Economic Studies in Inequality, Social Exclusion and Well-Being

Series Editor: Jacques Silber

Satya R. Chakravarty

Inequality, Polarization and Conflict

An Analytical Study

Springer

Economic Studies in Inequality, Social Exclusion and Well-Being

Volume 12

Series editor
Jacques Silber

More information about this series at http://www.springer.com/series/7140

Satya R. Chakravarty

Inequality, Polarization and Conflict

An Analytical Study

Springer

Satya R. Chakravarty
Economic Research Unit
Indian Statistical Institute
Kolkata, India

ISSN 2364-107X ISSN 2364-1088 (electronic)
Economic Studies in Inequality, Social Exclusion and Well-Being
ISBN 978-81-322-2165-4 ISBN 978-81-322-2166-1 (eBook)
DOI 10.1007/978-81-322-2166-1

Library of Congress Control Number: 2015933995

Springer New Delhi Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer (India) Pvt. Ltd. is part of Springer Science+Business Media (www.springer.com)

In memory of
Professor Nikhilesh Bhattacharya,
my favorite teacher and mentor

Preface

The major concern of my two earlier Springer books *Ethical Social Index Numbers* and *Inequality, Polarization and Poverty: Advances in Distributional Analysis* was measurement of inequality, poverty and well-being. Only one chapter of the second monograph was devoted to an analysis of income polarization. However, research on polarization has gained impetus in the last decade because of the pivotal role of polarization in analyzing the evolution of the distribution of income, economic growth and social conflicts. Policy advisers in many countries now insist on looking at polarization as a source of social conflict. In view of this, the present monograph makes a systematic treatment of theory and methodology of alternative notions of polarization and related issues. A wide coverage of inequality, polarization and conflict is provided in the book. It gives an overall view of the recent developments in the subject.

There are two approaches to the measurement of income polarization: bipolarization and multi-polar polarization. According to the first approach, polarization is the shrinkage of the middle class; on the other hand, the later approach regards polarization as clustering around local means of the distribution, wherever these local means are located on the income scale. In order to make a clear distinction between inequality and polarization, in Chap. 1 there will be a discussion on income inequality measurement. Then Chap. 2 of the monograph goes on to analyze alternative approaches to the measurement of bipolarization rigorously.

An analysis of multi-polar polarization indices is presented in an axiomatic framework in Chap. 3. Then in Chap. 4, there will be a formal discourse on reduced-form indices which are increasingly related to between-group component and decreasingly related to within-group component of a subgroup decomposable inequality index. Social polarization refers to the widening of gaps between specific subgroups of people in terms of their social circumstances and opportunities. Chapter 5 of this monograph studies social polarizations using a rigorous and analytical structure.

It is now well-known that human well-being is a multidimensional phenomenon. While some of the dimensions correspond to ratio scale variables (e.g., income, wealth), dimensions like health and literacy are represented by ordinal variables. Study of polarization for an ordinal dimension of human welfare is the subject of Chap. 6 of the book. Chapter 7 of the book analyzes the question of the effects of inequality, fractionalization and polarization on social conflict in a broad structure.

I am indebted to Nachiketa Chattopadhyay, Conchita D'Ambrosio, Bhargav Maharaj, Amita Majumder, Sonali Roy and Claudio Zoli for the benefit I derived from them as my coauthors. I gave seminars on several sections of the book at Bar-Ilan University, Ramat-Gan, Israel; Bocconi University, Milan, Italy; Statistics-Mathematics Unit of Indian Statistical Institute, Kolkata, India; Jawaharlal Nehru University, New Delhi, India; University of International Business and Economics, Beijing, China; and Yokohama National University, Yokohama, Japan. I am grateful to the seminar participants for their comments and suggestions. I have also interacted with Rolf Aaberge, Sabina Alkire, Yoram Amiel, Tony Atkinson, Charles Blackorby, Walter Bossert, Francois Bourguignon, Frank A. Cowell, Koen Decancq, Joseph Deutsch, David Donaldson, Jean-Yves Duclos, Bhaskar Dutta, Indranil Dutta, Udo Ebert, Gary S. Fields, Marc Fleurbaey, James E. Foster, Tomoki Fujii, Carlos Gradin, Nanak C. Kakwani, Ravi Kanbur, Serge-Christophe Kolm, Peter J. Lambert, Casilda Lasso de la Vega, Maria Ana Lugo, Francois Maniquet, Laurence Roope, Amartya K. Sen, Tony Shorrocks, Jacques Silber, Kai-Yuen Tsui, Gaston Yalonetzky, Shlomo Yitzhaki and Buhong Zheng. It is a pleasure for me to express my sincere gratitude to all of them.

I thank Md. Aslam, Debasmita Basu, Nandish Chattopadhyay, Ranajoy Guha Neogi, Doyel Kayal and Pradip Maiti for generating the figure files in different chapters. I also thank my wife Sumita for carefully reading the manuscript and my son Ananyo for helpful cooperation.

Kolkata, India Satya R. Chakravarty

Contents

About the Author

Satya R. Chakravarty is a Professor of Economics at the Indian Statistical Institute, Kolkata, India. He has articles published in many internationally known journals and edited volumes on welfare issues, cooperative game theory, industrial organization and mathematical finance; and books published by Cambridge University Press and Springer. He is an Associate Editor of *Social Choice and Welfare*, a member of the Editorial Board of *Journal of Economic Inequality* and a Co-editor of *Economics –E Journal*. He worked as a consultant to the Asian Development Bank, an external reviewer of the World Bank, and also as an adviser of the National Council of Social Policy Evaluation, Mexico. He was awarded the Mahalanobis memorial prize by the Indian Econometric Society in 1994 and is a fellow of the Human Development and Capability Association.

Chapter 1
Measuring Income Inequality

1.1 Introduction

Inequality in an income distribution in a society delineates disparities of incomes among the individuals in the society. Indicators of inequality are often employed to judge the distributional effects of a particular economic policy or evaluate a particular distribution. For instance, government policy advisors may be interested in knowing whether implementation of a suggested economic policy has led the economy to a lower level of inequality over a certain period of time. In order to reduce social tensions or conflicts, a society's objective may be to reduce the level of inequality that currently exists between different ethnic or social subgroups. Which particular social subgroup or region is a major source of current level of income discrepancy in the country? Has a particular ethnic subgroup in the society become more cohesive because of reduction of inequality in the subgroup? For any partitioning of the population with respect to some socioeconomic attribute, does more overall inequality, measured by a subgroup decomposable index, make the society more polarized in the sense that there is higher between-group inequality but lower within-group inequality so that the between-group component is dominant over the within-group component? Will a highly progressive tax system be able to make the income distribution more equitable and generate sufficient funds for financing the provision of a public good?

In order to answer all such questions and related enquiries, a rigorous discussion on the measurement of inequality is necessary. This is the objective of this chapter. After presenting some preliminaries in the next section, we discuss the axioms for an index of inequality in Sect. 1.3. This discussion will enable us to make a systematic comparison between indices of inequality and polarization. There will be a discussion on ethical approaches to the measurement of inequality, including stochastic dominance, in Sect. 1.4 because it will be useful for developing a similar approach to the measurement of bipolarization. Since subgroup decomposable

S.R. Chakravarty, *Inequality, Polarization and Conflict*, Economic Studies in Inequality, Social Exclusion and Well-Being 12, DOI 10.1007/978-81-322-2166-1_1

inequality indices form the basis of reduced-form polarization indices and the related ordering presented in Chap. 4, we analyze such inequality indices in Sect. 1.5.

1.2 Preliminaries

For a population of size n, an income distribution is represented by a vector $x = (x_1, x_2, \ldots x_n) \in D^n$, where D^n is the nonnegative part of the n-dimensional Euclidean space R^n with the origin deleted. Here, x_i denotes the income of individual i of the population. We can write D^n explicitly as $D^n = R^n_+/\{0.1^n\}$, where R^n_+ is the nonnegative part of the n-dimensional Euclidean space, 1^n is the n-coordinated vector of ones, and n is any arbitrary positive integer. The set of all possible income distributions is given by $D = \underset{n \in N}{\cup} D^n$, where N is the set of positive integers. Let D^n_+ be the positive part of D^n. The sets of all possible income distributions corresponding to R^n_+ and D^n_+ are denoted by R_+ and D_+, respectively. Observe that for all $n \in N$, each of the three sets D^n, R^n_+ and D^n_+ is convex, that is, if x and y are any two elements of any of these sets, then $tx + (1-t)y$ is also an element of that set, where $0 \leq t \leq 1$ is arbitrary.

Unless specified, we will assume that D is the set of all possible income distributions. We will adopt the following notation. For all $n \in N$, for all $x \in D^n$, $\lambda(x)$ (or, simply λ) stands for the mean of $x, \frac{1}{n}\sum_{i=1}^{n} x_i$. For all $n \in N$, for all $x \in D^n$, let $\hat{x}$ be the illfare-ranked or nondecreasingly ordered permutation of x, that is, $\hat{x}_1 \leq \hat{x}_2 \leq \ldots \leq \hat{x}_n$. The distribution $\bar{x}$ is used to denote the welfare-ranked or nonincreasingly ordered permutation of x, that is, $\bar{x}_1 \geq \bar{x}_2 \geq \ldots \geq \bar{x}_n$, where $x \in D^n$ is arbitrary. By an inequality index, we mean a nonconstant function $I : D \to R^1_+$. This general definition of an inequality index allows inequality comparisons of distributions of income whose totals as well as population sizes are different. If the domain of I is simply D^n, then we can make only comparisons of inequality for a fixed population size n. Inequality is not defined if $n = 1$. Consequently, we assume that $n \geq 2$.

Definition 1.1 A function $H : D \to R^1$ is called concave if for all $n \in N, x, y \in D^n$ and for all $0 \leq t \leq 1, H(tx + (1-t)y) \geq tH(x) + (1-t)H(y)$. The function $H : D^n \to R^1$ is called strictly concave if $H(tx + (1-t)y) > tH(x) + (1-t)H(y)$ for all $0 < t < 1$ and for all $x, y \in D^n$, where $x \neq y$. The function $H : D^n \to R^1$ is defined as convex (strictly convex) if $-H : D^n \to R^1$ is concave (strictly concave).

Definition 1.2 A function $H : D \to R^1$ is called S-concave if for all $n \in N, x \in D^n$ and for all bistochastic matrices A of order $n, H(xA) \geq H(x)$, where an $n \times n$ matrix A with nonnegative entries is called a bistochastic matrix order n if each of its rows and columns sums to unity.[1] Strict S-concavity of H requires that the weak inequality is to be replaced by a strictly inequality whenever xA is not a reordering or permutation of x. A function $H : D \rightarrow R^1$ is defined as S-convex (strictly S-convex) if $-H : D^n \rightarrow R^1$ is S-concave (strictly S-concave).

Definition 1.3 A function $H : D \rightarrow R^1$ is called symmetric if all $n \in N$, $x \in D^n$, $H(x) = H(y)$, where y is any permutation of x, that is, $y = x\Pi$, where Π is any permutation matrix of order n.

Symmetry requires invariance of the value of the function under reordering of incomes. It is an anonymity principle. All S-concave functions are symmetric.

Definition 1.4 For all $n \in N, x, y \in D^n$, we say that x is obtained from y by a Pigou (1912)–Dalton (1920) progressive transfer (progressive transfer, for short), which we denote by xTy, if for some i, j and $c > 0$,

$$x_i = y_i + c \leq x_j,$$
$$x_j = y_j - c,$$

and $x_k = y_k$ for all $k \neq i, j$.

That is, x is obtained from y by a transfer of c units of income from a rich person j to a poor person i that does not make the donor poorer than the recipient. Equivalently, we can say that y is obtained from x by a regressive transfer.

Definition 1.5 For all $n \in N, x, y \in D^n$, x is said to be obtained from y by a simple increment if $y_j + c = x_j$ for some j and $x_i = y_i$ for all $i \neq j$, where $c > 0$.

That is, x and y are identical except that the j^{th} income in x is obtained by increasing the corresponding income in y by the amount c. We denote this by the inequality $x \geq y$.

Definition 1.6 A function $H : D \rightarrow R^1$ is called increasing in individual arguments (increasing, for short) if for all $n \in N$, $x, y \in D^n$, $H(x) > H(y)$ whenever x is obtained from y by a simple increment.

Definition 1.7 For any $x \in D^n$, $y \in D^{nl}$, where each income in x appears l times in y, is called an $l-$fold replication of x, where $l \geq 2$ is an integer.

For ordered distributions, ordering should be maintained in the replicated versions as well. For instance, if $x \in D^n$, then the income distribution $\hat{y} = (\hat{x}_1, \hat{x}_1, \ldots, \hat{x}_1, \hat{x}_2, \ldots \hat{x}_2, \ldots, \hat{x}_n)$, where each $\hat{x}_i$ appears l times, is a $l-$ fold replication of $\hat{x}$.

Definition 1.8 A function $H : D \rightarrow R^1$ is called population replication invariant if for all $n \in N$, $x \in D^n$, $H(x) = H(y)$, where $y \in D^{nl}$ is an $l-$ fold replication of x, $l \geq 2$ being any integer.

[1] A bistochastic matrix order n with exactly one positive entry in each row and column is called a permutation matrix of order n.