Springer Series in Synergetics

Springer Complexity

Alexander E. Hramov
Alexey A. Koronovskii
Valeri A. Makarov
Alexey N. Pavlov
Evgenia Sitnikova

Wavelets in Neuroscience

Springer

Springer Complexity

Springer Complexity is an interdisciplinary program publishing the best research and academic-level teaching on both fundamental and applied aspects of complex systems – cutting across all traditional disciplines of the natural and life sciences, engineering, economics, medicine, neuroscience, social and computer science.

Complex Systems are systems that comprise many interacting parts with the ability to generate a new quality of macroscopic collective behavior the manifestations of which are the spontaneous formation of distinctive temporal, spatial or functional structures. Models of such systems can be successfully mapped onto quite diverse "real-life" situations like the climate, the coherent emission of light from lasers, chemical reaction-diffusion systems, biological cellular networks, the dynamics of stock markets and of the internet, earthquake statistics and prediction, freeway traffic, the human brain, or the formation of opinions in social systems, to name just some of the popular applications.

Although their scope and methodologies overlap somewhat, one can distinguish the following main concepts and tools: self-organization, nonlinear dynamics, synergetics, turbulence, dynamical systems, catastrophes, instabilities, stochastic processes, chaos, graphs and networks, cellular automata, adaptive systems, genetic algorithms and computational intelligence.

The three major book publication platforms of the Springer Complexity program are the monograph series "Understanding Complex Systems" focusing on the various applications of complexity, the "Springer Series in Synergetics", which is devoted to the quantitative theoretical and methodological foundations, and the "SpringerBriefs in Complexity" which are concise and topical working reports, case-studies, surveys, essays and lecture notes of relevance to the field. In addition to the books in these two core series, the program also incorporates individual titles ranging from textbooks to major reference works.

Editorial and Programme Advisory Board

Henry Abarbanel, Institute for Nonlinear Science, University of California, San Diego, USA

Dan Braha, New England Complex Systems Institute and University of Massachusetts Dartmouth, USA

Péter Érdi, Center for Complex Systems Studies, Kalamazoo College, USA and Hungarian Academy of Sciences, Budapest, Hungary

Karl Friston, Institute of Cognitive Neuroscience, University College London, London, UK

Hermann Haken, Center of Synergetics, University of Stuttgart, Stuttgart, Germany

Viktor Jirsa, Centre National de la Recherche Scientifique (CNRS), Université de la Méditerranée, Marseille, France

Janusz Kacprzyk, System Research, Polish Academy of Sciences, Warsaw, Poland

Kunihiko Kaneko, Research Center for Complex Systems Biology, The University of Tokyo, Tokyo, Japan

Scott Kelso, Center for Complex Systems and Brain Sciences, Florida Atlantic University, Boca Raton, USA

Markus Kirkilionis, Mathematics Institute and Centre for Complex Systems, University of Warwick, Coventry, UK

Jürgen Kurths, Nonlinear Dynamics Group, University of Potsdam, Potsdam, Germany

Andrzej Nowak, Department of Psychology, Warsaw University, Poland

Linda Reichl, Center for Complex Quantum Systems, University of Texas, Austin, USA

Peter Schuster, Theoretical Chemistry and Structural Biology, University of Vienna, Vienna, Austria

Frank Schweitzer, System Design, ETH Zurich, Zurich, Switzerland

Didier Sornette, Entrepreneurial Risk, ETH Zurich, Zurich, Switzerland

Stefan Thurner, Section for Science of Complex Systems, Medical University of Vienna, Vienna, Austria

Springer Series in Synergetics

Founding Editor: H. Haken

The Springer Series in Synergetics was founded by Herman Haken in 1977. Since then, the series has evolved into a substantial reference library for the quantitative, theoretical and methodological foundations of the science of complex systems.

Through many enduring classic texts, such as Haken's *Synergetics* and *Information and Self-Organization*, Gardiner's *Handbook of Stochastic Methods*, Risken's *The Fokker Planck-Equation* or Haake's *Quantum Signatures of Chaos*, the series has made, and continues to make, important contributions to shaping the foundations of the field.

The series publishes monographs and graduate-level textbooks of broad and general interest, with a pronounced emphasis on the physico-mathematical approach.

For further volumes:
http://www.springer.com/series/712

Alexander E. Hramov • Alexey A. Koronovskii •
Valeri A. Makarov • Alexey N. Pavlov •
Evgenia Sitnikova

Wavelets in Neuroscience

Springer

Alexander E. Hramov
Alexey A. Koronovskii
Research and Education Center
'Nonlinear Dynamics of Complex Systems'
Saratov State Technical University
Saratov, Russia

and

Department of Nonlinear Processes
Saratov State University
Saratov, Russia

Valeri A. Makarov
Department of Applied Mathematics
Complutense University
Madrid, Spain

Alexey N. Pavlov
Physics Department
Saratov State University
Saratov, Russia

and

Research and Education Center
'Nonlinear Dynamics of Complex Systems'
Saratov State Technical University
Saratov, Russia

Evgenia Sitnikova
Institute for Higher Nervous Activity and Neurophysiology
Russian Academy of Sciences
Moscow, Russia

ISSN 0172-7389
ISBN 978-3-662-43849-7 ISBN 978-3-662-43850-3 (eBook)
DOI 10.1007/978-3-662-43850-3
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014945977

© Springer-Verlag Berlin Heidelberg 2015
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To our parents

Preface

If people do not believe that mathematics is simple,
it is only because they do not realize
how complicated life is

John von Neumann

About 30 years ago Jean Morlet introduced for the first time the notion of a wavelet as a soliton-like function. At the beginning he applied this function to the analysis of backscattered seismic signals, but soon he realized that wavelets have a significantly broader field of possible applications. In 1981, Alexander Grossmann interpreted wavelets as coherent states and gave an elegant proof of Morlet's reconstruction algorithm. Since then this technique has witnessed explosive growth and it now represents a universal mathematical tool with useful applications in many scientific and engineering studies.

Originally wavelets emerged as an alternative to the classical spectral analysis based on the Fourier transform, such as windowed Fourier analysis or the Gabor transform. In order to improve processing of transient components in complex signals, Morlet decided to replace Gabor functions, which have a fixed duration, by new building blocks or time–frequency atoms, which can have an arbitrarily small duration. Later this concept led to new insights and a mathematically rigorous foundation.

Nowadays, there is no doubt that the introduction of wavelets theory was one of the most important events in mathematics over the past few decades. This is probably the only concept that has been applied in practically all fields of basic science. Moreover, wavelets are widely used for image recognition and compression, for analysis and synthesis of complex signals, in studies of turbulent flows and biological data, etc.

This book is devoted to application of wavelet-based methods in neuroscience. We have attempted to illustrate how wavelets may provide new insight into the complex behavior of neural systems at different levels: from the microscopic dynamics of individual cells (e.g., analysis of intracellular recordings) to the macroscopic level of widespread neuronal networks (e.g., analysis of EEG and MEG recordings). Our main aim has been to show how and where wavelet-based tools can gain an advantage over classical approaches traditionally used in neuroscience. We hope that the logical structure of the book as regards content (from micro to macro scale) represents a new approach to experiential data analysis and could be helpful in everyday use. The book describes several examples obtained by the authors in experimental neuroscience.

The book results from a long-term cooperation between research groups at Saratov State University, Saratov State Technical University, Universidad Complutense de Madrid, and the Moscow Institute of Higher Nervous Activity and Neurophysiology of the Russian Academy of Science. We want to express our sincere gratitude to Prof. V. S. Anishchenko and Prof. D. I. Trubetskov for their constant support, scientific exchange, and interest in our work. We thank our collaborators A. Brazhe, N. Brazhe, D. Dumsky, V. Grubov, G. van Luijtelaar, A. Luttjohann, A. Moreno, E. Mosekilde, A. Nazimov, A. Ovchinnikov, F. Panetsos, C. M. van Rijn, O. Sosnovtseva, A. Tupitsyn, and J. A. Villacorta-Atienza with whom we have worked on different aspects of neural dynamics over the last decade. Our special thanks go to Prof. J. Kurths who has encouraged us to write this book. We acknowledge fruitful discussions with our colleagues A. Balanov, I. Belykh, V. Kazantsev, I. Khovanov, A. Neiman, G. Osipov, V. Ponomarenko, M. Prokhorov, and V. Raevskiy. We also extend our warmest thanks to the Rector of Saratov State Technical University Prof. I. Pleve for support and help with preparation of this book. Finally, we would like to express our sincere gratitude to our families for their constant support and inspiration.

Over the years, our studies in the field of wavelets have been supported by the Russian Foundation of Basic Research (Russia), the Russian Scientific Foundation (Russia), the Ministry of Education and Science of Russian Federation (Russia), the U.S. Civilian Research and Development Education (USA), the BrainGain Smart Mix Program of the Netherlands Ministry of Economic Affairs (the Netherlands), and the Dynasty Foundation (Russia).

Saratov, Russia	Alexander E. Hramov
Saratov, Russia	Alexey A. Koronovskii
Madrid, Spain	Valeri A. Makarov
Saratov, Russia	Alexey N. Pavlov
Moscow, Russia	Evgenia Sitnikova
July 2014	

Contents

Acronyms

ANN	Artificial neural networks
BCI	Brain–computer interface
CNS	Central nervous system
CSD	Current source density
CWT	Continuous wavelet transform
DWT	Discrete wavelet transform
ECoG	Electrocorticogram
EEG	Electroencephalogram
FFT	Fast Fourier transform
GAERS	Genetic rats with absence epilepsy
HF	High frequency
HPF	High-pass filter
ICA	Independent component analysis
icEEG	Intracranial EEG
ISI	Interspike interval
LF	Low frequency
LFP	Local field potential
MEG	Magnetoencephalogram
NN	Neuronal network
OSDS	On-line SWD detection system
PCA	Principal component analysis
PSTH	Peristimulus time histogram
PWAF	Parametric wavelet sorting with advanced filtering
RTN	Reticular thalamic nucleus
rWF	Representative waveform
SD	Standard deviation
sdEEG	Subdural EEG
SI	Somatosensory
SWD	Spike-wave discharge
TC	Thalamocortical
TSS	Time scale synchronization

VPM	Ventroposteromedial
WAG/Rij	Wistar Albino Glaxo/Rijswijk
WF	Waveform
WMSPC	Wavelet method with superparamagnetic clustering
wICA	Wavelet independent component analysis
WNN	Wavelet neuronal network
WPOD	Wavelet power over domain
WSAC	Wavelet shape-accounting classifier
WSC	Wavelet-based spike classifier
WT	Wavelet transform

Chapter 1
Mathematical Methods of Signal Processing in Neuroscience

Abstract This chapter offers a brief introduction to the novel advanced mathematical methods of analysis and processing of neurophysiological data. First, we give the rationale for the development of specific mathematical approaches for decoding information from non-stationary neurophysiological processes with time-varying features. Second, we focus on the development of mathematical methods for automatic processing and analysis of neurophysiological signals, more specifically, in the development of brain–computer interfaces (BCIs). Finally, we give an overview of the main applications of wavelet analysis in neuroscience, from the *microlevel* (the dynamics of individual cells or intracellular processes) to the *macrolevel* (dynamics of large-scale neuronal networks in the brain as a whole, ascertained by analyzing electro- and magnetoencephalograms).

1.1 General Remarks

Neurodynamics is a contemporary branch of interdisciplinary neuroscience that examines mechanisms of the central nervous system based on the mutual experience of chemists, biologists, physicists, mathematicians, and specialists in the nonlinear theory of oscillations, waves, and dynamical chaos [1–6]. Practical applications of modern methods in neuroscience facilitate an interdisciplinary approach to brain functions and attract experts in experimental and theoretical neurobiology, psychophysiology, cognitive neuroscience, biophysics, physics, nonlinear dynamics, etc. This interdisciplinary collaboration provides unique methods for analyzing the functional activity of the central nervous system (CNS) that focus on the basic principles of the neuronal dynamics of individual cells and neural networks.

Recent progress in understanding molecular and ionic mechanisms of neuronal activity [7] encourages further investigation of certain key problems in modern physics, such as exploration of the functional properties and principles of information coding, as well as its representation and the processing of sensory data in the central nervous system. Perception and information processing are

A.E. Hramov et al., *Wavelets in Neuroscience*, Springer Series in Synergetics,
DOI 10.1007/978-3-662-43850-3_1, © Springer-Verlag Berlin Heidelberg 2015

important functions of the CNS. Visual, acoustic, tactile, and gustatory stimuli are transformed by the sensory receptors of the first order neurons into a sequence of electrical pulses. These first-order sensory neurons are therefore involved in primary processing of sensory information [8–12]. Sensory information is then passed through relay stations (brain stem and thalamic nuclei) that transform and convolve the information code, until finally it reaches the cerebral cortex which shapes the "fingerprint" of the external world [13, 14]. At each subsequent stage, the processes of information transfer become increasingly difficult to study. The question of how the totality of nervous impulses (action potentials or spikes) generated by single neurons can reflect the full complexity and diversity of the external world remains one of the biggest challenges in fundamental science [13, 15–17].

Experimental methods have recently been developed for registering the neuronal activity underlying processes of information encoding–decoding at different levels of the nervous system—from molecular changes in membrane properties of receptor cells to changes in the local (electrical) field potentials in the cerebral cortex. Traditional and noninvasive methods for registering electrical brain activity, such as electroencephalography (EEG) with electrodes arranged on the skin of the head, offer several advantages, and this method is still commonly used in neurophysiology and medicine. EEG is often used in various studies of brain functions in humans and animals [18, 19]. There are also invasive methods using implanted electrodes which provide better spatial resolution, and these are advantageous when examining neuronal activity in small groups of neurons in superficial (cortex) and deep (subcortical) structures. Another advantage of invasive recording techniques is that implanted electrodes can also be used for electrical stimulation with different research purposes, e.g., suppression of epileptic discharges [20–22]. The relatively new noninvasive recording technique known as magnetic encephalography (MEG) has become more popular over the last few years, because it provides better spatial resolution than EEG and better quality of signals reflecting brain activity [23–25].

1.2 Nonstationarity of Neurophysiological Data

Despite technical progress in developing new methods of data acquisition in experimental neurophysiology, mathematical methods of experimental data analysis could not be readily applied, and this may impede further progress. In the vast majority of experimental studies in neuroscience, only a few statistical methods of data analysis are used, e.g., calculation of the mean spike frequency, construction of various correlation characteristics and distribution functions, etc. Traditional methods of statistical analysis are undoubtedly useful, but most of them unable to evaluate the relevant information regarding complex processes in the CNS. In order to illustrate this fact, we give an example that demonstrates the response of a sensory neuron to periodic stimulation. From a mechanical point of view, the response of

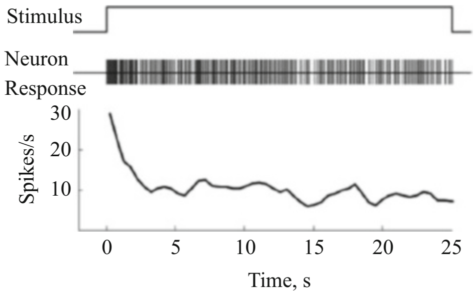

Fig. 1.1 Illustration of adaptation reaction of neuronal firing activity to a repeated stimulation. This neuron was recorded in a rat in the trigeminal sensory nuclear complex which receives tactile information from vibrissae. Stimulation was performed by periodic mechanical deflection of one whisker by a series of short directed air puffs (duration of each air pulse 5 ms). *From top to bottom*: start and end of stimulation by the sequence of periodic impulses, firing activity of a single neuron (train of spikes), and dynamics of the mean spike frequency (averaging over a sliding time window of 500 ms duration)

the neuron to a sequence of equal external stimuli could be identical, so periodic stimulation of a neuron with a series of impulses could elicit a periodic sequence of spikes (action potentials, for example, 2 or 3 spikes per stimulus). However, in the experimental situation, we often obtain time- and activity-dependent variations in the neuron's response (the neuron does not demonstrate an equal response to repeated identical stimuli) which reflect neuronal plasticity. The phenomenon of synaptic neuronal plasticity (the basic mechanism underlying memory and learning) reflects adaptation to external afferent activity modified by the internal characteristics of individual cells and the global dynamics of the wider neuronal network interactions [26, 27]. It is known that a neuron can even stop responding to the next stimulus from a certain moment.

Figure 1.1 illustrates the adaptive response of a neuron of the trigeminal complex to periodic stimulation. Maximum neuron activity (27 spikes/s) is observed at the onset of stimulation; it falls to an average of 10 spikes/s within a few seconds and varies thereafter, exhibiting a slow negative drift. On the one hand, such behavior of a living cell makes it extremely difficult to define characteristic forms/patterns of neural activity associated with the peculiar properties of a given stimulus. On the other hand, such complexity in neuronal activity encourages the development of more relevant (complex) methods of data analysis, in addition to the simple description of statistical characteristics of neuronal responses that is one of the tasks of neurodynamics. We conclude that more specific mathematical methods must be applied, such as wavelets [28–30], the Hilbert–Huang transform [31–33], and the Wigner–Ville transform [34–36], which are more suitable for decoding information about non-stationary processes with time-varying features.

1.3 Wavelets in Basic Sciences and Neuroscience

Wavelet analysis [28, 37–40] is unique in the sense that even the first practical application to neurophysiological data analysis produced prominent results [29,41–45]. For this reason, it is considered a very powerful analytical tool for studying the dynamics of neural systems.

Wavelet terminology was introduced in the 1980s [37,46,47]. This mathematical approach was initially proposed as an alternative to classical spectral analysis based on the Fourier transform. Wavelet theory is considered to be one of the most important events in mathematics of the past decades. Indeed, it appears to be the sole new mathematical concept that was immediately recognized as a tool in practically all branches of basic science (first and foremost, in physics and related disciplines) and many technical fields [30,48–55]. In fact, introduction of the wavelet theory itself was not entirely unexpected. It was developed to meet the very real needs of experimental investigations, particularly in geophysics and seismology. Contemporary wavelet analysis combines various pre-existing ideas and methods. For example, fast wavelet transform algorithms are based on the subband coding ideology known from radio and electric engineering [56]. Some ideas were borrowed from physics (coherent states [57], etc.) and mathematics (studies on Caldéron–Zygmund integral operators [58]). Wavelet analysis is logically related to the theory of diffusion differential equations [59].

Today, wavelets are widely used for the analysis and synthesis of various signals, image processing and recognition, compression of large volumes of information, digital filtration, the study of fully developed turbulence, and the solution of certain differential equations. This list can certainly be extended [54, 59–67]. The new theory aroused great interest from the very beginning. According to well-known estimates [48], since the 1990s, the number of publications using wavelets in physics has been growing continuously. The number of references to Internet sources containing the term "wavelet" has reached several million. In fundamental science, this mathematical approach is mostly applied to study complex temporally nonstationary or spatially nonhomogeneous nonlinear processes. Wavelet analysis is well adapted for studying the complex structure of signals from living systems, since other traditional computation techniques can be applied only to processes with time (or space)-constant parameters (i.e., stationary in time or spatially homogeneous). Despite the fact that wavelet analysis has long been regarded as a standard tool for studying complex processes and practical application of this method in neuroscience and medicine is just beginning, prognoses for its successful application are rather optimistic. In this monograph we highlight recent advances made by practical application of wavelet in neurodynamics and neurophysiology.

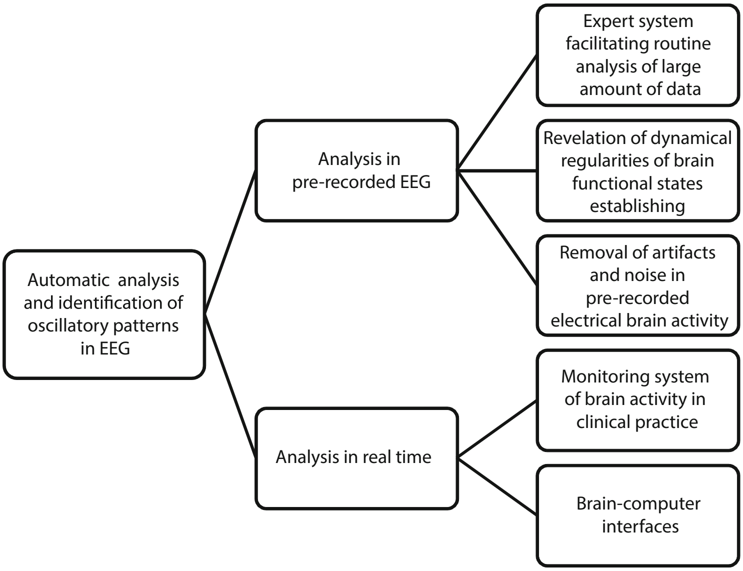

Fig. 1.2 Wavelet-based methods of automatic EEG diagnostics, processing, and analysis

1.4 Automatic Processing of Experimental Data in Neuroscience

An important field of wavelet applications in neurophysiology and neuroscience is the development of methods for automatic processing and analysis of brain signals. Electrical signals that can be recorded from the brain (EEG) represent a linear mixture of coexisting oscillatory components, i.e., nonlinear effects do not complicate the process of recognition. The development of expert systems for automatic EEG analysis is of particular interest for both fundamental neuroscience and clinical practice due to a wide spectrum of possible applications (classified in Fig. 1.2). One must distinguish between on-line and off-line analysis. Automatic (i.e., without the attention and control of an operator) analysis of pre-recorded EEG signals (*off-line* diagnostics) aims to reduce routine work, for example, to suppress artifacts in the recorded EEG. EEG analysis in real time (*on-line*) aims at fast detection of certain EEG events and the organization of closed-loop control systems. Clinically-oriented applications are the most effective field of on-line analysis of neurophysiological signals, including EEG monitoring with predictive diagnostic purposes, e.g., for the suppression of epileptic activity, the so-called spike–wave discharges [20].

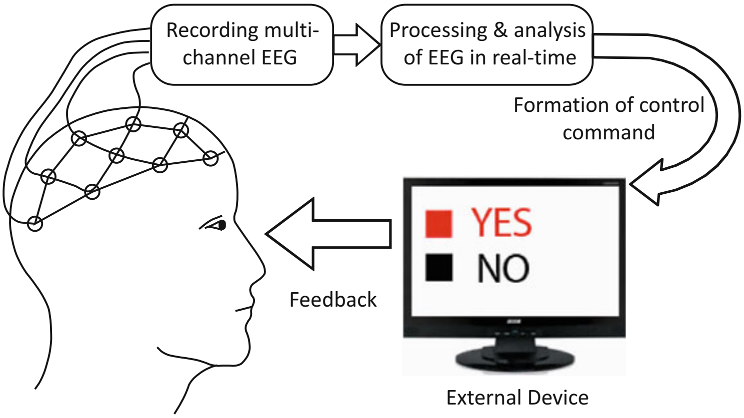

Fig. 1.3 General scheme of a simple brain–computer interface. Modern IBC is a system that registers and analyzes signals of electrical brain activity (usually EEG) from the user and "converts" them into a "machine" command for external device control. The central point of such a system is the development of algorithms for real-time recognition of EEG patterns corresponding to certain cogitative operations. Note the importance of the feedback loop in the BCI. This is necessary to adapt the aforementioned algorithms to recognize the specific patterns of electrical brain activity based on EEG features. Also the operator (user) must learn to evoke and control the relevant mental state, which is impossible without the use of feedback

1.5 Brain–Computer Interfaces

One of the most exciting applications of wavelets is to use it for *mental control* of brain functions, which, as a matter of fact, is a new form of human–computer interaction [68, 69]. The specific dynamics of electrical brain activity characterizes mental activity that includes compilation of imaginary commands ("mental action"). This "mental action" is associated with specific changes in the time–frequency characteristics and spatial structure of EEG [70–73]. In the brain–computer interface, mental control systems must perform the following steps (see Fig. 1.3):

- Recognize and select characteristic changes in the EEG (event-related *oscillatory patterns*).
- Decrypt their meaning (associated with a specific operation).
- Convert this meaning into commands for hardware control.

Mental control systems should be able to solve two main problems. First, the technical problem of precise recognition of an EEG pattern, subsequent formulation of a "command", and transmission to control. Second, cognitive and psychological tasks in which the operator (a person) should learn to keep specific mental states that can be recognized from analysis of the spatial-temporal structure of his/her EEG. An additional problem is that the system should work in real time. Earlier control systems were suggested to use information about complex physical activity expressed as body movements of the operator, e.g., the trajectory when moving a hand in the process of equipment handling. These interfaces encountered many problems, including registration of complex information, isolation of relevant information from the general data stream, and correct interpretation. Besides that, such interfaces require a system of sensors for registration of motor activity and a wireless device for data transmission from operator to computer. Therefore, simple brain–computer interfaces (BCI) are of particular interest, such as interfaces that are able to monitor electrical brain activity and detect the mental intentions of the operator. For example, simple stimulus–symbol interfaces conceived by the operator [74, 75] open up new prospects for resolving the problem of mental control.

Thus, algorithms of automatic EEG pattern recognition associated with specific cogitative operations in real time help to effectively perform the first step (pattern recognition) in brain–computer interfaces. Wavelet-based methods are perfectly suited to pattern recognition tasks [76–79].

Note that brain–computer interfaces have already been used as an alternative to traditional devices for inputting information into the computer. So for certain categories of users, for example, people with motor function disabilities, this way of interacting with the computer can improve their quality of life, at least partly, opening the way to a full-fledged life in society [80–83]. One of the first successfully worked BCIs was developed at Emory University by Roy Bakay and Phillip Kennedy, who used implanted depth electrodes in the brain motor center of a paralyzed 53-year-old patient, who was able to move the cursor on a computer screen, and thus communicate with doctors (writing several simple sentences) [84]. Rapid progress in neuroscience and technology suggests that brain–computer interfaces could be widely used for control of artificial limbs, manipulators, and robot technical devices (for example, wheelchairs), and also in the gaming industry [85–88].

1.6 Topics to Consider

A mathematically rigorous description of wavelet analysis can be found in numerous textbooks and monographs (see, for example, [28, 53, 55, 60, 89–93]) as well as in reviews in scientific journals [17, 51, 52, 94]. This book focuses on the new possibilities provided by the wavelet approach for decoding information from signals recorded on the level of individual neurons and groups of neurons, as well as neural network activity. A large number of the aforementioned scientific publications aimed to identify the most important problems in the field of wavelet applications to neurodynamics and neurophysiology. On this topic, we distinguish the following three areas of wavelet applications in neuroscience:

- **Microlevel** (cellular/intracellular)—wavelet analysis of the dynamics of individual cells or intracellular processes.
- **Mesolevel** (groups of cells)—analysis of information processes in small neuronal ensembles.

- **Macrolevel** (brain activity)—analysis of macrodynamics in widespread neural networks (EEG/MEG, neuroimaging data).

This monograph discusses the progress made on each of these levels in a consistent manner. The book contains seven chapters:

- Chapter 2 provides a mathematical introduction to wavelet analysis, including the basic concepts and definitions of wavelet theory, and considers practically significant questions related to effective numerical implementation of the wavelet transform (both, discrete and continuous). Special attention is paid to the importance of the relationship between wavelet and Fourier analysis. This chapter specifically addresses those readers who are not familiar with the mathematical concepts of complex signal processing.

The next two chapters describe methods for wavelet investigation of neurophysiological systems.

- Chapter 3 discusses the application of wavelets for analysis of cellular dynamics at the microscopic level (individual cells or intracellular processes). This chapter also presents the principles for analyzing the information from a single cell, using electrical signals of individual neurons.
- Chapter 4 describes the main aspects of the wavelet analysis of a variety of impulse shapes (action potentials) of individual neurons using extracellular records of single-unit neuronal activity. We consider different approaches to classifying neuronal impulses in terms of their configuration, some based solely on wavelets, and others involving combined methods, such as wavelet neural networks.

The last three chapters of the book consider the macrodynamics of neuronal networks using wavelet analysis of electroencephalograms (EEGs).

- Chapter 5 considers the main definitions and principles of electroencephalography that are required for a better understanding of Chaps. 6 and 7. We describe general physical and mathematical approaches to time–frequency analysis of rhythmic EEG activity using continuous wavelet transforms. We also review some recent achievements of wavelet-based studies of electrical brain activity, including (i) time–frequency analysis of EEG structure, (ii) automatic detection of oscillatory patterns in pre-recorded EEG, (iii) classification of oscillatory patterns, (iv) real-time detection of oscillatory patterns in EEG, (v) detection of synchronous states of electrical brain activity, (vi) artifact suppression/rejection in multichannel EEG, and (vii) the study of cognitive processes.
- Chapter 6 describes some results of time–frequency analysis of EEG structure using the continuous wavelet transform. In this chapter we pay special attention to technical and computational details of time–frequency analysis of neurophysiological signals (EEG of animals and humans). This chapter also presents wavelet analysis of hypersynchronous rhythmic activity in multichannel EEG, characterizing the onset of absence epilepsy in patients.

- Chapter 7 considers basic problems of automatic diagnostics and processing of EEG. We discuss the wavelet-based techniques in order to fully automatize "routine" operations, such as visual inspection of EEG. In addition, we exhibit examples of practical applications of wavelet methods for automatic analysis of pre-recorded EEG and MEG signals (*off-line* diagnostics), and also some examples of EEG analysis in real-time (*on-line*). We also discuss the principles of fast and precise detection of transient events in EEG and the organization of closed-loop control systems that can be used in BCI. Finally, we consider methods of artifact suppression in multichannel EEG based on a combination of wavelets and independent component analysis

This book is based primarily on the fundamental results in neurodynamics obtained recently by the authors—physicists, mathematicians, and biologists in close collaboration with specialists in experimental neurophysiology. At the same time, the book contains a relatively complete bibliography (over 400 sources) characterizing the application of wavelets in neurophysiological research. In general, this book overviews theoretical and practical knowledge and, in our opinion, demonstrates the advantages of powerful analytical tools and novel mathematical methods of signal processing and nonlinear dynamics in order to address neurophysiological problems. Moreover, wavelet analysis helps to reveal important information and facilitates a deeper understanding of the investigated phenomena. More intensive studies in this area can contribute to interdisciplinary interactions between physics, nonlinear dynamics, applied mathematics, and neurophysiology and promote further mutual research in these areas.

References

1. G. Buzsaki, A. Draguhn, Neuronal oscillations in cortical networks. Science **304**, 1926 (2004)
2. H.D. Abarbanel, M.I. Rabinovich, A. Selverston, M.V. Bazhenov, R. Huerta, M.M. Sushchik, L.L. Rubchinskii, Synchronisation in neural networks. Phys.–Usp. **39**(4), 337 (1996)
3. V.I. Nekorkin, Nonlinear oscillations and waves in neurodynamics. Phys.–Usp. **51**(3), 295 (2008)
4. B.P. Bezruchko, V.I. Ponomarenko, M.D. Prokhorov, D.A. Smirnov, P.A. Tass, Modeling nonlinear oscillatory systems and diagnostics of coupling between them using chaotic time series analysis: applications in neurophysiology. Phys.–Usp. **51**(3), 304 (2008)
5. M.I. Rabinovich, M.K. Muezzinoglu, Nonlinear dynamics of the brain: emotion and cognition. Phys.–Usp. **53**(4), 357 (2010)
6. M.I. Rabinovich, K.J. Friston, P. Varona (eds.), *Principles of Brain Dynamics: Global State Interactions* (MIT, Cambridge, 2012)
7. H.C. Tuckwell, *Introduction to Theoretical Neurobiology* (Cambridge University Press, Cambridge, 1988)
8. W.J. Freeman, *Mass Action in the Nervous System* (Academic, New York, 1975)
9. L.S. da Fernando, Neural mechanisms underlying brain waves: from neural membranes to networks. Electroencephalogr. Clin. Neurophysiol. **79**, 81 (1991)
10. G.R. Ivanitskii, A.B. Medvinskii, M.A. Tsyganov, From the dynamics of population autowaves generated by living cells to neuroinformatics. Physics-Uspekhi **37**(10), 961 (1994)

11. W.J. Freeman, Mesoscopic neurodynamics: from neuron to brain. J. Physiol. (France) **94**, 303 (2000)
12. G.N. Borisyuk, R.M. Borisyuk, Y.B. Kazanovich, G.R. Ivanitskii, Models of neural dynamics in brain information processing—the developments of ‘the decade’. Phys.–Usp. **45**(10), 1073 (2002)
13. A. Villacorta-Atienza, M.G. Velarde, V.A. Makarov, Compact internal representation of dynamic situations: neural network implementing the causality principle. Biol. Cybern. **103**, 285 (2010)
14. V.A. Makarov, A. Villacorta-Atienza, in *Recurrent Neural Networks for Temporal Data Processing* (INTECH’2011, Shanghai), ed. by H. Cardot (2011), pp. 81–102
15. N.P. Castellanos, V.A. Makarov, Recovering EEG brain signals: artifact suppression with wavelet enhanced independent component analysis. J. Neurosci. Methods **158**, 300 (2006)
16. A. Villacorta-Atienza, M.G. Velarde, V.A. Makarov, Compact internal representation of dynamic situations: neural network implementing the causality principle. Biol. Cybern. **103**, 285 (2010)
17. A.N. Pavlov, A.E. Hramov, A.A. Koronovskii, Y.E. Sitnikova, V.A. Makarov, A.A. Ovchinnikov, Wavelet analysis in neurodynamics. Phys.–Usp. **55**(9), 845 (2012)
18. P.L. Nunez, K. Srinivasan, *Electric Fields of the Brain: The Neurophysics of EEG* (Oxford University Press, New York, 1981)
19. E. Niedermeyer, F.L. da Silva, *Electroencephalography: Basic Principles, Clinical Applications, and Related Fields* (Lippincot Williams & Wilkins, Philadelphia, 2004)
20. A. Berenyi, M. Belluscio, D. Mao, G. Buzsaki, Closed-loop control of epilepsy by transcranial electrical stimulation. Science **337**(6095), 735 (2012)
21. A. Luttjohann, E.L.M. van Luijtelaar, The dynamics of cortico-thalamo-cortical interactions at the transition from pre-ictal to ictal LFPs in absence epilepsy. Neurobiol. Dis. **47**, 49 (2012)
22. A. Luttjohann, J.M. Schoffelen, E.L.M. van Luijtelaar, Peri-ictal network dynamics of spike–wave discharges: phase and spectral characteristics. Exp. Neurol. **239**, 235 (2013)
23. M. Hämäläinen, R. Hari, R.J. Ilmoniemi, J. Knuutila, O.V. Lounasmaa, Magnetoencephalography: theory, instrumentation, and applications to noninvasive studies of the working human brain. Rev. Mod. Phys. **65**, 413 (1993)
24. P. Hansen, M. Kringelbach, R. Salmelin (eds.), *MEG: An Introduction to Methods* (Oxford University Press, New York, 2010)
25. I. Westmijse, P. Ossenblok, B. Gunning, E.L.M. van Luijtelaar, Onset and propagation of spike and slow wave discharges in human absence epilepsy: a MEG study. Epilepsia **50**, 2538 (2009)
26. E. Ahissar, P.M. Knutsen, Object localization with whiskers. Biol. Cybern. **98**, 449 (2008)
27. V.A. Makarov, A.N. Pavlov, A.N. Tupitsyn, F. Panetsos, A. Moreno, Stability of neural firing in the trigeminal nuclei under mechanical whisker stimulation. Comput. Intell. Neurosci. **2010**, 340541 (2010)
28. I. Daubechies, *Ten Lectures on Wavelets* (SIAM, Philadelphia, 1992)
29. A. Aldroubi, M. Unser, *Wavelets in Medicine and Biology* (CRC, Boca Raton, 1996)
30. J.C. Van den Berg (eds.), *Wavelets in Physics* (Cambridge University Press, Cambridge, 2004)
31. N.E. Huang, Z. Shen, S.R. Long, A new view of nonlinear water waves: the Hilbert spectrum. Ann. Rev. Fluid Mech. **31**, 417 (1999)
32. N.E. Huang, Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung, H.H. Liu, The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc. R. Soc. A: Math. Phys. Eng. Sci. **454**, 903 (1998)
33. N.E. Huang, Z. Shen, *Hilbert–Huang Transform and Its Applications* (World Scientific, Singapore, 2005)
34. B. Boashash, in *Advances in Spectrum Analysis and Array Processing*, ed. by S. Haykin (Prentice Hall, Englewood Cliffs, 1990), pp. 418–517
35. S. Qian, D. Chen, *Joint Time–Frequency Analysis* (Prentice Hall, Upper Saddle River, 1996)
36. W. Mecklenbrauker, F. Hlawatsch, *The Wigner Distribution: Theory and Applications in Signal Processing* (Elsevier, Amsterdam, 1997)

37. A. Grossman, J. Morlet, Decomposition of Hardy function into square integrable wavelets of constant shape. SIAM J. Math. Anal. **15**(4), 273 (1984)
38. M.B. Ruskai, G. Beylkin, R. Coifman, I. Daubechies, S.G. Mallat, Y. Meyer, L. Raphael, *Wavelets and Their Applications and Data Analysis* (Jones and Bartlett, Boston, 1992)
39. Y. Meyer, *Wavelets: Algorithms and Applications* (SIAM, Philadelphia, 1993)
40. Y. Meyer, *Wavelets and Operators* (Cambridge University Press, Cambridge, 1992)
41. J.J. Benedetto, A.I. Zayed, *Sampling, Wavelets, and Tomography* (Birkháuser, Boston, 2004)
42. J.C. Letelier, P.P. Weber, Spike sorting based on discrete wavelet transform coefficients. J. Neurosci. Methods **101**, 93 (2000)
43. E. Hulata, R. Segev, E. Ben-Jacob, A method for spike sorting and detection based on wavelet packets and Shannon's mutual information. J. Neurosci. Methods **117**, 1 (2002)
44. Q.R. Quiroga, Z. Nadasdy, Y. Ben-Shaul, Unsupervised spike detection and sorting with wavelets and superparamagnetic clustering. Neural Comput. **16**, 1661 (2004)
45. R.Q. Quiroga, A. Kraskov, T. Kreuz, P. Grassberger, Perfomance of different synchronization measures in real data: a case study on electroencephalographic signals. Phys. Rev. E **65**, 041903 (2002)
46. J. Morlet, G. Arens, E. Fourgeau, D. Glard, Wave propagation and sampling theory. Part I. Complex signal and scattering in multilayered media. Geophysics **47**(2), 203 (1982)
47. J. Morlet, G. Arens, E. Fourgeau, D. Giard, Wave propagation and sampling theory. Part II. Sampling theory and complex waves. Geophysics **47**(2), 222 (1982)
48. P.S. Addison, *The Illustrated Wavelet Transform Handbook: Introductory Theory and Applications in Science Engineering, Medicine and Finance* (Institute of Physics Publishing, Bristol, 2002)
49. G. Kaiser, *A Friendly Guide to Wavelets* (Springer/Birkhauser, Boston, 1994)
50. S.G. Mallat, *A Wavelet Tour of Signal Processing* (Academic, New York, 1998)
51. N.M. Astaf'eva, Wavelet analysis: basic theory and some applications. Phys.–Usp. **39**(11), 1085 (1996)
52. I.M. Dremin, O.V. Ivanov, V.A. Nechitailo, Wavelets and their uses. Phys.–Usp. **44**(5), 447 (2001)
53. A.A. Koronovskii, A.E. Hramov, *Continuous Wavelet Analysis and Its Applications* (Fizmatlit, Moscow, 2003)
54. V.G. Anfinogentov, A.A. Koronovskii, A.E. Hramov, Wavelet analysis and its applications for examination of behaviour of nonlinear dynamical systems of different nature. BRAS: Phys. **64**(12), 2383 (2000)
55. B. Torresani, *Continuous Wavelet Transform* (Savoire, Paris, 1995)
56. M. Vetterli, J. Kovacevic, *Wavelets and Subband Coding* (Prentice Hall, Englewood Cliffs, 1995)
57. S.T. Ali, J.P. Antoine, J.P. Gazeau, *Coherent States, Wavelets and Their Generalizations* (Springer, New York, 1999)
58. Y. Meyer, R. Coifman, *Calderon–Zygmund and Multilinear Operators* (Cambridge University Press, Cambridge, 1997)
59. D.E. Postnov, Evaluation of a continuous wavelet transform by solving the Cauchy problem for a system of partial differential equations. Comput. Math. Math. Phys. **46**(1), 73 (2006)
60. J.J. Benedetto, M. Frazier, *Wavelets: Mathematics and Applications* (CRC, Boca Raton, 1994)
61. R. Gencay, F. Selcuk, B. Whitcher, *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics* (Academic, San Diego, 2001)
62. T. Strutz, *Bilddatenkompression. Grundlagen, codierung, JPEG, MPEG, wavelets* (Vieweg Braunschweig, Wiesbaden, 2002)
63. J.S. Walker, *A Primer on Wavelets and Their Scientific Applications* (CRC, Boca Raton, 1999)
64. C.L. da Fontoura, J.R.M. Cesar, *Shape Analysis and Classification: Theory and Practice* (CRC, Boca Raton, 2001)
65. S. Jaffard, Y. Meyer, R. Ryan, *Wavelets: Tools for Science and Technology* (SIAM, Philadelphia, 2001)

66. M.V. Wickerhauser, *Adapted Wavelet Analysis from Theory to Software* (A.K. Peters, Wellesley, 1994)
67. E.B. Postnikov, E.A. Lebedeva, Decomposition of strong nonlinear oscillations via modified continuous wavelet transform. Phys. Rev. E **82**(5), 057201 (2010)
68. C. Guger, H. Ramoser, G. Pfurtscheller, Real-time EEG analysis for a brain–computer interface (BCI) with subject-specific spatial patterns. IEEE Trans. Neural Syst. Rehabil. Eng. **8**(4), 447 (2000)
69. S.G. Mason, G.E. Birch, A general framework for brain–computer interface design. IEEE Trans. Neural Syst. Rehabil. Eng. **11**(1), 70 (2003)
70. S. Makeig, S. Enghoff, T.P. Jung, T.J. Sejnowski, A natural basis for efficient brain-actuated control. IEEE Trans. Neural Syst. Rehabil. Eng. **8**, 208 (2000)
71. N.E. Sviderskaya, T.N. Dashinskaja, G.V. Taratunova, Spatial organization of EEG activation during the creative processes. J. High. Nerv. Act. **51**, 393 (2001)
72. N.E. Sviderskaya, *Spatial Organization of Electroencephalogram* (VGMA Press, Voronezh, 2008)
73. N.E. Sviderskaya, A.G. Antonov, Effect of individual psychological characteristics on the spatial organization of EEG during nonverbally-divergent mindset. Hum. Physiol. **34**(5), 34 (2008)
74. L.A. Farwell, E. Donchin, Talking off the top of your head: toward a mental prosthesis utilizing event related brain potentials. EEG Clin. Neurophysiol. **70**, 510 (1988)
75. J.N. Mak, Y. Arbel, J.W. Minett, L.M. McCane, B. Yuksel, D. Ryan, D. Thompson, L. Bianchi, D. Erdogmus, Optimizing the P300-based brain–computer interface: current status, limitations and future directions. J. Neural Eng. **8**, 025003 (2011)
76. M. Huang, P. Wu, Y. Liu, L. Bi, H. Chen, Application and contrast in brain–computer interface between Hilbert–Huang transform and wavelet transform, in *The 9th International Conference for Young Computer Scientists (ICYCS'08)*, Zhang Jia Jie, 18–21 Nov 2008, pp. 1706–1710
77. M.R. Kousarrizi, A.A. Ghanbari, M. Teshnehlab, M. Aliyari, A. Gharaviri, Feature extraction and classification of EEG signals using wavelet transform, SVM and artificial neural networks for brain–computer interfaces, in *International Joint Conference on Bioinformatics, Systems Biology and Intelligent Computing, 2009 (IJCBS'09)*, Shanghai, 3–5 Aug 2009, pp. 352–355
78. T. Bassani, J.C. Nievola, Pattern recognition for brain–computer interface on disabled subjects using a wavelet transformation, in *IEEE Symposium on Computational Intelligence in Bioinformatics and Computational Biology (CIBCB'08)*, Sun Valley Idaho, 15–17 Sept 2008, pp. 180–186
79. V.M. Vaughan, Guest editorial brain–computer interface technology: a review of the second international meeting. IEEE Trans. Neural Syst. Rehabil. Eng. **11**(2), 94 (2003)
80. U. Hoffmann, J.M. Vesin, T. Ebrahimi, K. Diserens, An efficient P300-based brain–computer interface for disabled subjects. J. Neurosci. Methods **167**(1), 115 (2008)
81. G. Pires, U. Nunes, M. Castelo-Branco, Comparison of a rowcolumn speller vs. a novel lateral single-character speller: assessment of BCI for severe motor disabled patients. Clin. Neurophysiol. **123**, 1168 (2012)
82. N.V. Manyakov, N. Chumerin, A. Combaz, M.M. van Hulle, Comparison of classification methods for P300 brain–computer interface on disabled subjects. Comput. Intell. Neurosci. **2011**, 519868 (2011)
83. S. Lu, C. Guan, H. Zhang, Unsupervised brain–computer interface based on intersubject information and online adaptation. IEEE Trans. Neural Syst. Rehabil. Eng. **17**, 135 (2009)
84. P.R. Kennedy, R.A. Bakay, Restoration of neural output from a paralyzed patient by a direct brain connection. Neuroreport **9**, 1707 (1998)
85. J.R. Wolpaw, Brain–computer interfaces as new brain output pathways. J. Physiol. **579**(Part 3), 613 (2007)
86. A.Y. Kaplan, S.L. Shishkin, I.P. Ganin, I.A. Basyul, A.Y. Zhigalov, Adapting the P300-based brain–computer interface for gaming: a review. IEEE Trans. Comput. Intell. AI Games (Special Issue on Brain/Neuronal–Computer Games Interfaces and Interaction) **5**(2), 141 (2013)

87. I.P. Ganin, S.P. Shishkin, A.G. Kochetkova, Y.A. Kaplan, Brain–computer interface on the base of “wave P300”: study of the effect of stimulus number in the sequence of their presentation. Hum. Physiol. **38**(2), 5 (2012)
88. G. Edlinger, C. Guger, Social environments, mixed communication and goal-oriented control application using a brain–computer interface, in *Proceedings of the International Conference UAHCI 2011*, Orlando. LNCS, vol. 6766 (2011), pp. 545–554
89. D.E. Newland, *An Introduction to Random Vibrations, Spectral and Wavelet Analysis* (Wiley, New York, 1993)
90. M. Holschneider, *Wavelets: An Analysis Tool* (Oxford University Press, Oxford, 1995)
91. C. Blatter, *Wavelets: A Primer* (A.K. Peters, Natick, 1998)
92. M. Farge, J.C. Hunt, J.C. Vassilicos, *Wavelets, Fractals and Fourier Transforms* (Oxford University Press, Oxford, 1995)
93. D.B. Percival, A.T. Walden, *Wavelet Methods for Time Series Analysis* (Cambridge University Press, Cambridge/New York, 2000)
94. B.P. van Milligen, E. Sánchez, T. Estrada, C. Hidalgo, B. Brãnas, B. Carreras, L. Garsia, Wavelet bicoherence: a new turbulence analysis tool. Phys. Plasmas **2**(8), 3017 (1995)