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M.N.S. Swamy, Concordia University

मातृभ्यो नमः
पितृभ्यो नमः
गुरुभ्यो नमः

To
My parents Boddu and Dalamma,
My beloved wife Sarojini,
and My mentor Prof. M.N.S. Swamy

Preface

The life of people has changed tremendously in view of the rapid growth of mobile and wireless communication. Channel coding is the heart of digital communication and data storage. The traditional block codes and conventional codes are commonly used in digital communications. To approach the theoretical limit for Shannon's channel capacity, the length of a linear block code or constant lengths of convolutional codes have to be increased, which in turn makes the decoder complexity to become high and may render it physically unrealizable. The powerful turbo and LDPC codes approach the theoretical limit for Shannon's channel capacity with feasible complexity for decoding. MIMO communications is a multiple antenna technology which is an effective way for high speed or high reliability communications. The MIMO can be implemented by space-time coding. However, a single book which can serve as a textbook for Bachelor and Master students on this topic is lacking in the market.

In this book, many illustrative examples are included in each chapter for easy understanding of the coding techniques. An attractive feature of this book is the inclusion of MATLAB-based examples with codes to encourage readers to implement on their personal computers and become confident of the fundamentals and gain more insight into coding theory. In addition to the problems that require analytical solutions, MATLAB exercises are introduced to the reader at the end of each chapter.

The book is divided into 11 chapters. Chapter 1 introduces the basic elements of a digital communication system, statistical models for wireless channels, capacity of a fading channel, Shannon's noisy channel coding theorem and the basic idea of coding gain. Chapter 2 gives an overview of the performance analysis of different modulation techniques, and also deals with the performance of different diversity combining techniques in a multi-channel receiver. Chapter 3 introduces Galois fields and polynomials over Galois fields. Chapter 4 covers linear block codes including RS codes because of their popularity in burst error correction in wireless networks. Chapter 5 discusses the design of a convolutional encoder and Viterbi decoding algorithm for the decoding of convolutional codes, as well as the performance analysis of convolutional codes over AWGN and Rayleigh fading channels. In this chapter, punctured convolutional codes are also discussed. Chapter 6 provides a treatment of the design of turbo codes, BCJR algorithm for iterative decoding of turbo codes, and performance analysis of turbo codes. Chapter 7 focuses on the design and analysis of Trellis-coded modulation schemes using both the conventional and turbo codes. Chapter 8 describes the design of low parity check codes (LDPC), decoding algorithms and performance analysis of LDPC codes. The erasure correcting codes like Luby transform (LT) codes and Raptor codes are described in Chap. 9. Chapter 10 provides an in-depth study of multiple-input multiple-output (MIMO) systems in which multiple antennas are used both at the transmitter and at the receiver. The design of space-time codes and implementations of MIMO systems are discussed in Chap. 11.

Salient features of this book are as follows:

- Provides comprehensive exposure to all aspects of coding theory for wireless channels with clarity and in an easy way to understand
- Provides an understanding of the fundamentals, design, implementation and applications of coding for wireless channels
- Presents illustration of coding techniques and concepts with several fully worked numerical examples
- Provides complete design examples and implementation
- Includes PC-based MATLAB *m*-files for the illustrative examples are included in the book.

The motivation in writing this book is to include modern topics of increasing importance such as turbo codes, LDPC codes and space-time coding in detail, in addition to the traditional RS codes and convolutional codes, and also to provide a comprehensive exposition of all aspects of coding for wireless channels. The text is integrated with MATLAB-based programs to enhance the understanding of the underlying theories of the subject. These MATLAB codes are free to download from the book's page on Springer.com.

This book is written at a level suitable for undergraduate and master students in electronics and communication engineering, electrical and computer engineering, computer science, and applied physics as well as for self-study by researchers, practicing engineers and scientists. Depending on the chapters chosen, this text can be used for teaching a one or two semester course on coding for wireless channels. The prerequisite knowledge of the readers in principles of digital communication is expected.

K. Deergha Rao

Contents

About the Author

K. Deergha Rao is director and professor in the Navigational Electronics Research and Training Unit (NERTU), University College of Engineering, Osmania University, Hyderabad, India. Earlier, he was a postdoctoral fellow and part-time professor at the Department of Electronics and Communication Engineering, Concordia University, Montreal, Canada. He has executed several research projects for premium Indian organizations such as Defence Research and Development Organization (DRDO), Hindustan Aeronautical Limited (HAL) and Bharat Electronics Limited (BEL). His teaching areas are digital signal processing, digital image processing, coding theory for wireless channels and MIMO wireless communications, whereas his research interests include GPS signal processing, wireless channel coding, blind equalization, robust multiuser detection, OFDM UWB signal processing, MIMO SFBC OFDM, image processing, cryptosystems and VLSI signal processing. Professor Rao has presented papers at IEEE international conferences several times in the U.S.A., Switzerland and Russia. He has more than 100 publications to his credit, including more than 60 publications in IEEE journals and conference proceedings. He is a senior member of IEEE and has served as chairman of communications and signal processing societies joint chapter of IEEE Hyderabad section. He is currently a member of the IEEE SPS chapters committee. He was awarded 2013 IETE K.S. Krishnan Memorial Award for the best system-oriented paper. He has served as Communications Track Chair for IEEE INDICON 2011 held at Hyderabad. He is an editorial board member of the International Journal of Sustainable Aviation(Inderscience Publishers, U.K.). He has coauthored a book, Digital Signal Processing (Jaico Publishing House, India).

Chapter 1
Introduction

In this chapter, a digital communication system with coding is first described. Second, various wireless communication channels, their probability density functions, and capacities are discussed. Further, Shannon's noisy channel coding theorem, channel coding principle, and channel coding gain are explained. Finally, some application examples of channel coding are included.

1.1 Digital Communication System

A communication system is a means of conveying information from one user to another user. The digital communication system is one in which the data are transmitted in digital form. A digital communication system schematic diagram is shown in Fig. 1.1. The source coding is used to remove redundancy from source information for efficient transmission. The transmitted signal power and channel bandwidth are the key parameters in the design of digital communication system. Using these parameters, the signal energy per bit (E_{b}) to noise power spectral density (N_0) ratio is determined. This ratio is unique in determining the probability of bit error, often referred to as bit error rate (BER). In practice, for a fixed E_{b}/N_0, acceptable BER is possible with channel coding. This can be achieved by adding additional digits to the transmitted information stream. These additional digits do not have any new information, but they make it possible for the receiver to detect and correct errors thereby reducing the overall probability of error.

1.2 Wireless Communication Channels

1.2.1 Binary Erasure Channel (BEC)

Erasure is a special type of error with known location. The BEC transmits one of the two binary bits 0 and 1. However, an erasure '*e*' is produced when the receiver receives an unreliable bit. The BEC channel output consists of 0, 1, and *e* as shown

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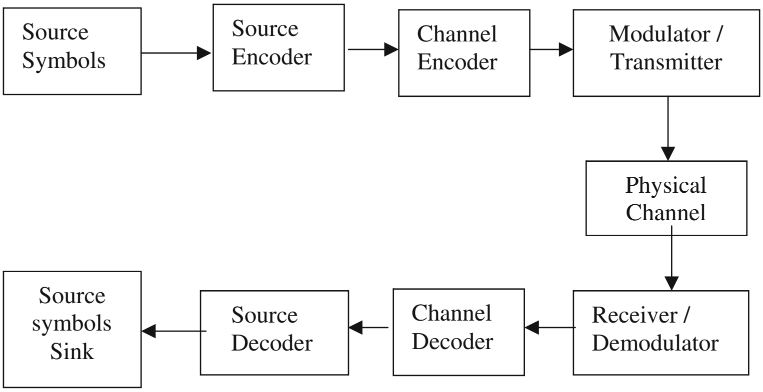


Fig. 1.1 Digital communication system with coding

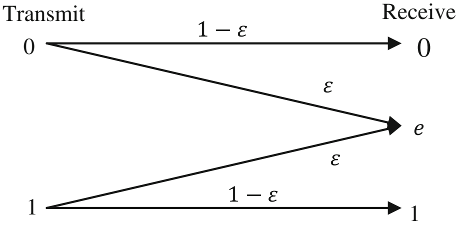


Fig. 1.2 Binary erasure channel

in Fig. 1.2. The BEC erases a bit with probability ε, called the erasure probability of the channel. Thus, the channel transition probabilities for the BEC are

$$\left.\begin{aligned} P(y=0|x=0) &= 1-\varepsilon, \\ P(y=e|x=0) &= \varepsilon, \\ P(y=1|x=0) &= 0, \\ P(y=0|x=1) &= 0, \\ P(y=e|x=1) &= \varepsilon, \\ P(y=1|x=1) &= 1-\varepsilon. \end{aligned}\right\} \tag{1.1}$$

1.2.2 Binary Symmetric Channel (BSC)

The BSC is discrete memoryless channel that has binary symbols both in the input and output. It is symmetric because the probability for receiving 0 when 1 is transmitted is same as the probability for receiving 1 when 0 is transmitted. This

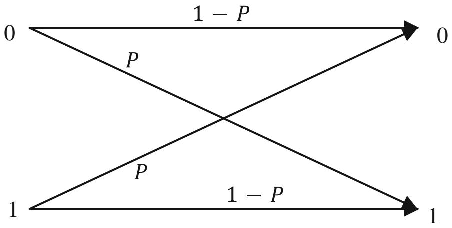


Fig. 1.3 Binary symmetric channel

probability is called the crossover probability of the channel denoted by P as shown in Fig. 1.3. The probability for no error, i.e., receiving the same as transmitted, is $1-P$. Hence, the channel transition probabilities for the BSC are

$$\left.\begin{aligned} P(y=0|x=0) &= 1-P, \\ P(y=0|x=1) &= P, \\ P(y=1|x=0) &= P, \\ P(y=1|x=1) &= 1-P, \end{aligned}\right\} \tag{1.2}$$

1.2.3 Additive White Gaussian Noise Channel

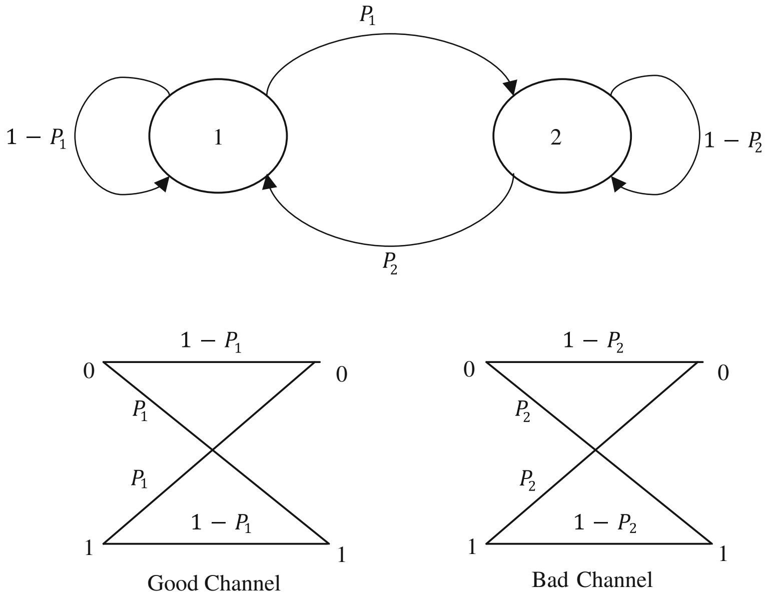
In an AWGN channel, the signal is degraded by white noise η which has a constant spectral density and a Gaussian distribution of amplitude. The Gaussian distribution has a probability density function (pdf) given by

$$P_{\mathrm{df}}(\eta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\eta^2}{2\sigma^2}\right) \tag{1.3}$$

where σ^2 is the variance of a Gaussian random process.

1.2.4 Gilbert–Elliott Channel

For bursty wireless channels, the Gilbert–Elliott (GE) channel [1, 2] is one of the simplest and practical models. The GE channel is a discrete-time stationary model as shown in Fig. 1.4 with two states: one bad state or burst state '2' wherein a BSC resides with high error probabilities $(1-P_2)$ and the other state is a good state '1' wherein a BSC resides with low error probabilities $(1-P_1)$.



Good Channel

Bad Channel

Fig. 1.4 A two-state channel

Another common GE example is that the BEC resides in a bad state with ε close to unity and assigns erasures to all of the bits transmitted during the high-error-rate (bad) state.

1.2.5 Fading Channel

In the radio channel, the received power is affected by the attenuations due to the combinations of the following effects:

1. *The Path loss*: It is the signal attenuation. The power received by the receiving antenna decreases when the distance between transmitter and receiver increases. The power attenuation is proportional to $(\text{distance})^{\alpha}$, where α values range from 2 to 4. When the distance varies with time, the path loss also varies.
2. *The Shadowing loss*: It is due to the absorption of the radiated signal by scattering structure. It is derived from a random variable with lognormal distribution.
3. *The Fading loss*: The combination of multipath propagation and the Doppler frequency shift produces the random fluctuations in the received power which gives the fading losses.

1.2.6 Fading

Fading gives the variations of the received power along with the time. It is due to the combination of multipath propagation and the Doppler frequency shift which gives the time-varying attenuations and delays that may degrade the communication system performance. The received signal is a distorted version of the transmitted signal which is a sum of the signal components from the various paths with different delays due to multipath and motion.

Let T_s be the duration of a transmitted signal and B_x be the signal bandwidth. The fading channel can be classified based on coherence time and coherence bandwidth of the channel. The coherence time and coherence bandwidth of a channel are defined as follows:

Doppler spread: The significant changes in the channel occur in a time T_c whose order of magnitude is the inverse of the maximum Doppler shift B_D among the various paths, called the *Doppler spread* of the channel.

The coherence time of the channel T_c is

$$T_c \triangleq \frac{1}{B_D} \tag{1.4}$$

Delay spread: The maximum among the path delay differences, a significant change occurs when the frequency change exceeds the inverse of T_D, called the *delay spread* of the channel.

The coherence bandwidth of the channel B_c is as follows:

$$B_c \triangleq \frac{1}{T_D} \tag{1.5}$$

The classification fading channels is shown in Fig. 1.5.

The fast fading causes short burst errors which are easy to correct. The slow fading will affect many successive symbols leading to long burst errors. Due to energy absorption and scattering in physical channel propagation media, the transmitted signal is attenuated and becomes noisy. The attenuation will vary in mobile communications based on the vehicle speed, surrounding trees, buildings, mountains, and

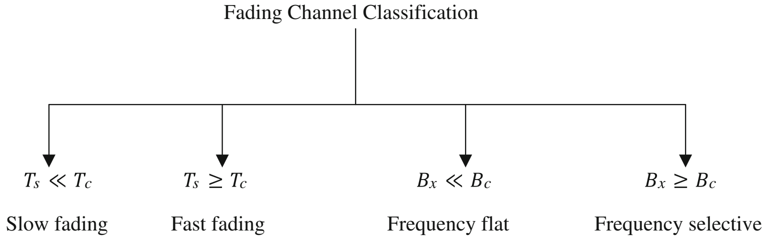


Fig. 1.5 Classification of fading channels

terrain. Based on the receiver location, moving receiver signals interfere with one another and take several different paths. As such, the wireless channels are called multipath fading channels. Hence, the additive white Gaussian noise (AWGN) assumption for wireless channels is not realistic. Thus, the amplitudes in wireless channel are often modeled using Rayleigh or Rician probability density function.

The most common fading channel models are as follows:

1. Flat independent fading channel
2. Block fading channel

In flat independent fading channel, the attenuation remains constant for one symbol period and varies from symbol to symbol. Whereas in block fading channel, the attenuation is constant over a block of symbols and varies from block to block.

1.3 Statistical Models for Fading Channels

1.3.1 Probability Density Function of Rician Fading Channel

When the received signal is made up of multiple reflective rays plus a significant line of sight (non-faded) component, the received envelope amplitude has a Rician probability density function (PDF) as given in Eq. (1.6), and the fading is referred to as Rician fading.

$$\begin{aligned} P_{\mathrm{df}}(x) &= \frac{x}{\sigma^2}\exp\left(-\frac{(x^2+A^2)}{2\sigma^2}\right)I_0\left(\frac{xA}{\sigma^2}\right) && \text{for } x \ge 0, A \ge 0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{1.6}$$

where x is the amplitude of the received faded signal, I_0 is the zero order modified Bessel function of the first kind, and A denotes the peak magnitude of the non-faded signal component called the specular component. The Rician PDF for different values of sigma and $A = 1$ is shown in Fig. 1.6.

1.3.2 Probability Density Function of Rayleigh Fading Channel

Rayleigh fading occurs when there are multiple indirect paths between the transmitter and the receiver and no direct non-fading or line of sight (LOS) path. It represents the worst case scenario for the transmission channel. Rayleigh fading assumes that a received multipath signal consists of a large number of reflected waves with independent and identically distributed phase and amplitude. The envelope of the received carrier signal is Rayleigh distributed in wireless communications [3].

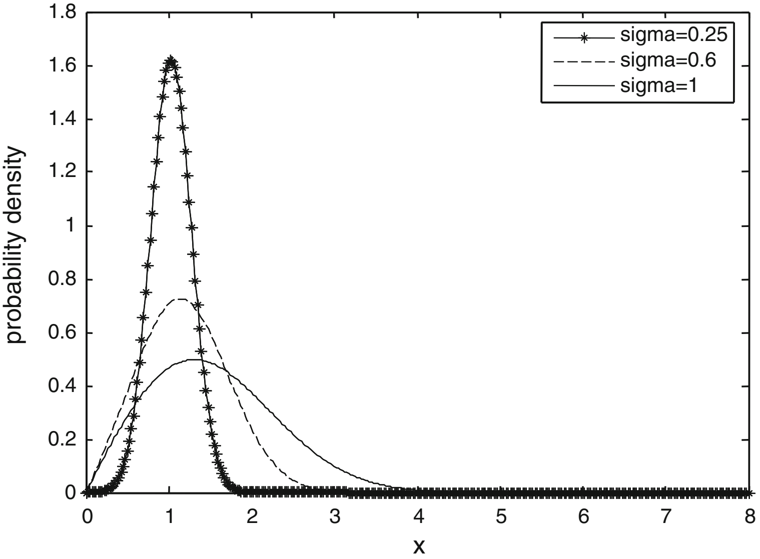


Fig. 1.6 Probability density of Rician fading channel

As the magnitude of the specular component approaches zero, the Rician PDF approaches a Rayleigh PDF expressed as follows:

$$\begin{aligned} P_{\mathrm{df}}(x) &= \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \text{for } x \geq 0 \\ &= 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{aligned} \tag{1.7}$$

The Rayleigh PDF for different values of sigma is shown in Fig. 1.7.

Additive white Gaussian noise and Rician channels provide fairly good performance corresponding to an open country environment, while Rayleigh channel, which best describes the urban environment fading, provides relatively worse performance.

1.3.3 Probability Density Function of Nakagami Fading Channel

The Nakagami model is another very popular empirical fading model [4]

$$P_{\mathrm{df}}(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{2\sigma^2}\right)^m r^{2m-1} \mathrm{e}^{-m\frac{r^2}{2\sigma^2}} \tag{1.8}$$

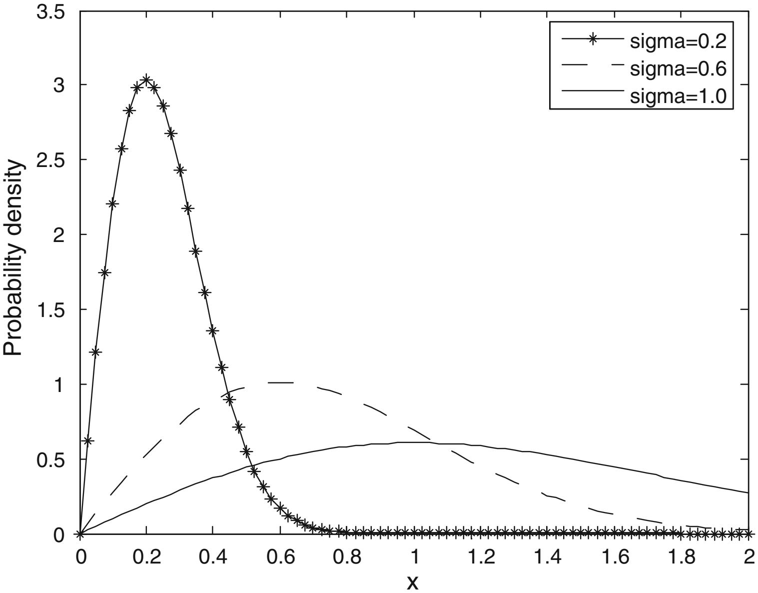


Fig. 1.7 Probability density of Rayleigh fading channel

where $\sigma^2 = \frac{1}{2}E[r^2]$, $\Gamma(.)$ is the gamma function, $m \geq \frac{1}{2}$ is the fading figure.

The received instantaneous power r^2 satisfies a gamma distribution. The phase of the signal is uniformly distributed in $[0, 2\pi)$. The Nakagami distribution is a general model obtained from experimental data fitting, and its shape is very similar to that of the Rice distribution. The shape parameter ‘m’ measures the severity of fading.

When

$m = 1$, it is Rayleigh fading.
$m \rightarrow \infty$, it is AWGN channel; that is, there is no fading.
$m > 1$, it is close to Rician fading.

However, due to lack of the physical basis, the Nakagami distribution is not as popular as the Rician and Rayleigh fading models in mobile communications. Many other fading channel models are discussed in Kuhn [5].

1.4 Channel Capacity

Channel capacity can be defined as the maximum rate at which the information can be transmitted over a reliable channel.

$$\text{Spectral or Bandwidth Efficiency} = \frac{\text{Transmission rate}}{\text{Channel Band width}} = \frac{R_s\mathcal{H}}{B} \text{ bits/s/Hz} \tag{1.9}$$

where R_s is the symbol rate, and $\mathcal{H}$ is the entropy.

The channel capacity is also known as Shanon's capacity can be defined as the average mutual information for a channel with energy constraint.

1.4.1 Channel Capacity of Binary Erasure Channel

The channel capacity of BEC is

$$\mathbf{C}_{\mathrm{BEC}} = 1 - \varepsilon \tag{1.10}$$

ε is the probability of a bit erasure, which is represented by the symbol e.

1.4.2 Channel Capacity of Binary Symmetric Channel

The Channel capacity of BSC is as follows:

$$\mathbf{C}_{\mathrm{BSC}} = 1 - \mathcal{H}(P) \tag{1.11}$$

$\mathcal{H}(P)$ is the binary entropy function given by Ryan and Lin [6]

$$\mathcal{H}(P) = -P\log_2(P) - (1-P)\log_2(1-P) \tag{1.12}$$

P is the probability of a bit error.

1.4.3 Capacity of AWGN Channel

An AWGN channel can be expressed by the following input–output relationship

$$y = x + \eta \tag{1.13}$$

where x is the transmitted source signal, y denotes the output of the channel, *and* η is a real Gaussian process with zero mean, variance $\sigma_\eta^2 = E[\eta^2]$, and two sided power spectral density $\frac{N_0}{2}$. The mutual information $I(x; y)$ with constraint on the energy of the input signal can be expressed as follows:

$$I(x,y) = \mathcal{H}(y) - \mathcal{H}(\eta) \tag{1.14}$$

where $\mathcal{H}(y)$ is the entropy of the channel output, and $\mathcal{H}(\eta)$ is the entropy of the AWGN. Since $\sigma_y^2 = \sigma_x^2 + \sigma_\eta^2$, the entropy $\mathcal{H}(y)$ is bounded by $\frac{1}{2}\log_2 \pi e(\sigma_x^2 + \sigma_\eta^2)$ and thus

$$\begin{aligned} I(x,y) &\leq \frac{1}{2}\log_2 \pi e(\sigma_x^2 + \sigma_\eta^2) - \frac{1}{2}\log_2 \pi e \sigma_\eta^2 \\ &= \frac{1}{2}\log_2(1 + \frac{\sigma_x^2}{\sigma_\eta^2}) \end{aligned} \tag{1.15}$$

The mutual information $I(x,y)$ is maximum when the input x is a real Gaussian process with zero mean and variance σ_x^2. The capacity of the channel is the maximum information that can be transmitted from x to y by varying the PDF P_{df} of the transmit signal x. The signal-to-noise ratio (SNR) is defined by

$$\text{SNR} \triangleq \frac{\sigma_x^2}{\sigma_\eta^2} \tag{1.16}$$

Thus, the capacity of an AWGN channel is given by

$$\mathbf{C} = \frac{1}{2}\log_2(1 + \text{SNR}) \text{ bits/s/Hz} \tag{1.17}$$

Since $\sigma_{\text{x}}^2 = BE_{\text{s}}$ and $\sigma_\eta^2 = B\frac{N_0}{2}$, Eq. (1.17) can be rewritten as follows:

$$\mathbf{C} = \frac{1}{2}\log_2\left(1 + 2\frac{E_{\text{s}}}{N_0}\right) \text{ bits/s/Hz} \tag{1.18}$$

where B is the bandwidth, E_{s} denotes the symbol energy, and N_0 represents the noise spectral density.

If x and η are independent complex Gaussian processes, the channel capacity can be expressed as follows:

$$\mathbf{C} = \log_2(1 + \text{SNR}) \text{ bits/s/Hz} \tag{1.19}$$

Since $\sigma_{\text{x}}^2 = BE_{\text{s}}$ and $\sigma_\eta^2 = BN_0$ for complex Gaussian process, Eq. (1.19) can be rewritten as follows:

$$\mathbf{C} = \log_2\left(1 + \frac{E_{\text{s}}}{N_0}\right) \text{ bits/s/Hz} \tag{1.20}$$

Example 1.1 What is the capacity of a channel with an SNR of 20 dB.

Solution $\mathbf{C} = \log_2(1+20) = 6.65$ bits/s/Hz.

The capacity is increasing as a log function of the SNR, which is a slow increase. Clearly, increasing the capacity by any significant factor takes an enormous amount of power.

1.4.4 Channel Capacity of Gilbert–Elliott Channels

The channel capacity of GE Channel is given by Ryan and Lin [6]

$$\mathbf{C}_{\mathrm{GE}} = \sum_{s=1}^{S} P_{\mathrm{s}} \mathbf{C}_{\mathrm{s}} \tag{1.21}$$

where P_{s} is the probability of being state in s state, and $\mathbf{C}_{\mathrm{s}}$ is the capacity of the channel in s state.

1.4.5 Ergodic Capacity of Fading Channels

A slow flat fading channel with AWGN can be expressed by the following input–output relationship

$$y = hx + \eta \tag{1.22}$$

where x is the transmitted source signal, y denotes the output of the channel, η is the AWGN, and h is a Gaussian random variable with Rician or Rayleigh PDF.

The fading channel model given in Eq. (1.22) can be seen as a Gaussian channel with attenuation h. If h is assumed to be an ergodic process, the capacity of the fading channel is the Ergodic capacity computed by the following expression

$$\mathbf{C} = E\left[\log_2\left(1 + h^2 \mathrm{SNR}\right)\right] \text{ bits/s/Hz} \tag{1.23}$$

where the expectation $E[\cdot]$ is with respect to random variable h. If $E[h^2] = 1$, Eq. (1.23) is always less than AWGN channel capacity since $E[f(X)] \leq f(E[X])$ according to Jensen inequality. If h has Rayleigh PDF, computation of Eq. (1.24) yields [5]

$$\mathbf{C} = \log_2 e \cdot \exp\left(\frac{1}{\mathrm{SNR}}\right) \cdot \mathrm{expint}\left(\frac{1}{\mathrm{SNR}}\right) \text{ bits/s/Hz} \tag{1.24}$$

where

$$\text{expint}\ (x) \triangleq \int_{x}^{\infty} \frac{e^{t}}{t} \mathrm{d}t$$

which is the capacity of the independent Rayleigh fading channel with no constraint on the constellation of the input signal. The following MATLAB program is written and used to compute the AWGN channel capacity in AWGN and the ergodic capacity of a Rayleigh fading channel.

Program 1.1: MATLAB program to compute capacity of AWGN channel and ergodic capacity of Rayleigh fading channel with channel state information (CSI).

```
% capacity of AWGN channel and ergodic capacity of Rayleigh fading
channel with %channel state information (CSI).
clear all
close all
SNRdB = [-10:0.1:30];
SNRlin = 10.^(SNRdB/10);
C_AWGN = log2 (1 + SNRlin);% AWGN
C_Rayleigh = log2(exp(1)) * exp (1 ./ SNRlin ) .* expint( 1 ./ SNRlin);%
Rayleigh
plot(SNRdB, C_AWGN, '-', SNRdB, C_Rayleigh, '--');
xlabel(' SNR(dB)'), ylabel('{\it Capacity} (bit/s/Hz)');
legend('AWGN', 'Rayleigh fading' );
```

The SNR versus capacity plot obtained from the above MATLAB program is shown in Fig. 1.8. From Fig. 1.8, it can be observed that there is a much lower performance difference between the capacities of AWGN and Rayleigh channels. This is highly indicative that the coding of fading channels will yield considerable coding gain for large SNR.

Example 1.2 For large SNR's, verify that the SNR required to obtain the same ergodic capacity for the AWGN channel and the independent Rayleigh fading channel differs by 2.5 dB.

Solution AWGN channel capacity is given by $\mathbf{C} = \log_2(1 + \text{SNR})$ bits/s/Hz.

For large SNRs, the above equation can be approximated as follows:

$$\mathbf{C} = \log_2(\text{SNR})\ \text{bits/s/Hz}$$

The ergodic capacity in Rayleigh fading channel is given by

$$\mathbf{C} = \log_2 e \cdot \exp\left(\frac{1}{\text{SNR}}\right) \cdot \text{expint}\ \left(\frac{1}{\text{SNR}}\right)$$