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Brigitte Falkenburg · Margaret Morrison
Editors

Why More Is Different

Philosophical Issues in Condensed Matter
Physics and Complex Systems

 Springer

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Chapter 1

Introduction

Brigitte Falkenburg and Margaret Morrison

This volume on philosophical issues in the physics of condensed matter fills a crucial gap in the overall spectrum of philosophy of physics. Philosophers have generally focused on emergence in debates relating to the philosophy of mind, artificial life and other complex biological systems. Many physicists working in the field of condensed matter have significant interest in the philosophical problems of reduction and emergence that frequently characterise the complex systems they deal with. More than four decades after Philip W. Anderson's influential paper *More is Different* (Anderson 1972) and his well known exchange with Steven Weinberg in the 1990s on reduction/emergence, philosophers of physics have begun to appreciate the rich and varied issues that arise in the treatment of condensed matter phenomena. It is one of the few areas where physics and philosophy have a genuine overlap in terms of the questions that inform the debates about emergence. In an effort to clarify and extend those debates the present collection brings together some well-known philosophers working in the area with physicists who share their strong philosophical interests.

The traditional definition of emergence found in much of the philosophical literature characterizes it in the following way: A phenomenon is emergent if it cannot be reduced to, explained or predicted from its constituent parts. One of the things that distinguishes emergence in physics from more traditional accounts in philosophy of mind is that there is no question about the “physical” nature of the emergent phenomenon, unlike the nature of, for example, consciousness. Despite these differences the common thread in all characterizations of emergence is that it depends on a hierarchical view of the world; a hierarchy that is ordered in some fundamental way. This hierarchy of levels calls into question the role of reduction

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in relating these levels to each other and forces us to think about the relation of parts and wholes, explanation, and prediction in novel ways.

In discussing this notion of a “hierarchy of levels” it is important to point out that this is not necessarily equivalent to the well known fact that phenomena at different scales may obey different fundamental laws. For instance, while general relativity is required on the cosmological scale and quantum mechanics on the atomic, these differences do not involve emergent phenomena in the sense described above. If we characterise emergence simply in terms of some “appropriate level of explanation” most phenomena will qualify as emergent in one context or another. Emergence then becomes a notion that is defined in a relative way, one that ceases to have any real ontological significance. In true cases of emergence we have generic, stable behaviour that *cannot* be explained in terms of microphysical laws and properties. The force of “cannot” here refers not to ease of calculation but rather to the fact that the micro-physics fails to provide the foundation for a physical explanation of emergent behaviour/phenomena. Although the hierarchical structure is certainly present in these cases of emergence the ontological status of the part/whole relation is substantially different.

What this hierarchical view suggests is that the world is ordered in some fundamental way. Sciences like physics and neurophysiology constitute the ultimate place in the hierarchy because they deal with the basic constituents of the world—fundamental entities that are not further reducible. Psychology and other social sciences generally deal with entities at a less fundamental level, entities that are sometimes, although not always, characterised as emergent. While these entities may not be reducible to their lower level constituents they are nevertheless *ontologically* dependent on them. However, if one typically identifies explanation with reduction, a strategy common across the sciences, then this lack of reducibility will result in an accompanying lack of explanatory power. But, as we shall see from the various contributions to this volume, emergent phenomena such as superconductivity and superfluidity, to name a few, are also prevalent in physics. The significance of this is that these phenomena call into question the reliance on reduction as the ultimate form of explanation in physics and that everything can be understood in terms of its micro-constituents and the laws that govern them.

The contributions to the collection are organized in three parts: reduction, emergence, and the part-whole-relation, respectively. These three topics are intimately connected. The reduction of a whole to its parts is typical of explanation and the practices that characterise physics; novel phenomena typically emerge in complex compound systems; and emergence puts limitations on our ability to see reduction as a theoretical goal. In order to make these relations transparent, we start by clarifying the concepts of reduction and emergence. The first part of the book deals with general issues related to reduction, its scope, concepts, formal tools, and limitations. The second part focuses on the characteristic features of emergence and their relation to reduction in condensed matter physics. The third deals with specific models of the part-whole-relation used in characterizing condensed matter phenomena.

1.1 Reduction

Part I of the book embraces four very different approaches to the scope, concepts, and formal tools of reduction in physics. It also deals with the relation between reduction and explanation, as well as the way limitations of reduction are linked with emergence. The first three papers are written by condensed matter physicists whose contributions to the collection focus largely on reduction and its limitations. The fourth paper, written by a philosopher-physicist, provides a bridge between issues related to reduction in physics and more philosophically oriented approaches to the problem.

On the Success and Limitations of Reductionism in Physics by Hildegard Meyer-Ortmanns gives an overview of the scope of reductionist methods in physics and beyond. She points out that in these contexts ontological and theoretical reduction typically go together, explaining the phenomena in terms of interactions of smaller entities. Hence, for her, ontological and theoretical reduction are simply different aspects of methodological reduction which is the main task of physics; a task that aims at explanation via part-whole relations (ontological reduction) and the construction of theories describing the dynamics of the parts of a given whole (theoretical reduction). This concept of “methodological” reduction closely resembles what many scientists and philosophers call “mechanistic explanation” (see Chap. 13). The paper focuses on the underlying principles and formal tools of theoretical reduction and illustrates them with examples from different branches of physics. She shows how the same methods, in particular, the renormalization group approach, the “single step” approaches to pattern formation, and the formal tools of quantum field theory, are used in several distinct areas of research such as particle physics, cosmology, condensed matter physics, and biophysics. The limitations of methodological reduction in her sense are marked by the occurrence of strong emergence, i.e., non-local phenomena which arise from the local interactions of the parts of a complex system.

Barbara Drossel’s contribution reminds us that the thorny problem of theoretical reduction in condensed matter physics deals, in fact, with three theories rather than two. *On the Relation between the Second Law of Thermodynamics and Classical and Quantum Mechanics* reviews the foundations of the thermodynamic arrow of time. Many physicists and philosophers take for granted that the law of the increase in entropy is derived from classical statistical mechanics and/or quantum mechanics. But how can irreversible processes be derived from reversible deterministic laws? Drossel argues that all attempts to obtain the second law of thermodynamics from classical mechanics include additional assumptions which are extraneous to the theory. She demonstrates that neither Boltzmann’s H-theorem nor the coarse graining of phase-space provide a way out of this problem. In particular, coarse graining as a means for deriving the second law involves simply specifying the state of a system in terms of a finite number of bits. However, if we regard the concept of entropy as based on the number of possible microstates of a closed system, then this approach obviously begs the question. She emphasizes that

quantum mechanics also fails to resolve the reduction problem. Although the Schrödinger equation justifies the assumption of a finite number of possible microstates, it does not explain the irreversibility and stochasticity of the second law.

Joachim Ankerhold addresses another complex reduction problem in the intersection of quantum mechanics, thermodynamics and classical physics, specifically, the question of how classical behaviour emerges from the interactions of a quantum system and the environment. The well-known answer is that the dissipation of the superposition terms into a thermal environment results in decoherence. *Dissipation in Quantum Mechanical Systems: Where is the System and Where is the Reservoir?* shows that issues surrounding this problem are not so simple. Given that the distinction between a quantum system and its environment is highly problematic, the concept of an open quantum system raises significant methodological problems related to ontological reduction. Condensed matter physics employs ‘system + reservoir’ models and derives a reduced density operator of the quantum system in order to describe decoherence and relaxation processes. The ‘system + reservoir’ picture depends on the epistemic distinction of the relevant system and its irrelevant surroundings. But, due to quantum entanglement it is impossible to separate the system and the reservoir, resulting in obvious limitations for the naïve picture. The paper shows that the model works only for very weak system-reservoir interactions based on a kind of perturbational approach; whereas in many other open quantum systems it is difficult to isolate any “reduced” system properties. However, due to a separation of time scales the appearance of a (quasi-) classical reduced system becomes possible, even in the deep quantum domain.

Rafaella Hillerbrand in her contribution entitled *Explanation via Microreduction: On the Role of Scale Separation for Quantitative Modelling* argues that scale separation provides the criterion for specifying the conditions under which ontological reduction can be coupled with theoretical or explanatory reduction. She begins by clarifying the philosophical concepts of reduction. The distinction between “ontological” and “explanatory” reduction employed here is based on the opposition of ontology and epistemology, or the distinction between what there is and what we know. Ontological reduction is “micro-reduction”, similar to Meyer-Ortmann’s concept of ontological reduction. Theoretical reduction is based on knowledge and can be further divided into “epistemic” reduction (tied to the DN- or deductive nomological model of explanation), and explanatory reduction in a broader sense, the main target of Hillerbrand’s investigation. Her paper discusses scale separation and the role it plays in explaining the macro features of systems in terms of their micro constituents. She argues that scale separation is a necessary condition for the explanatory reduction of a whole to its parts and illustrates this claim with several examples (the solar system, the laser, the standard model of particle physics, and critical phenomena) and a counter-example (fluid dynamic turbulence). Her main conclusion is that micro-reduction with scale separation gives rise to a special class of reductionist models.

1.2 Emergence

The papers in Part II take a closer look at the limitations of reduction in order to clarify various philosophical aspects of the concept of emergence. According to the usual definition given above, emergent phenomena arise out of lower-level entities, but they cannot be reduced to, explained nor predicted from their micro-level base. Given that solids, fluids and gases consist of molecules, atoms, and subatomic particles, how do we identify emergent phenomena as somehow distinct from their constituents? What exactly is the relation between the micro- and the macro-level, or the parts and the emergent properties of the whole? A crucial concept is autonomy, that is, the independence of the emergent macro-properties. But the term “emergent” means that such properties are assumed to arise out of the properties and/or dynamics of the parts. How is this possible and what does this mean for the autonomy or independence of emergent phenomena?

Margaret Morrison focuses on the distinction of epistemic and ontological independence in characterizing emergence and how this is distinguished from explanatory and ontological reduction. *Why and How is More Different?* draws attention to the fact that the traditional definition of emergence noted above can be satisfied on purely epistemological grounds. However, taking account of Anderson’s seminal paper we are presented with a notion of emergence that has a strong ontological dimension—that the whole is *different* from its parts. Since the phenomena of condensed matter physics are comprised of microphysical entities the challenge is to explain how this part/whole relation can be compatible with the existence of ontologically independent macro-properties; the properties we characterize as emergent. For example, all superconducting metals exhibit universal properties of infinite conductivity, flux quantization and the Meissner effect, regardless of the microstructure of the metal. However, we typically explain superconductivity in terms of the micro-ontology of Cooper pairing, so in what sense are the emergent properties independent/autonomous? Understanding this micro-macro relation is crucial for explicating a notion of emergence in physics. Morrison argues that neither supervenience nor quantum entanglement serve to explain the ontological autonomy of emergent phenomena. Nor can theoretical descriptions which involve approximation methods etc., explain the appearance of generic, universal behaviour that occurs in phase transitions. The paper attempts a resolution to the problem of ontological independence by highlighting the role of spontaneous symmetry breaking and renormalization group methods in the emergence of universal properties like infinite conductivity.

Robert Battermann’s contribution entitled *Autonomy and Scales* also addresses the problem of autonomy in emergent behaviour but from a rather different perspective, one that has been ignored in the philosophical literature. He focuses on a set of issues involved in modelling systems across many orders of magnitude in spatial and temporal scales. In particular, he addresses the question of how one can explain and understand the relative autonomy and safety of models at continuum scales. He carefully illuminates why the typical battle line between reductive

“bottom-up” modelling and ‘top-down’ modelling from phenomenological theories is overly simplistic. Understanding the philosophical foundations implicit in the physics of continuum scale problems requires a new type of modelling framework. Recently multi-scale models have been successful in showing how to upscale from statistical atomistic/molecular models to continuum/hydrodynamics models. Batterman examines these techniques as well as the consequences for our understanding of the debate between reductionism and emergence. He claims that there has been too much focus on what the actual fundamental level is and whether non-fundamental (idealized) models are dispensable. Moreover, this attention to the “fundamental” is simply misguided. Instead we should focus on proper modeling techniques that provide bridges across scales, methods that will facilitate a better understanding of the relative autonomy characteristic of the behavior of systems at large scales.

Paul Humphreys paper *‘More is Different ... Sometimes’* presents a novel and intriguing interpretation of Philip Anderson’s seminal paper *‘More Is Different’*. While Anderson’s paper is explicit in its arguments for the failure of construction methods in some areas of physics, Humphreys claims that it is inexplicit about the consequences of those failures. He argues that as published, Anderson’s position is obviously consistent with a reductionist position but, contrary to many causal claims, does not provide evidence for the existence of emergent phenomena. Humphreys defines various emergentist positions and examines some recent undecidability results about infinite and finite Ising lattices by Barahona and by Gu et al. He claims that the former do not provide evidence for the existence of ontologically emergent states in real systems but they do provide insight into prediction based accounts of emergence and the limits of certain theoretical representations. The latter results bear primarily on claims of weak emergence and provide support for Anderson’s views. Part of the overall problem, Humphreys argues, is that one should not move from conclusions about the failure of constructivism and undecidability to conclusions about emergence without an explicit account of what counts as an entity being emergent and why. The failure of constructivism in a particular instance is not sufficient for emergence in the sense that the inability in practice or in principle to compute values of a property is insufficient for the property itself to count as emergent. He leaves as an open question the pressing problem of determining what counts as a novel physical property.

Continuing with the attempt to clarify exactly what it at stake in the characterization of emergent phenomena, Sorin Bangu’s paper *Neither Weak, Nor Strong? Emergence and Functional Reduction* draws attention to the long history behind the clarification of the concept of emergence, especially in the literature on the metaphysics of science. Notions such as ‘irreducibility’, ‘novelty’ and ‘unpredictability’ have all been invoked in an attempt to better circumscribe this notoriously elusive idea. While Bangu’s paper joins that effort, it also contributes a completely different perspective on the clarificatory exercise. He carefully examines a class of familiar physical processes such as boiling and freezing, processes generically called ‘phase transitions’ that are characteristic of what most philosophers and physicists take to be paradigm cases of emergent phenomena. Although he is broadly sympathetic to

some aspects of the traditional characterization, the paper questions what kind of emergence these processes are thought to instantiate. Bangu raises this issue because he ultimately wants to depart from the orthodoxy by claiming that the two types of emergence currently identified in the literature, ‘weak’ and ‘strong’, do not adequately characterize the cases of boiling and freezing. The motivation for his conclusion comes from an application of Kim’s (1998, 1999, 2006) ‘functional’ reduction model (F-model). When applied to these cases one finds that their conceptual location is undecided with respect to their ‘emergent’ features. As it turns out, their status depends on how one understands the idealization relation between the theories describing the macro-level (classical thermodynamics) and the micro-level (statistical mechanics) reality.

1.3 Parts and Wholes

Part III consists of four papers that focus on the part-whole-relation in order to shed light on the methods, successes and limitations of ontological reduction in condensed matter physics and beyond. The first two contributions discuss the explanatory power of the many-body systems of condensed matter physics but with a very different focus in each case. The last two papers investigate the dynamical aspects of the part-whole relation and their ontological consequences. Today, ontological reduction is often characterised in terms of “mechanistic explanation”. A mechanism typically consists of some type of causal machinery according to which the properties of a whole are caused by the dynamic activities of the parts of a compound system. In that sense the papers in this section of the book deal, broadly speaking, with the successes and limitations of mechanistic explanation, even though the term is only used specifically in Kuhlman’s paper.

Andreas Hüttemann, Reimer Kühn, and Orestis Terzidis address the question of whether there is an explanation for the fact that, as Fodor put it, the micro-level “converges on stable macro-level properties”, and whether there are lessons from this explanation for similar types of issues. *Stability, Emergence and Part-Whole-Reduction* presents an argument that stability in large (but non-infinite) systems can be understood in terms of statistical limit theorems. They begin with a small simulation study of a magnetic system that is meant to serve as a reminder of the fact that an increase of the system size leads to reduced fluctuations in macroscopic properties. Such a system exhibits a clear trend towards increasing stability of macroscopic (magnetic) order and, as a consequence, the appearance of ergodicity breaking, i.e. the absence of transitions between phases with distinct macroscopic properties in finite time. They describe the mathematical foundation of the observed regularities in the form of limit theorems of mathematical statistics for independent variables (Jona-Lasinio 1975) which relates limit theorems with key features of large scale descriptions of these systems. Generalizing to coarse-grained descriptions of systems of interacting particle systems leads naturally to the incorporation of renormalization group ideas. However, in this case Hüttemann et al. are mainly

interested in conclusions the RNG approach allows one to draw about system behaviour away from criticality. Hence, an important feature of the analysis is the role played by the finite size of actual systems in their argument. Finally, they discuss to what extent an explanation of stability is a reductive explanation. Specifically they claim to have shown that the reductionist picture, according to which the constituents' properties and states determine the behaviour of the compound system, and the macro-phenomena can be explained in terms of the properties and states of the constituents, is neither undermined by stable phenomena in general nor by universal phenomena in particular.

Axel Gelfert's contribution *Between Rigor and Reality: Many-Body Models in Condensed Matter Physics* focusses on three theoretical dimensions of many-body models and their uses in condensed matter physics: their structure, construction, and confirmation. Many-body models are among the most important theoretical 'workhorses' in condensed matter physics. The reason for this is that much of condensed matter physics aims to explain the macroscopic behaviour of systems consisting of a large number of strongly interacting particles, yet the complexity of this task requires that physicists turn to simplified (partial) representations of what goes on at the microscopic level. As Gelfert points out, because of the dual role of many-body models as models of physical systems (with specific physical phenomena as their explananda) as well as mathematical structures, they form an important sub-class of scientific models. As such they can enable us to draw general conclusions about the function and functioning of models in science, as well as to gain specific insight into the challenge of modelling complex systems of correlated particles in condensed matter physics. Gelfert's analysis places many-body models in the context of the general philosophical debate about scientific models (especially the influential 'models as mediators' view), with special attention to their status as mathematical models. His discussion of historical examples of these models provides the foundation for a distinction between different strategies of model construction in condensed matter physics. By contrasting many-body models with phenomenological models, Gelfert shows that the construction of many-body models can proceed either from theoretical 'first principles' (sometimes called the *ab initio* approach) or may be the result of a more constructive application of the formalism of many-body operators. This formalism-based approach leads to novel theoretical contributions by the models themselves (one example of which are so-called 'rigorous results'), which in turn gives rise to cross-model support between models of different origins. A particularly interesting feature of Gelfert's deft analysis is how these different features allow for exploratory uses of models in the service of fostering model-based understanding. Gelfert concludes his paper with an appraisal of many-body models as a specific way of investigating condensed matter phenomena, one that steers a middle path 'between rigor and reality'.

Brigitte Falkenburg investigates the ontological status of quasi-particles that emerge in solids. Her paper *How Do Quasi-Particles Exist?* shows that structures which emerge within a whole may, in fact, be like the parts of that whole, even though they seem to be higher-level entities. Falkenburg argues that quasi-particles are real, collective effects in a solid; they have the same kinds of physical properties

and obey the same conservation laws and sum rules as the subatomic particles that constitute the solid. Hence, they are ontologically on a par the electrons and atomic nuclei. Her paper challenges the philosophical view that quasi-particles are fake entities rather than physical particles and counters Ian Hacking's reality criterion: "If you can spray them, they exist". Because of the way quasi-particles can be used as markers etc. in crystals, arguments against their reality tend to miss the point. How, indeed, could something that contributes to the energy, charge etc. of a solid in accordance with the conservation laws and sum rules be classified as "unreal"? In order to spell out the exact way in which quasi-particles exist, the paper discusses their particle properties in extensive detail. They are compared in certain respects to those of subatomic matter constituents such as quarks and the virtual field quanta of a quantum field. Falkenburg concludes that quasi-particles are ontologically on par with the real field quanta of a quantum field; hence, they are as real or unreal as electrons, protons, quarks, photons, or other quantum particles. Her contribution nicely shows that the questions of scientific realism cannot be settled without taking into account the emergent phenomena of condensed matter physics, especially the conservation laws and sum rules that connect the parts and whole in a hierarchical view of the physical world.

Meinard Kuhlmann's paper addresses the important issue of mechanistic explanations which are often seen as the foundation for what is deemed explanatory in many scientific fields. Kuhlmann points out that whether or not mechanistic explanations are (or can be) given does not depend on the science or the basic theory one is dealing with but rather on the type of object or system (or 'object system') under study and the specific explanatory target. As a result we can have mechanistic and non-mechanistic explanations in both classical and quantum mechanics. *A Mechanistic Reading of Quantum Laser Theory* shows how the latter is possible. Kuhlmann's argument presents a novel approach in that quantum laser theory typically proceeds in a way that seems at variance with the mechanistic model of explanation. In a manner common in the treatment of complex systems, the detailed behaviour of the component parts plays a surprisingly subordinate role. In particular, the so-called "enslaving principle" seems to defy a mechanistic reading. Moreover, being quantum objects the "parts" of a laser are neither located nor are they describable as separate entities. What Kuhlmann shows is that despite these apparent obstacles, quantum laser theory provides a good example of a mechanistic explanation in a quantum-physical setting. But, in order to satisfy this condition one needs to broaden the notion of a mechanism. Although it is tempting to conclude that these adjustments are ad hoc and question-begging, Kuhlmann expertly lays out in detail both how and why the reformulation is far more natural and less drastic than one may expect. He shows that the basic equations as well as the methods for their solution can be closely matched with mechanistic ideas at every stage. In the quantum theory of laser radiation we have a decomposition into components with clearly defined properties that interact in specific ways, dynamically producing an organization that gives rise to the macroscopic behavior we want to explain. He concludes the analysis by showing that the structural

similarities between semi-classical and quantum laser theory also support a mechanistic reading of the latter.

Most of the contributions to this volume were presented as talks in a workshop of the Philosophy Group of the German Physical Society (DPG) at the general spring meeting of the DPG in Berlin, March 2012. Additional papers were commissioned later. We would like to thank the DPG for supporting the conference from which the present volume emerged, and Springer for their interest in the publication project and for allowing us the opportunity to put together a volume that reflects new directions in philosophy of physics. A very special thank you goes to Angela Lahee from Springer, who guided the project from the initial proposal through to completion. In addition to her usual duties she wisely prevented us from giving the volume the amusing but perhaps misleading title “Condensed Metaphysics” (as an abbreviation of “The Metaphysics of Condensed Matter Physics”). Not only did she offer many helpful suggestions for the title and the organisation of the book, but showed tremendous patience with the usual and sometimes unusual delays of such an edition. Finally we would like to thank each of the authors for their contributions as well as their willingness to revise and reorganise their papers in an effort to make the volume a novel and we hope valuable addition to the literature on emergence in physics.

Part I

Reduction

Chapter 2

On the Success and Limitations of Reductionism in Physics

Hildegard Meyer-Ortmanns

2.1 Introduction

Natural sciences, and in particular physics, can look back over a track record of increasing predictive power with regard to the outcome of time evolutions, control, as well as the design of experiments of far-reaching technological and practical importance. But, their success has also brought deeper insights into the underlying laws that govern a wide variety of phenomena. Without doubt this success is based on methodological reductionism, i.e., the attempt to reduce explanations to smaller constituents (although not necessarily the smallest) and to explain phenomena completely in terms of interactions between fundamental entities. Included in the scope of methodological reductionism is theoretical reductionism, wherein one theory with limited predictive power can be obtained as a limiting case of another theory, just as Newtonian mechanics is included in general relativity. From the beginning we should emphasize that reductionism does not preclude emergent phenomena. It allows one to predict some types of emergent phenomena, as we shall see later, even if these phenomena are not in any sense the sum of the processes from which they emerge.

In the following, emergence is understood as involving new, sometimes novel properties of a whole that are not shared by its isolated parts. Emergent phenomena generated this way are therefore intrinsically nonlocal. Within the reductionistic approach we understand them as a result of local interactions, as characteristic of approaches in physics. Emergent phenomena definitely extend beyond simple formation of patterns, such as those in mass and pigment densities. Functionality may be an emergent property as well, as in cases where systems are built up of cells, the fundamental units of life. In our later examples, we shall not refer to “weak emergence”, where a phenomenon is predicted as a result of a model.

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Instead, we shall usually mean “strong emergence”, where nonlocal phenomena arise from local interactions.

Emergent features are not restricted to patterns in an otherwise homogeneous background. “Being alive” is also an emergent property, arising from the cell as the fundamental unit of life. The very notion of complexity is a challenging one. In our context, systems are considered genuinely complex if they show behavior that cannot be understood by considering small subsystems separately. Our claim is a modest one—it is not that complex systems can be understood in all their facets by analyzing them locally, but that complexity can often be reduced by identifying local interactions. Moreover, we do not adopt the extreme view which considers complex systems as inherently irreducible, thereby requiring a holistic approach. The art is to focus on just those complex features that can be reduced and broken up into parts. Why this is not a fruitless endeavor is the topic of the Sect. 2.2.

Section 2.2 deals with the “recipes” responsible for the success. They are abstract guiding principles as well as the use of symmetries, such as the principle of relativity and Lorentz covariance, leading to the theory of special relativity; the equivalence principle and covariance under general coordinate transformations, leading to the theory of general relativity, as well as the gauge principle and invariance under local gauge transformations (complemented by the Higgs mechanism for the electroweak part), leading to the standard model of elementary particle physics. These theories have extraordinary predictive power for phenomena that are governed by the four fundamental interactions; three of them involve the realm of subatomic and atomic physics at one end of the spatial scale, while gravity becomes the only relevant interaction on cosmic scales, where it determines the evolution of the universe.

Interactions on macro or intermediate mesoscopic scales, like the nano and microscales, are in principle produced by the fundamental interactions when composite objects are formed. In practice, they can be derived using a phenomenological approach that involves models valid on this particular scale. Beyond the very formulation of these models, reductionism becomes relevant as soon as one tries to bridge the scales, tracing phenomena on the macroscale back to those on the underlying scales. “Tracing back” means predicting changes on the macro and mesoscopic scales produced by changes on the microscale. A computational framework for performing these bridging steps is the renormalization group approach of Kogut and Wilson (1974), Wilson (1975) and Kadanoff (1977). The framework of the renormalization group goes far beyond critical phenomena, magnetism, and spin systems (see Sect. 2.2.2.1).¹ More generally, but very much in the spirit of the renormalization group, we now have what is called multiscale analysis, with applications in a variety of different realms. In general, it involves links between different subsystems, with each subsequent system having fewer degrees of freedom than its predecessor. The new system may still be

¹ For further applications, see also Meyer-Ortmanns and Reisz (2007).

complex, but the iterative nature of the procedure gradually reduces the complexity (see Sect. 2.2.2.4 below).

Sometimes one is in the fortunate situation where no intermediate steps are needed to bridge the scales from micro to macro behaviour. This can happen when static spatial patterns form on large scales according to rules obeyed by the constituents on the smaller scale, or when shock waves propagate over large distances and transport local changes. We shall illustrate pattern formation with applications as different as galaxy formation in the universe as well as spots and stripes on animals in the realm of living systems. We shall further use dynamical pattern formation in evolving strains of bacteria to illustrate increasing mathematical complexity, as more and more features are simultaneously taken into account. This leads us to conclude that any candidate for an equation of “everything” will be constrained to describe only “something”, but not the whole (see Sect. 2.2.4).

One may wonder why there is in general a need for bridging the scales in intermediate steps. Why not use a single step by exploiting modern computer facilities? After all, it is now possible to simulate a virus in terms of its atomic constituents (an example will be sketched in Sect. 2.2.5). The very same example we use to illustrate the power of up-to-date computer simulations could in principle also serve to demonstrate typical limitations of reductionism. Reductionism, pushed to its extreme, makes the description clumsy. It does not identify the main driving mechanisms on intermediate scales that underlie the results on larger scales. Reductionism then falls short of providing explanations in terms of simple mechanisms, which is what we are after. A more serious worry is that new aspects, properties, features, and interpretations may emerge on the new scale that a computer experiment may inevitably miss. In a fictive dialogue we debate the positions of an extreme reductionism with a more moderate version. As an example of the moderate version, we consider DNA from the perspective of physics and computer science. Even if there are no equations of theories that deserve the attribute “of everything”, or if a multitude of disciplines must be maintained in the future, one may still wonder whether some further steps towards a universal theory of complex systems are possible. Such steps will be sketched in Sect. 2.4.

2.2 On the Success of Reductionism

2.2.1 *Symmetries and Other Guiding Principles*

Physical theories are primarily grounded in experiment in that they are proposed to reproduce and predict experimental outcomes. What distinguishes them from optimized fits of data sets is their range of applicability and their predictive power. Some of these theories deserve to be classified as fundamental. To this class belongs the theories of special, general relativity and the standard model of particle physics.

In this section we would like to review some guiding principles that led to their construction, restricting what would otherwise be a multitude of models to a limited set.

2.2.1.1 The Special Relativity Principle

According to Albert Einstein the Special Relativity Principle postulates that all inertial frames are totally equivalent for the performance of all physical experiments, not only mechanical ones, but also electrodynamics. (In this way, Einstein was able to eliminate absolute space as the carrier of light waves and electromagnetic fields.) Insisting in particular on the constancy of the velocity of light propagation in all inertial frames, one is then led in a few steps to the conclusion that the coordinates of two inertial frames must be related by Lorentz transformations. (First one can show that the transformations must be linear, then one can reduce considerations to special transformations in one space direction, and finally one shows that the well-known γ -factor takes its familiar form $\gamma = 1/\sqrt{1 - v^2/c^2}$.)² If a physical law is invariant under these special Lorentz transformations, and also under spatial rotations and translations in space and time, it holds in any inertial system. The corresponding transformations between two arbitrary inertial systems are then Poincaré transformations. The reductionism arising from the special relativity principle (including the constancy of the velocity of light) leads to the restriction to formulate laws in inertial frames in flat space as equations between tensors under Poincaré transformations. In particular, it restricts the choice of Lagrangians, such as the Lagrangian of electrodynamics, to scalars under these transformations.

2.2.1.2 The Equivalence Principle and General Relativity

Einstein wanted to eliminate “absolute space” in its role in distinguishing inertial frames as those in which the laws take a particularly simple form. He put the equivalence principle at the center of his considerations. According to the (so-called) weak equivalence principle, inertial and gravitational mass are proportional for *all* particles, so that all particles experience the same acceleration in a given gravitational field. This suggests absorbing gravity into geometry, the geometry of space-time, to which all matter is exposed. The equivalence principle led Einstein to formulate his general relativity theory. From Newton’s theory, it was already known that mechanics will obey the same laws in a freely falling elevator as in a laboratory that is not accelerated and far away from all attracting masses. Einstein extrapolated this fact to hold, not only for the laws of mechanics, but so that all local, freely falling, nonrotating labs are fully equivalent for the performance of *all* experiments.

² For the derivation see, for example, Rindler (1969).

(Therefore the simple laws from inertial systems now hold everywhere in space, but only locally, so that special relativity also becomes a theory that is supposed to hold only locally.) This extrapolation amounts to the postulate that the equations in curved space-time should be formulated as tensor equations under general coordinate transformations, where curved space-time absorbs the effect of gravity. Due to the homogeneous transformation behavior of tensors, the validity of tensor equations in one frame ensures their validity in another frame, related by general coordinate transformations. This postulate finally led Einstein to the theory of general relativity that has been confirmed experimentally to a high degree of accuracy.

2.2.1.3 Gauge Theories of the Fundamental Interactions

In the previous sections on the relativity principle, the postulated symmetries referred to transformations of the space-time coordinates and restricted the form of physical laws. In the theories of strong, weak, and electromagnetic interactions, we have to deal with internal symmetries. Here it is not only the right choice of the symmetry group which is suggested by conserved matter currents, but also the prescription of how to implement the dynamics of matter and gauge fields that lead to the construction of gauge theories and finally to the standard model of particle physics.

Hermann Weyl was the first to consider electromagnetism as a local gauge theory of the $U(1)$ symmetry group (Weyl 1922). Let us first summarize the steps in common to the construction of electromagnetic, strong, and weak interactions as a kind of “recipe”. As result of Noether’s theorem, one can assign a global (space-independent) continuous symmetry to a conserved matter current. The postulate of local gauge invariance then states that the combined theory of matter and gauge fields should be invariant under local (that is space-dependent) gauge transformations. Obviously, a mass term, which is bilinear in the matter fields $\psi, \bar{\psi}$ and contains a partial derivative, violates this invariance. To compensate for the term that is generated from the derivative of the space dependent phase factors in the gauge transformations, one introduces a so-called minimal coupling between the matter fields and the gauge fields, replacing the partial derivative by the covariant derivative in such a way that the current is covariantly conserved. It remains to equip the gauge fields with their own dynamics and construct the gauge field strengths in such a way that the resulting Lagrangian is invariant under local gauge transformations.

Let us demonstrate these steps in some more detail. Under local gauge transformations, matter fields $\psi_c(x)$ transform according to

$$\psi_c(x) \rightarrow (\exp(-i\Theta^a(x)T_a))_{cc'}\psi_{c'}(x) \equiv (g(x)\psi)_c(x). \quad (2.1)$$

Here T_a are the infinitesimal generators of the symmetry group $SU(n)$ in the fundamental representation, $\Theta^a(x)$ are the space dependent group parameters,

$a = 1, \dots, N$, with N the dimension of the group, and $c, c' = 1, \dots, n$, where n characterizes the symmetry group $SU(n)$. Gauge fields $\mathcal{A}_{\mu, cc'}(x) = \bar{g}A_{\mu}^a(x)(T_a)_{cc'}$, which are linear combinations of the generators T_a , with \bar{g} a coupling constant, transform inhomogeneously according to

$$\mathcal{A}'_{\mu}(x) = g(x)(\mathcal{A}_{\mu} - i\hat{\partial}_{\mu})g^{-1}(x). \quad (2.2)$$

In general, this equation should be read as a matrix equation. In the language of differential geometry, the gauge field corresponds to a connection, it allows one to define a parallel transport of charged vector fields $\psi_c(x)$ from one space-time point x , along a path \mathcal{C} to another point y . This parallel transport can then be used to compare vector fields from different space-time points in one and the same local coordinate system. It thus leads to the definition of the covariant derivative D_{μ} :

$$[(\hat{\partial}_{\mu} + i\mathcal{A}_{\mu})\psi(x)]_c dx^{\mu} =: (D_{\mu}\psi)_c(x) dx^{\mu}. \quad (2.3)$$

The path dependence of the parallel transport is described infinitesimally by the field strength tensor $\mathcal{F}_{\mu\nu}(x)$ with $\mathcal{F}_{\mu\nu}(x) = \bar{g}F_{\mu\nu}^a(x)T_a$. In terms of gauge fields, $F_{\mu\nu}^a(x)$ is given by

$$F_{\mu\nu}^a(x) = \hat{\partial}_{\mu}A_{\nu}^a(x) - \hat{\partial}_{\nu}A_{\mu}^a(x) - \bar{g}f^{abc}A_{\mu}^b(x)A_{\nu}^c(x), \quad (2.4)$$

with structure constants f^{abc} specific to the gauge group. For $U(1)$, the last term vanishes, whence it is characteristic of the nonabelian gauge groups. In geometric terms, the field strength tensor is given by the commutator of the covariant derivatives

$$D_{\mu}D_{\nu} - D_{\nu}D_{\mu} = i\mathcal{F}_{\mu\nu}(x). \quad (2.5)$$

The last equation reflects the fact that the parallel transport is path dependent if there is a non-vanishing field strength, in very much the same way as the parallel transport of a vector in Riemannian space depends on the path if the space is curved. The field strength of the gauge fields then transforms under local gauge transformations $g(x)$ according to the adjoint representation of the symmetry group:

$$\mathcal{F}_{\mu\nu}(x) \rightarrow g(x)\mathcal{F}_{\mu\nu}(x)g^{-1}(x). \quad (2.6)$$

This construction principle leads for (quantum) electrodynamics to the familiar Lagrange density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}^{(l)}(x)(i\gamma^{\mu}D_{\mu} - M_l)\psi^{(l)}(x), \quad (2.7)$$

with $D_\mu = \hat{\partial}_\mu - ieA_\mu$. By construction it is invariant under the local $U(1)$ transformations given by

$$\begin{aligned} A_\mu &\rightarrow A_\mu(x) + \hat{\partial}_\mu \Theta(x), \\ \psi^{(l)}(x) &\rightarrow e^{ie\Theta(x)} \psi^{(l)}(x), \\ \bar{\psi}^{(l)}(x) &\rightarrow \bar{\psi}^{(l)}(x) e^{-ie\Theta(x)}, \end{aligned} \quad (2.8)$$

where $\Theta(x)$ is a space-dependent phase, l labels the electron or muon, ψ is a Dirac spinor representing the matter fields, and A_μ represents the photons. For quantum chromodynamics, the resulting Lagrange density takes the same form as in (2.7):

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \bar{\psi}(x)(i\gamma^\mu D_\mu - M)\psi(x), \quad (2.9)$$

where we have suppressed the indices of the mass matrix M and the quark fields ψ . Note that ψ here carries a multi-index α, f, c , where α is a Dirac index, f a flavor index, and c a color index, and all indices are summed over in \mathcal{L} . The gauge transformations (2.1) can be specialized to $T_a = \frac{1}{2}\lambda_a$, with $a = 1 \dots, 8$ and λ_a the eight Gell-Mann matrices, and $c, c' = 1, 2, 3$, for the three colors of the $SU(3)$ color symmetry. The covariant derivative takes the form $D_\mu = \hat{\partial}_\mu - ig\frac{\lambda_a}{2}A_\mu^a(x)$, where the gauge fields A_μ^a now represent the gluon fields mediating the strong interaction, and the field strength tensor $F_{\mu\nu}^a$ is given by (2.4) with structure constants f^{abc} from $SU(3)$. Note that the quadratic term in (2.4) represents the physical fact that gluons are also self-interacting, in contrast to photons. So in spite of the same form of (2.7) and (2.9), the physics thereby represented is as different as are quantum electrodynamics and quantum chromodynamics.

Finally, the combined action of electromagnetic and weak interactions is constructed along the same lines, with an additional term in the action that implements the Higgs mechanism, to realize the spontaneous symmetry breaking of $SU(2)_w \times U(1)$ to $U(1)_e$ (where the subscript w stands for “weak” and e for “electromagnetic”) and give masses to the vector bosons W^+ , W^- and Z mediating the weak interactions.

The similarities between the local gauge theories and general relativity become manifest in the language of differential geometry and point to the deeper reasoning behind what we called initially a “recipe” for how to proceed. In summary, the pendants in local gauge theories and general relativity are the following:

- The local space $\mathcal{H}(x)$ of charged fields $\psi(x)$ with unitary structure corresponds to the tangential space with local metric $g_{\mu\nu}(x)$ and Lorentz frames.
- The local gauge transformations correspond to general coordinate transformations.
- The gauge fields $A_{\mu,cc'}(x)$, defining the connection in the parallel transport, correspond to the Christoffel symbols, which describe the parallel transport of tangential vectors on a Riemannian manifold.