizes Leibniz's reflections and deliberations.[1] One of our principal aims in editing the present volume is to address the interrelations between mathematics and philosophy as far as possible without drawing on grand reconstructions which in the past all too often were based on insufficient evidence or what scholars conceived of as ad hoc programmatic stances, a typical example being Leibniz's so easily misunderstood pronouncement: "My metaphysics is all mathematics, so to speak, or could

[1] In an exchange with Basnage de Bauval, Leibniz revealed his intention to publish his correspondance with Arnauld and advanced what was to be expected from the content of his letters in these terms: "Il y aura un melange curieux de pensées philosophiques et Mathematiques qui auront peut-estre quelque fois la grace de la nouveauté"; Leibniz to Basnage de Bauval, 3/13 January 1696 (A II, 3, 121).

N. B. Goethe (✉)
School of Philosophy, National University of Cordoba, Cordoba, Argentina
e-mail: ngoethe@ffyh.unc.edu.ar

P. Beeley
Faculty of History, University of Oxford, Oxford, UK
e-mail: philip.beeley@history.ox.ac.uk

D. Rabouin
Laboratoire SPHERE, UMR 7219, CNRS—Université Paris Diderot, Paris, France
e-mail: davidrabouin2@gmail.com

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become so".[2] The difficulties presented by such reconstructions were already apparent when they emerged at the beginning of the last Century, during the second "Leibniz Renaissance" (the first having occurred in the eighteenth Century). Commentators such as Léon Brunschvicg followed an approach already adopted by the neo-Kantian philosopher Cassirer. Searching to defend him from attacks by Russell and Couturat[3], Brunschvicg criticized those who tended to confuse Leibniz's merely programmatic pronouncements with the position or rather positions which he actually maintained, which Brunschvicg termed his "real logic":

> We do not have the right to claim that Leibniz's philosophy is, properly stated, unambiguously and without ulterior motive, a *panlogism*. It would necessitate, in effect, that the relation of the predicate to the subject be achieved. In fact, the principles of 'the real logic, or a certain general analysis independent of algebra', as Leibniz put it in a letter to Malebranche, bring us back from traditional logic to differential calculus. The alternative expressed here was not completely satisfying for Leibniz in respect of his philosophical ambitions: for him, just as in the case of geometry for Descartes, differential calculus was only the most convincing 'sample' of his method, and he never gave up the project of a system of universal logic, in which the new mathematics would enter as a particular case. This is beyond doubt, but it only concerns, once more, the dream of what leibnizianism should be according to Leibniz—a dream condemned to be lost in the clouds of a tireless imagination and that for two centuries were believed to be without fruit.[4]

But despite such criticism, Brunschvicg himself (as a reflection of his time) offered his own reconstruction. He was convinced that it was possible to start from a coherent set of theses thus setting the ground for what he conceived of as Leibniz's "mathematical philosophy", while accepting that tensions and even inconsistencies might possibly remain. As a matter of fact, the use of such reading strategies was not uncommon until fairly recently amongst scholars seeking to elucidate from a variety of intellectual perspectives the way in which mathematics and philosophy are interrelated in Leibniz's thought.[5] To a certain extent, the assumptions underlying

[2] Leibniz to L'Hospital, 27 December 1694 (A III, 6, 253): "Ma metaphysique est toute Mathematique pour dire ainsi, ou la pourroit devenir".

[3] See Russell (1903).

[4] Brunschvicg (1912, 204). Unless otherwise stated, all the translations are ours.

[5] Concerning Leibniz scholarship in the twentieth Century, see Albert Heinekamp (1989) who distinguished three main lines of study: first, the view that focuses on the ideal of system ("à la recherche du vrai système leibnizien"); second, the defense of the "structuralist" reading ("les interprétations structuralistes"); third, the view that denies any systematic structure in Leibniz's philosophy ("refus du caractère systématique de la philosophie leibnizienne") which, according to Heinekamp, begins to be present only in the 80's. The first line of reading may be regarded as the most widely represented amongst scholars interested in studying Leibniz from the perspective of the interrelations between mathematics and philosophy. Amongst French scholars, Serres (1968) and Belaval (1960) may be mentioned as cases where the indirect impact of mid-twentieth Century foundational philosophy of mathematics and logic can be detected. One could also mention the work of G.-G. Granger (1981), who emphasizes the epistemic value of Leibniz's guiding ideas at the basis of his mathematical contributions (vis-à-vis the work of other great seventeenth Century contributions to mathematical analysis) but also sees Leibniz' mathematical work as a possible anticipation of modern non-standard analysis. For a contextual study of the development of formal logic in the late nineteenth and early twentieth Century and the exact role played by Leibniz's

such reconstructions often prevented the study of the interrelations between mathematics and philosophy in their own right.

A further difficulty with such approaches to the study of Leibniz's thought was that it motivated scholars to make sometimes arbitrary choices in his mathematical and philosophical writings without any consideration of the time and material context of production. This tendency comes to light paradigmatically in the selection of unpublished material practiced by past editors. As a matter of fact, it was precisely there where the problem started. As Couturat already noted, previous editors selected from the Leibniz's *Nachlass* the most relevant pieces to be published according to their specific intellectual interest; but unavoidably, similar objections could be made against the editor of *Opuscules et fragment inédits*.[6] While B. Russell's attempt at systematic reconstruction flatly ignored Leibniz's mathematical contributions, it is noteworthy that Cassirer and Brunschvicg, as reflected in the passage quoted above, mainly focused on the elaboration of the differential calculus taking it to be essential to understanding the interrelations between mathematics, physics, and metaphysics.[7] On the other hand, Couturat was originally motivated by G. Peano's references to Leibniz "logical insights" and anticipations to search amongst his unpublished notes for Leibniz's many experiments with "formal calculi" and other programmatic sketches related to his goal to design new working tools—which Leibniz called "characteristics"—as well as any material deemed relevant to the vision of a universal grammar, and universal mathematics with logic as the sustaining link.[8]

As noted, such lines of research by proceeding selectively led not only to the introduction of arbitrary divisions in Leibniz's writings, often ignoring chronological order, but sometimes even entailed opposing readings of one and the same section of his works. For instance, the very same texts on *analysis situs* could be interpreted either along the lines of conceptual analysis (by commentators such as Cassirer) or along the lines of formal calculus and logical theory of relations (by commentators such as Couturat).

A last difficulty presented by this time-honored approach was its pretention to propose a picture of Leibniz's philosophy as a whole. As Dietrich Mahnke emphasized already in the early 1920s, it left readers with the unfortunate impression of facing a choice between different 'paintings' of Leibniz, depending on whether or not mathematics was involved in the drawn portrait. Typical examples were, on the one hand, the project to which Mahnke gave the name "universal mathematics",

work as a possible anticipation of modern approaches in logic and mathematics, see Peckhaus (1997). Despite revealing historical studies, the 'logicist' trend is still represented explicitly in recent times, for instance, by Sasaki (2004, 405), who goes so far as to speak of "Leibniz's 'logicist-formalist' philosophy of mathematics".

[6] Couturat (1903), Preface.

[7] See Russell's Preface to the second edition of his book on Leibniz, Russell (1937): in composing his original book, Russell conceded that he ignored all material relevant to Leibniz's mathematical studies and contributions, but still insisted that his "interpretation of Leibniz's philosophy is still the same" as in 1900.

[8] See Couturat (1903), Preface, and Peckhaus (1997).

as dealt with in various forms by Couturat, Cassirer, and Brunschvicg and, on the other hand, the so-called "metaphysics of individuation" which he identified with commentators such as Kabitz, Sickel, and Baruzi.[9] Interestingly enough, Couturat[10] himself warned against philosophers as well as mathematicians who ignored Leibniz's recommendations that "mathematicians have just as much need to be philosophers as philosophers to be mathematicians".[11]

Once again, the elements at the basis of all these interpretations are to be found in Leibniz's writings, as well as in his rich and extensive correspondence.

The present collection of essays aims to elucidate how these different aspects in Leibniz's thought relate to each other, evolving over time as his thinking unfolds. With this aim in mind, the papers in this volume take advantage of two fortunate circumstances. First, we are today in the privileged position of being able to take a fresh look at material which has long been available in conjunction with those letters and papers most recently published by the Academy edition. With the benefit of a considerable extended textual basis we propose to look at Leibniz's mathematical practice while at the same time exploring his goals and the underlying values and ideas that guided his problem-solving activities. For example, we examine his notes and interactions with others in the process of studying mathematics in Paris under the guidance of Huygens, but we are also interested in exploring how his mathematical experience evolved, transforming his earlier philosophical views. For Leibniz, thinking unfolds and takes place in time, a fact which is beautifully reflected in his writings. The second fortunate circumstance that motivates scholarly research on the interrelations between mathematics and philosophy in Leibniz's thought relates to today's growing interest in broadening the perspective of philosophy of mathematics, so that it engages historical case-studies. The new focus on the history of mathematical practice emphasizes precisely how such practice is intertwined with philosophical ideas. The notion of a specific area of study called "philosophy of mathematics" began to develop only in the early twentieth Century as an enterprise whose main concern was to deal with growing worries about foundational issues in mathematics. This logicist project left no room for historical case studies and the institutional contextualization of mathematical practice. Instead, it focused on deductive rigor, the elaboration of predicate logic, and the axiomatic method. Leaving behind such stringent formal concerns, the field has been opening up to include the

[9] However, even Mahnke tried to rescue the idea of system by proposing a view which was conceived as a synthesis of both leading interpretations at his time in his book *Leibnizens Synthese von Universalmathematik und Individualmetaphysik* (Mahnke 1925).

[10] See Couturat (1901, vii): "Les philosophes, séduits à bon droit par sa métaphysique, n'ont accordé que peu d'attention à ses doctrines purement logiques, et n'ont guère étudié son projet d'une Caractéristique universelle, sans doute à cause de la forme mathématique qu'il revêtait. D'autre part, les mathématiciens ont surtout vu dans Leibniz l'inventeur du Calcul différentiel et intégral, et ne se sont pas occupés de ses théories générales sur la valeur et la portée de la méthode mathématique, ni de ses essais d'application de l'Algèbre à la Logique, qu'ils considéraient dédaigneusement comme de la métaphysique. Il en est résulté que ni les uns ni les autres n'ont pleinement compris les principes du système, et n´ont pu remonter jusqu´à la source d´où découlent à la fois le Calcul infinitésimal et la Monadologie".

[11] Leibniz to Nicolas Malebranche, 13/23 March 1699 (A II, 3, 539): "Les Mathematiciens ont autant besoin d'estre philosophes, que les philosophes d´estre Mathematiciens".

study of the work of the research mathematician, and how that work interacts with philosophical ideas and other cultural ingredients in broader historical context. This is the most welcome setting to return to the study of Leibniz, the research mathematician, who insisted upon the need to think philosophically while immersed in mathematical practice.

2 Encountering Mathematics in Paris

Although Leibniz had good political reasons for travelling to Paris in March 1672, it was the intellectual culture and above all the presence of some of the then greatest mathematical minds in Europe which persuaded him to prolong his stay, interrupted by a short visit to London, until October 1676.[12] In a letter written some two years after he had returned to Germany in order to take up his position as court counsellor and librarian in Hanover, he talks of devoting himself with an "almost limitless passion" to mathematics during those four heady years in the French capital.[13]

Leibniz's initiation to mathematics is of course associated primarily with Christiaan Huygens. On numerous occasions in later life he expresses his considerable intellectual debt to the Dutch savant.[14] However, it was some time after Leibniz's arrival in Paris before the two men actually met. Until late summer 1672, Leibniz was preoccupied with official tasks which his patron Johann Christian von Boineburg had assigned to him: the Egyptian plan, which Leibniz had himself devised in order to divert Louis XIV's military ambitions away from Europe, and the recovery of Boineburg's French rent and pension. Nonetheless, by September Leibniz had been introduced to Antoine Arnauld and Pierre de Carcavi, and soon thereafter there were encounters with the astronomers Giovanni Cassini and Ole Rømer.[15] This was the challenging intellectual environment he had long desired:

> Paris is a place where it is difficult to distinguish oneself: one finds the most capable men of the time in every kind of scientific endeavour and much effort and a little robustness is necessary in order to establish one's reputation.[16]

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[12] Leibniz to Duke Johann Friedrich, autumn 1679 (A II, 1 (2006), 761); Leibniz to Fabri, beginning of 1677(A II, 1 (2006), 442); Leibniz to Conring, 24 August 1677 (A II, 1 (2006), 563).

[13] Leibniz to the Pfalzgräfin Elisabeth, November 1678 (A II, 1 (2006), 66).

[14] See for example Leibniz, *De solutionibus problematic catenarii vel funicularis in Actis Junii A. 1691. aliisque a Dn. I. B. propositis* (GM V, 255); *Historia et origo calculi differentialis* (GM V, 398); Leibniz to Huygens, first half of October 1690 (A III, 4, 598); Leibniz to Remond, 10 January 1714 (GP III, 606): "Il est vray que je n'entray dans les plus profondes [sc. mathematiques] qu'apres avois conversé avec M. Hugens à Paris".

[15] See Antognazza (2009, 140–141).

[16] Leibniz to Duke Johann Friedrich, 21 January 1675 (A I, 1, 491−492): "Paris est un lieu, ou il est difficile de se distinguer: on y trouve les plus habiles hommes du temps, en toutes sortes des sciences, et il faut beaucoup de travail, et un peu de solidité, pour y establir sa reputation". See also Leibniz to Gallois, first half of December 1677 (A III, 2, 293−294); Leibniz to Bignon, 9/19 October 1693 (A I, 10, 590) .

It was not until the autumn that Leibniz was able to meet with Huygens for the first time. For the Dutch savant, effectively entrusted by Colbert with the planning and organization of the Académie Royale des Sciences, this was not a meeting with an absolute stranger. Leibniz was already becoming known in the Republic of Letters as a man of prodigious learning, who besides possessing exceptional knowledge in law and philosophy was "mathematically very inclined, and well versed in physics, medicine, and mechanics".[17] But, more specifically, Huygens's attention had been drawn to the promising young man from Germany almost a year and a half before they actually met. The Bremen-born secretary of the Royal Society, Henry Oldenburg, eager to promote the growth of the new science in Germany, had spoken enthusiastically of Leibniz in his letters. In his most recent communication, he referred to Leibniz's two tracts on motion, the *Hypothesis physica nova* and the *Theoria motus abstracti*, both of which with his help had been reprinted in London under the auspices of the Royal Society in 1671. Oldenburg's description of Leibniz was clearly intended to serve as an introduction:

> He seems of no ordinary intelligence, but is one who has examined minutely what great men, both ancient and modern, have had to say about Nature, and finding that plenty of difficulties remain, has set to work to resolve them. I cannot tell you how far he has succeeded, but I dare affirm that his ideas deserve consideration.[18]

Knowing full well that Leibniz had first been motivated to write on the theory of motion after he had read the laws of motion published in the *Philosophical Transactions* by John Wallis, Christopher Wren, and Huygens himself, Oldenburg proceeded to quote a passage from Leibniz questioning the conformity of the laws presented by Huygens and Wren to the abstract concepts of motion.

3 The Mathematical Novice

It is important to recognize that the young man initiated in mathematics in the autumn of 1672 was, as Oldenburg emphasized, steeped in both ancient and modern philosophy, while having a sound knowledge of jurisprudence and Protestant and Catholic theology. By contrast, as far as mathematics was concerned, Leibniz brought with him little more than what he had been able to glean from introductory

[17] Boineburg to Conring, 22 April 1670, Gruber (1745, II, 1286−1287): "Leibnizio literae tuae maximo sunt solatio. Est iuvenis 24 annorum, Lipsiensis, Juris Doctor: imo doctus supra quam vel dici potest, vel credi, Philosophiam omnem percallet, veteris et novae felix ratiocinator. Scribendi facultate apprime armatus. Mathematicus, rei naturalis, medicinae, mechanicae omnis sciens et percupidus; assiduus et ardens".

[18] Oldenburg to Huygens, 28 March 1671, Hall and Hall (1965−1986, VII, 537−538/538−539): "Il ne semble pas un Esprit du commun, mais qui ait esplusché ce que les grands hommes, anciens et modernes ont commenté sur la Nature, et trouvant bien de difficultez qui restent, travaillé d'y satisfaire. Je ne vous scaurois pas dire comment il y ait reussi; j'oseray pourtant affirmer que ses pensees meritent d'estre considerées." See also Oldenburg to Huygens, 8 November 1670, Hall and Hall (1965−1986, VII, 239−240/241−242).

textbooks of Harsdörffer or Cardano and from the mathematical exploits of Thomas Hobbes—an author he had read avidly while he was in Mainz. Although he described the two tracts on motion of his youth on one occasion to Nicolas Malebranche as "the beginnings of his mathematical studies"[19], he would later generally dismiss them precisely because of their lack of sophistication in exact science. When he arrived in Paris, Leibniz was to all intents and purposes a mathematical novice.

> The desire to do justice to the favourable opinion which people had of me led me by good fortune to find new ways of analysis and to make discoveries in mathematics, although I had scarcely thought about this science before I came to France, for philosophy and jurisprudence had previously been the object of my studies from which I produced a number of essays.[20]

It is probable that the first meeting between Huygens and Leibniz took place in the Dutch savant's rooms in the Royal Library in Paris. During the course of their exchange, Leibniz mentioned with the remarkable boldness typical of his youth that he had discovered a method for summing infinite series. This method was the fruit of investigations into the Euclidean axiom "The whole is greater than its part", to which his attention had been drawn in Mainz, after reading the first part of Hobbes's *De corpore*.[21] In Chap. 8, Hobbes argues that *Totum esse maius parte*, like all geometrical axioms, must be demonstrable.[22] Already then during his service at the court of Johann Philipp von Schönborn, Leibniz had considered *Totum esse maius parte* to be reducible to the only two types of unproved truths which he considered admissible, namely definitions and identities. By the time he met Huygens he had not only succeeded in producing a syllogistic proof that every part of a given magnitude is smaller than the whole, but also, using the principle of identity, he had developed his main theorem that the summation of consecutive terms of a series of differences could be carried out over an infinite number of terms—assuming only that the expected total sum approaches a finite limit.

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[19] Leibniz to Malebranche, end of January 1693 (A II, 2, 659): "Au commencement de mes etudes mathematiques je me fis une theorie du movement absolu, où supposant qu'il n'y avoit rien dans le corps que l'étendüe et l'impenetrabilité, je fis des regles du mouvement absolu que je croyois veritables, et j'esperois de les pouvoir concilier avec les phenomenes par le moyen du systeme des choses."

[20] Leibniz to Pellisson-Fontanier, 7 May 1691 (A I, 6, 195−196): "L'envie de me rendre digne de l'opinion favorable qu'on avoit de eue de moy, m'avoit fait faire quelques decouvertes dans les Mathematiques, quoyque je n'eusse gueres songé à cette science, avant que j'estois venu en France, la philosophie et la jurisprudence ayant esté auparavant l'objet de mes études dont j'avois donné quelques essais." See also Leibniz to Duke Johann Friedrich, 29 March 1679 (A I, 2, 155); Leibniz to Duke Ernst August, early 1680? (A I, 3, 32); Leibniz to Foucher, 1675 (A II, 1 (2006), 389); *De numeris characteristicis ad linguam universalem constituendam* (A VI, 4, 266).

[21] See Leibniz, *Historia et origo calculi differentialis* (GM V, 395).

[22] I, 8, § 25; Hobbes (1651, 72).

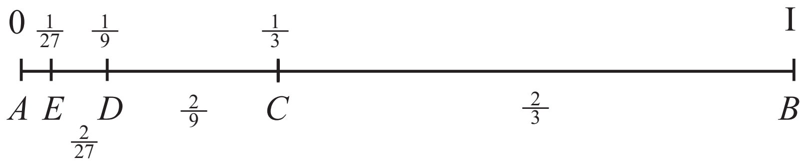
4 Early Successes in Paris

After listening to Leibniz's youthful deliberations, Huygens decided to put him to the test and asked him to determine the sum of the infinite series of reciprocal triangular numbers.[23]

$$1/1 + 1/3 + 1/6 + 1/10 + \text{etc}\,.$$

The result of this summation was already known to him, but he had not yet put this into print. Huygens also suggested that Leibniz consult two books which he had previously cited, but had not read: Wallis's *Arithmetica infinitorum* and the *Opus geometricum* of Grégoire de Saint-Vincent.

Developing a principle found in the *Opus Geometricum*, that the line segments representing terms of the geometrical progression must be considered to start from the same place, Leibniz recognized that the differences of consecutive terms are proportional to the original series.



From here can be read off

$$2/3 + 2/9 + 2/27 + \ldots = 1$$

Or, more generally

$$1/t + 1/t^2 + 1/t^3 + \ldots = 1/(t-1)$$

Decisively, Leibniz was able to show how conceptually a general method could be applied. Thus, by taking AB = 1, AC = 1/2, AD = 1/3, AE = 1/4, he achieved the relation

$$1/1.2 + 1/2.3 + 1/3.4 + 1/4.5 + \ldots = 1$$

and then, multiplying by 2, produced the result which Huygens had sought, namely

$$1/1 + 1/3 + 1/6 + 1/10 + \ldots = 2$$

[23] See Hofmann (1974, 15).

Writing to Oldenburg on 16/26 April 1673, Leibniz does not seek to hide his joy at this early success:

> But by my method I find the sum of the whole series continued to infinity, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28 etc.; indeed, I do not believe this to have been laid before the public previously for the reason that the very noble Huygens first proposed this problem to me, with respect to triangular numbers, and I solved it generally for numbers of all kinds much to the surprise of Huygens himself.[24]

Nor did Leibniz stop here, but also succeeded in obtaining the sum of the reciprocals of pyramidal numbers as well as the sum of reciprocal trigono-trigonal numbers.

$$D = 1 + 1/5 + 1/15 + 1/35 + 1/70 + \ldots = 4/3$$

The exuberance which Leibniz felt at achieving such early success—and being able to impress Huygens at the same time—can be gauged from the language he employed in what he evidently hoped would be his first mathematical publication, having already seen two letters to Oldenburg on his theory of motion published in the *Philosophical Transactions*. Most articles which appeared in the new scientific journals of the second half of the seventeenth century took the form of letters to the editor. It was therefore perfectly natural for Leibniz to set out some of his newly achieved mathematical results in a long letter to Jean Gallois, editor of the *Journal des Sçavans* and secretary of the Académie Royale des Sciences.[25] Unfortunately for Leibniz, and no doubt unbeknown to him at the time, the French journal temporarily ceased publication on 12 December 1672, that is to say, around the time his letter was sent. By the time publication was resumed on 1 January 1674, Leibniz's contribution would have been considered out of date, not least in view of the author's mathematical development during the intervening twelve months.

5 Mathematical and Philosophical Deliberations on Infinity

The *Accessio ad arithmeticam infinitorum*, as the letter to Gallois was entitled, provides evidence of the remarkable growth in Leibniz's understanding of the nature of concept of infinity compared to the views he had set out little over a year earlier in his *Theoria motus abstracti*. Whereas there he had approached the continuum ontologically, seeking to reconcile infinite divisibility with the actual existence of parts by postulating points in such a way that they could be conceived as constitutive entities, he now appeals to the argumentative force provided by genuine mathe-

[24] Leibniz to Oldenburg, 26 April 1673 (A III, 1, 83−89, 88): "At ego totius seriei in infinitum continuatae summam invenio methodo mea: 1/3 1/6 1/10 1/15 1/21 1/28 etc. in infinitum; quod jam publice propositum esse, vel ideo non credidi, quia a Nobilissimo Hugenio mihi primum propositum est hoc problema in numeris triangularibus; ego vero id non in triangularibus tantum, sed et pyramidalibus etc. et in universum in omnibus ejus generis numeris solvi ipso Hugenio mirante".

[25] See Bos (1978, 61).

matical proofs, such as those he had shown to Huygens, where there is an infinite progression within finite limits.

> He namely who is led by the senses will persuade himself that there cannot be a line of such shortness, that it contains not only an infinite number of points, but also an infinite number of lines (as an infinite number of actually separated parts) having a finite relation to what is given, unless demonstrations compel this.[26]

Part of what Leibniz sets out to achieve in the *Accessio* is to demonstrate that infinite number is impossible. Employing a strategy used in numerous other contemporary letters and papers, he develops his position in contrast to the position put forward by Galileo in the *Discorsi e dimostrazioni matematiche*, where infinite number, understood as the number of all numbers, is compared to unity. Galileo argued that every number into infinity had its own square, its own cube, and so on, and that therefore there must be as many squares and cubes as there are roots or integers, which however is impossible. The Pisan mathematician famously concludes from this that quantitative relations such as those of equality or "greater than" or "less than" do not apply when it comes to the infinite. That is to say, Galileo effectively negated the validity of the axiom *Totum esse maius parte* with respect to infinite numbers.

Leibniz compared Galileo's conclusion to Grégoire's negation of the validity of the axiom in horn angles in his *Opus geometricum*. In both cases, Leibniz found that it was a mistaken concept of infinity which had led to denying the universality of the axiom: "that this axiom should fail is impossible, or, to say the same thing in other words, the axiom never fails except in the case of null or nothing".[27] Precisely the universal validity of the axiom leads to the conclusion that infinite number is impossible, "it is not one, not a whole, but nothing". Employing an argument which is also found in contemporary algebraic studies, Leibniz is able to proclaim that not only is $0+0=0$, but also $0-0=0$. Consequently, an infinity which is produced from all units or which is the sum of all must in his view be regarded quite simply as nothing, about which, therefore, "nothing can be known or demonstrated, and which has no attributes".[28]

Alongside providing evidence of the relative sophistication of Leibniz's mathematical work by the end of 1672, the *Accessio* provides the earliest example of the intimate relation between philosophy and mathematics in his thought.[29] Right at the beginning, he asserts that the method of indivisibles is to be ranked among those

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[26] Leibniz for Gallois, end of 1672 (A II, 1 (2006), 342): "Quis enim sensu duce persuaderet sibi, nullam dari posse lineam tantae brevitatis, quin in ea sint non tantum infinita puncta, sed et infinitae lineae (ac proinde partes a se invicem separatae actu infinitae) rationem habentes finitam ad datam; nisi demonstrationes cogerent."

[27] *Ibid*, 349: "at Axioma illud fallere impossibile est, seu quod idem est, Axioma illud nunquam, ac non nisi in Nullo seu Nihilo fallit, Ergo Numerus infinitus est impossibilis, non unum, non totum, sed Nihil."

[28] Leibniz, *Mathematica* (A VII, 1, 657): "Nam $0+0=0$. Et $0–0=0$. Infinitum ergo ex omnibus unitatibus conflatum, seu summa omnium esr nihil, de quo scilicet nihil potest cogitari aut demonstrari, et nulla sunt attributa." See also *De bipartitionibus numerorum eorumque geometricis interpretationibus* (A VII, 1, 227).

[29] See Beeley (2009).