

Notes on Numerical Fluid Mechanics
and Multidisciplinary Design 125

Michel O. Deville · Jean-Luc Estivalezes
Vincent Gleize · Thien-Hiep Lê
Marc Terracol · Stéphane Vincent *Editors*

Turbulence and Interactions

Proceedings of the TI 2012 conference

 Springer

Notes on Numerical Fluid Mechanics and Multidisciplinary Design

Volume 125

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Editors

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ISSN 1612-2909

ISSN 1860-0824 (electronic)

ISBN 978-3-662-43488-8

ISBN 978-3-662-43489-5 (eBook)

DOI 10.1007/978-3-662-43489-5

Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014940791

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Preface

The “Turbulence and Interactions 2012” (TI2012) conference was held in La Salines-Bains on the island of La Réunion, France, on June 11th–14th, 2012. The scientific sponsors of the conference were

- Ecole Polytechnique Fédérale de Lausanne (EPFL),
- ERCOFTAC : European Research Community on Flow, Turbulence and Combustion,
- Institut de Mécanique et d’Ingénierie (I2M), Bordeaux,
- ONERA - The French Aerospace Lab,
- Université de La Réunion.

This third TI conference was very successful as it attracted 35 researchers from 6 countries. The magnificent venue and the beautiful weather helped the participants to discuss freely and casually, share ideas and projects, and spend very good times all together.

The organisers were fortunate in obtaining the presence of the following invited speakers: J.H. Chen (Sandia National Laboratories), B. Geurts (University of Twente), C. Wagner (The German Aerospace Center -DLR). The topics covered by the 23 contributed papers ranged from experimental results through theory to computations. They represent a snapshot of the state-of-the-art in turbulence research. The papers of the conference went through the usual reviewing process and the result is given in this book of Proceedings.

In the present volume, the reader will find the keynote lectures followed by the contributed talks given in alphabetical order of the first author.

The organisers of the conference would like to acknowledge the support of EPFL, I2M and ONERA.

Bordeaux, Lausanne, Paris, Toulouse
January 23rd, 2014.

Michel O. Deville
Jean-Luc Estivalezes
Vincent Gleize
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Marc Terracol
Stéphane Vincent

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Theory and Application of Regularization Modeling of Turbulence

Bernard J. Geurts

Abstract. Turbulence readily arises in numerous flows in nature and technology. The large number of degrees of freedom of turbulence poses serious challenges to numerical approaches aimed at simulating and controlling such flows. While the Navier-Stokes equations are commonly accepted to precisely describe fluid turbulence, alternative coarsened descriptions need to be developed to cope with the wide range of length and time scales. These coarsened descriptions are known as large-eddy simulations in which one aims to capture only the primary features of a flow, at considerably reduced computational effort. Such coarsening introduces a closure problem that requires additional phenomenological modeling. A systematic approach to the closure problem, known as regularization modeling, will be reviewed. Its application to turbulent mixing will be illustrated. Leray and LANS-alpha regularization are discussed in some detail.

1 Introduction

A new modeling approach for large-eddy simulation (LES) is obtained by combining a ‘regularization principle’ with an explicit filter and its inversion. This regularization approach allows a systematic derivation of the implied subgrid-model, which resolves the closure problem. The central role of the filter in LES is fully restored, i.e., both the interpretation of LES predictions in terms of direct simulation results as well as the corresponding subgrid closure are specified by the filter (Geurts & Holm, 2003). The regularization approach is illustrated with ‘Leray-smoothing’ of the nonlinear convective terms. In turbulent mixing the new, implied

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subgrid model performs favorably compared to the dynamic eddy-viscosity procedure (Geurts & Holm, 2006; Kuczaj & Geurts, 2007). The model is robust at arbitrarily high Reynolds numbers and correctly predicts self-similar turbulent flow development.

Accurate modeling and simulation of turbulent flow is a topic of intense ongoing research (Meneveau & Katz, 2000). Modern strategies for turbulent flow are aimed at reducing the dynamical complexity of the underlying system of partial differential equations while reliably predicting the primary flow phenomena. In large-eddy simulation (LES) these conflicting requirements are expressed by coarsening the description on the one hand and subgrid modeling on the other hand. The coarsening is achieved by spatial filtering (Germano, 1992) which externally specifies the physical detail that will ideally be retained in the LES solution. Maintaining the dynamical properties of the resolved large scales is approached by introducing subgrid modeling to deal with the closure problem that arises from filtering the nonlinear terms.

In the filtering approach to incompressible flow the specification of the basic convolution filter L is all that is required to uniquely define the relation between the unfiltered and filtered flow field as well as the closure problem for the so-called turbulent stress-tensor τ_{ij} . This situation is in sharp contrast with actual present-day large-eddy modeling in which the specification of the subgrid model for τ_{ij} as well as the comparison with reference direct numerical simulation (DNS) results is performed largely independent of the specific choice of the filter L .

In this paper we will formulate an alternative approach to large-eddy simulations which completely restores the two central roles of the basic filter L , i.e., providing an interpretation of LES predictions in terms of filtered DNS results as well as fully specifying all details of the subgrid model. The key elements in this new formulation are a ‘regularization principle’, a filter L and its (formal) inverse operator denoted by L^{-1} (Geurts, 1997).

The organization of this paper is as follows. In Section 2 we introduce the concept of regularization. Section 3 is devoted to the derivation of sub-filter models for large-eddy simulation from regularization principles. The assessment of the Leray model is presented in Section 4 and concluding remarks are collected in Section 5.

2 Turbulence Regularization

A regularization principle expresses the smoothing of the dynamics of the Navier-Stokes equations through a specific proposal for direct alteration of the nonlinear convective terms. This modeling differs significantly from traditional, less direct approaches, e.g., involving the introduction of additional eddy-viscosity contributions (Smagorinsky, 1963). The latter are clearly of a different physical nature and do not fully do justice to the intricate nonlinear transport structure of the filtered Navier-Stokes equations. The regularization principle gives rise to a basic mixed formulation involving both the filtered and unfiltered solution. Application of L and L^{-1} then allows to derive an equivalent representation solely in terms of the filtered

solution. This provides a unique identification of the implied subgrid model without any further external (ad hoc) input or mathematical-physical considerations of the closure problem. The regularization modeling approach is not only theoretically transparent and elegant, but it also gives rise to accurate LES predictions. In particular, we consider the implied subgrid model that arises from Leray's regularization principle (Leray, 1934). A comparison between the Leray model and dynamic subgrid modeling (e.g., (Vreman, Geurts & Kuerten, 1997)) will be made for turbulent mixing flow, both at moderate and at high Reynolds numbers.

In the filtering approach one assumes any normalized convolution filter $L : u_i \rightarrow \bar{u}_i$ where \bar{u}_i (u_i) denotes the filtered (unfiltered) component of the velocity field in the x_i direction. Filtering the Navier-Stokes equations yields

$$\partial_i \bar{u}_i + \partial_j (\bar{u}_j \bar{u}_i) + \partial_i \bar{p} - \frac{1}{Re} \partial_{jj} \bar{u}_k = -\partial_j \tau_{ij} \quad (1)$$

where the turbulent stress tensor $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ represents the closure problem and Re denotes the Reynolds number. Both the relation between u_i and \bar{u}_i as well as the properties of τ_{ij} are fully specified by L . In actual subgrid modeling for LES, the next step is to introduce a subgrid model $m_{ij}(\bar{\mathbf{u}})$ to approximate τ_{ij} . A variety of subgrid models has been proposed to capture dissipative, dispersive or similarity properties of τ_{ij} .

Many subgrid models are arrived at through a physical or mathematical reasoning which is only loosely connected to a specific filter L . As an example, the well-known Smagorinsky model (Smagorinsky, 1963) is given by $m_{ij}^S = -(C_S \Delta)^2 |S_{ij}(\bar{\mathbf{u}})| S_{ij}(\bar{\mathbf{u}})$ where the rate of strain tensor $S_{ij} = \partial_i \bar{u}_j + \partial_j \bar{u}_i$ and $|S_{ij}|^2 = S_{ij} S_{ij} / 2$. The only explicit reference to the filter, made in this model, is through the filter-width Δ . In actual simulations Δ is specified in terms of the grid-spacing h rather than in terms of L . Furthermore, the Smagorinsky constant C_S is determined independent of L , which further reduces any principal role for the filter. The situation is comparable for the 'tensor-diffusivity' model $m_{ij}^{TD} = C_{TD} \Delta_k^2 \partial_k \bar{u}_i \partial_k \bar{u}_j$, with Δ_k the filter-width in the x_k -direction (Clark, Ferziger & Ferziger, 1979). The coefficient C_{TD} is usually related to the normalized second moment $(L(x^2) - x^2) / \Delta^2$ of the filter L . For various popular filters such as the top-hat or the Gaussian filter one finds $C_{TD} = 1/12$, i.e., independent of the actual filter used. The role of the filter is in principle fully explicit in Bardina's similarity model $m_{ij}^B = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ (Bardina, Ferziger, & Reynolds, 1984). In actual simulations, however, one frequently adopts a wider explicit filter or a filter of a different type, to enhance smoothing properties of this model (Meneveau & Katz, 2000). Moreover, the model is sometimes multiplied by a constant C_B which is specified independently of any presumed filter (Salvetti & Banerjee, 1995). Finally, the successful dynamic subgrid modeling requires only the explicit specification of the so-called test-filter (Germano, Piomelli, Moin & Cabot, 1991). To retain the central Germano identity the test-filter can in principle be chosen independent of L , mainly requiring the specification of the filterwidth of the test-filter relative to Δ . Additional averaging over homogeneous directions, 'clipping' steps to stabilize actual simulations, and the fact

that the assumed base-models are themselves only loosely connected to L , also make the dynamic procedure rather insensitive to the specific assumed filter.

In contrast to these popular LES models, the regularization approach involves the introduction of a pair (L, L^{-1}) to fully specify the implied subgrid model as well as the interpretation of LES predictions in terms of reference DNS results. The selection of any other pair $(\mathcal{L}, \mathcal{L}^{-1})$ directly leads to its corresponding DNS interpretation and the associated subgrid model consistent with the regularization principle. This modeling strategy has a number of important benefits, addressing directly the nonlinear convective contributions and requiring no additional ‘external’ information such as model coefficients or the width of the test-filter. The regularization principle allows a transparent modeling in which the modeled system of equations can be made to share a number of fundamental properties with the Navier-Stokes equations, such as transformation symmetries, Kelvin’s circulation theorem, etc.. The implied subgrid model is quite simple to implement, with some technical complications arising from the construction of an accurate inverse operator L^{-1} .

3 Regularization to Derive Sub-filter Models

In this section we present two regularization principles and their translation to the corresponding sub-filter model for Large-Eddy Simulation. We begin with the Leray model and subsequently extend this to the LANS- α model.

Leray Modeling

To illustrate the regularization approach we consider the intuitively appealing and particularly simple Leray regularization in which the convective fluxes are replaced by $\bar{u}_j \partial_j u_i$, i.e., the solution \mathbf{u} is convected with a smoothed velocity $\bar{\mathbf{u}}$. Consequently, the nonlinear effects are reduced by an amount governed by the smoothing properties of L . The governing equations in the Leray formulation can be written as (Leray, 1934)

$$\partial_j \bar{u}_j = 0 \quad ; \quad \partial_t u_i + \bar{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re} \partial_{jj} u_i = 0 \quad (2)$$

Uniqueness and regularity of the solution to these equations have been established rigorously (Leray, 1934). The Leray formulation contains the unfiltered Navier-Stokes equations in the limiting case $L \rightarrow Id$, e.g., as $\Delta \rightarrow 0$ (Id denotes the identity). The unfiltered solution can readily be eliminated from (2) by using the inversion operator $u_j = L^{-1}(\bar{u}_j)$. After some calculation (2) can be written in the same way as the LES ‘template’ (1) in which τ_{ij} on the right hand side is replaced by the asymmetric, filtered similarity-type Leray model m_{ij}^L given by:

$$m_{ij}^L = L\left(\bar{u}_j L^{-1}(\bar{u}_i)\right) - \bar{u}_j \bar{u}_i = \overline{\bar{u}_j u_i} - \bar{u}_j \bar{u}_i \quad (3)$$

This model requires the explicit application of both L and L^{-1} . The tensor m_{ij}^L is not symmetric. However, the flow is governed by the divergence $\partial_j m_{ij}^L$ which

can be shown to transform covariantly under Galilean transformations and under a change to a uniformly rotating reference frame, as does $\partial_j \tau_{ij}$. For properly chosen filter, Leray solutions of the regularized Navier-Stokes equations behave better with respect to smoothness and boundedness. Correspondingly, the subgrid model (3) can be expected to yield similar benefits in a large-eddy context. The straightforward model $m_{ij} = L(L^{-1}(\bar{u}_i)L^{-1}(\bar{u}_j)) - \bar{u}_i\bar{u}_j$ does not provide sufficient smoothing and leads to unstable LES on coarse grids, at high Re .

LANS- α Regularisation by Kelvin Filtering

A regularisation principle which additionally possesses correct circulation properties may be obtained by starting from the following Kelvin theorem:

$$\frac{d}{dt} \oint_{\Gamma(\mathbf{u})} u_j dx_j - \frac{1}{Re} \oint_{\Gamma(\mathbf{u})} \Delta u_j dx_j = 0 \quad (4)$$

where $\Gamma(\mathbf{u})$ is a closed fluid loop moving with the Eulerian velocity \mathbf{u} . The unfiltered Navier-Stokes equations may be derived from (4) (Foias, Holm & Titi, 2001). This provides some of the inspiration to arrive at an alternative regularisation principle for Navier-Stokes turbulence (Foias, Holm & Titi, 2001). In fact, the basic regularisation principle was originally derived by applying Taylor's hypothesis of frozen-in turbulence in a Lagrangian averaging framework. In this framework, the fluid loop is considered to move with the smoothed transport velocity $\bar{\mathbf{u}}$, although the circulation velocity is still the unsmoothed velocity, \mathbf{u} . That is, in (4) we replace $\Gamma(\mathbf{u})$ by $\Gamma(\bar{\mathbf{u}})$; so the material loop Γ moves with the filtered transport velocity. From this filtered Kelvin principle, we may obtain the Euler-Poincaré equations governing the smoothed solenoidal fluid dynamics, with $\partial_j \bar{u}_j = 0$

$$\partial_t u_j + \bar{u}_k \partial_k u_j + u_k \partial_j \bar{u}_k + \partial_j p - \partial_j \left(\frac{1}{2} \bar{u}_k u_k \right) - \frac{1}{Re} \Delta u_j = 0 \quad (5)$$

Comparison with the Leray regularisation principle reveals two additional terms in (5). These terms guarantee the regularised flow to be consistent with the modified Kelvin circulation theorem in which $\Gamma(\mathbf{u}) \rightarrow \Gamma(\bar{\mathbf{u}})$. For LANS- α the analytical properties of the regularised solution are based on the energy balance for $\int \mathbf{u} \cdot L(\mathbf{u}) d^3x$.

The Euler-Poincaré equations (5) can also be rewritten in the form of the LES template. The extra terms that arise in (5) give rise to additional terms in the implied subgrid model:

$$\partial_t \bar{u}_i + \partial_j (\bar{u}_j \bar{u}_i) + \partial_i \bar{p} - \frac{1}{Re} \Delta \bar{u}_i = -\partial_j (\overline{\bar{u}_j u_i} - \bar{u}_j \bar{u}_i) - \frac{1}{2} (\overline{u_j \partial_i \bar{u}_j} - \bar{u}_j \partial_i u_j) \quad (6)$$

We observe that the Leray model reappears as part of the implied LANS- α subgrid model on the right-hand side of (6). Compared to the Leray model, the additional second term in the LANS- α model takes care of recovering the Kelvin circulation theorem for the smoothed solution. This formulation is given in terms of a general

filter L and its inverse. After some further rewriting it may be shown that this model can be formulated in conservative form, i.e., a tensor m_{ij}^α can be found such that the right hand side of (6) can be written as $-\partial_j m_{ij}^\alpha$. We illustrate this next for a particular filter.

The subgrid model presented in (6) can be specified further in case the filter L has the Helmholtz operator as its inverse, i.e., $u_i = L^{-1}(\bar{u}_i) = (1 - \alpha^2 \partial_{jj})\bar{u}_i = \text{He}_\alpha(\bar{u}_i)$. Then we recover the original LANS- α equations (Foias, Holm & Titi, 2001). The LANS- α model derives its name from the length-scale parameter $\alpha \approx \Delta/5$. After some rewriting, the following parameterisation for the turbulent stress tensor is obtained:

$$m_{ij}^\alpha = \alpha^2 \text{He}_\alpha^{-1} \left(\partial_k \bar{u}_i \partial_k \bar{u}_j + \partial_k \bar{u}_i \partial_j \bar{u}_k - \partial_i \bar{u}_k \partial_j \bar{u}_k \right) \quad (7)$$

The first term on the right-hand side is the *Helmholtz-filtered* tensor-diffusivity model. The second term combined with the first term, corresponds to Leray regularisation using Helmholtz inversion as filter. The third term completes the LANS- α model and maintains Kelvin's circulation theorem. In (7) an inversion of the Helmholtz operator He_α is required which implies application of the exponential filter. However, since the Taylor expansion of the exponential filter is identical at quadratic order to that of the top-hat or the Gaussian filters, one may approximate He_α^{-1} , e.g., by an application of the explicit top-hat filter, for reasons of computational efficiency.

Spectral Consequences of Regularization Modeling

The different regularisation models are known to have different effects on the tail of the resolved kinetic energy spectrum $E(k)$. In the Kolmogorov picture of homogeneous, isotropic turbulence an inertial range in which $E(k) \sim k^{-5/3}$ develops over an extended range of wavenumbers k up to a Kolmogorov wavenumber $k_\eta \sim 1/\eta$ where η is the viscous dissipation length-scale. This entire dynamic range needs to be properly captured in order to arrive at a reliable DNS. The Leray and LANS- α models give rise to a spectrum in which there is a smooth transition from a $-5/3$ power law to a much steeper algebraic decay, beyond wavenumbers $\sim 1/\Delta$. The sharper decrease of kinetic energy with wavenumber implies a corresponding strong reduction in required computational effort needed for the simulation of the relevant dynamic range. The LANS- α model displays a tail of the spectrum $\sim k^{-3}$ while the Leray model decays even more steeply, as $\sim k^{-13/3}$ (Foias, Holm & Titi, 2001). The steeper decay using the Leray model is directly reflected in the smoother impression of instantaneous solutions. Hence, through the selection of Δ a direct external control is achieved over the computational costs associated with the regularisation models. This is illustrated in figure 1. In case an energy range of, say, m decades is desired then all wavenumbers up to $k_L(m)$, $k_\alpha(m)$ and $k_{DNS}(m)$ need to be resolved for the Leray, LANS- α and DNS approaches respectively. This corresponds to a significant difference in the associated computational expense, while all three simulations would provide excellent accuracy at least for all wavenumbers up to $\sim 1/\Delta$.

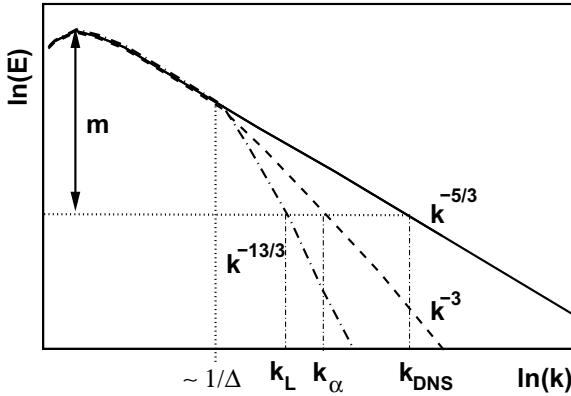


Fig. 1 Sketch of resolved kinetic energy spectrum in a homogeneous, isotropic turbulence, displaying a $-5/3$ tail in DNS (solid), a -3 tail in LES using the LANS- α model (dashed) and a $-13/3$ tail in LES using the Leray model (dash-dotted)

In the sequel we consider invertible numerical quadrature approximating the top-hat filter. In one dimension the numerical convolution filtering $\bar{u} = G * u$ corresponds to kernels

$$G(z) = \sum a_j \delta(z - z_j) ; |z_j| \leq \Delta/2 \quad (8)$$

In particular, we consider three-point filters with $a_0 = 1 - \alpha$, $a_1 = a_{-1} = \alpha/2$ and $z_0 = 0$, $z_1 = -z_{-1} = \Delta/2$. Here we use $\alpha = 1/3$ which corresponds to Simpson quadrature of the top-hat filter. In actual simulations the resolved fields are known only on a set of grid points $\{x_m\}_{m=0}^N$. The application of L^{-1} to a general discrete solution $\{\bar{u}(x_m)\}$ can be specified using discrete Fourier transformation as (Kuerten, Geurts, Vreman & Germano, 1999)

$$L^{-1}(\bar{u}_m) = \sum_{j=-n}^n \left(\frac{\alpha - 1 + \sqrt{1 - 2\alpha}}{\alpha} \right)^{|j|} \frac{\bar{u}_{m+rj/2}}{(1 - 2\alpha)^{1/2}} \quad (9)$$

where the subgrid resolution $r = \Delta/h$ is assumed to be even. An accurate and efficient inversion can be obtained with only a few terms, recovering the original signal to within machine accuracy with $n \approx 10$. The invertibility of L only refers to invertibility on the LES grid. Injection from a fine DNS grid to a coarse LES grid is not invertible. At fixed Δ , variation of the subgrid resolution r allows an independent control over flow-smoothing and numerical representation (Geurts & Fröhlich, 2002). Simulation results obtained in this way are properly smoothed for $k\Delta < 2\pi$. At constant Δ the inclusion of modes with higher wavenumber k in case $r > 1$ allows to approach the grid-independent solution to the ‘fixed- Δ ’ problem. However, the modes with $k > 2\pi/\Delta$ are not properly smoothed in the sense of Leray; the Fourier transform of the kernel G does not reduce in amplitude for large $k\Delta$ but rather, it oscillates between fixed limits. To achieve a genuine PDE result, Leray analysis

requires correct smoothing by the filter also at high wavenumber. The present results are limited to the modes with $k\Delta < 2\pi$ and in subsequent illustrations we restrict ourselves to this range.

4 Assessment of Leray Model

To assess the Leray model the turbulent mixing layer is simulated in a volume ℓ^3 at various Re adopting a fourth order accurate finite volume discretization and explicit four-stage, second-order accurate Runge-Kutta time-stepping. We compare predictions with those obtained using the dynamic subgrid model, which was shown to be among the most accurate models in a comparative study of the same turbulent mixing layer reported in (Vreman, Geurts & Kuerten, 1997).

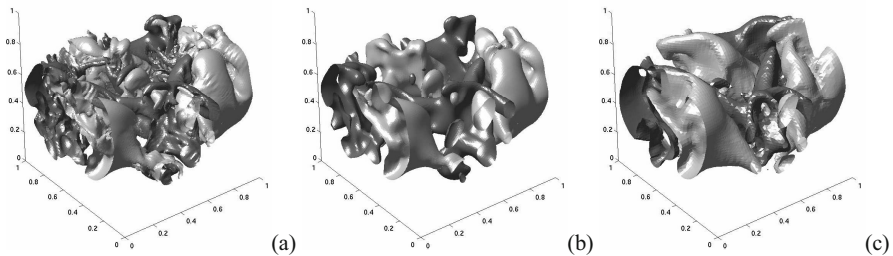


Fig. 2 Normal velocity component u_2 at time $t = 80$, (a): DNS, (b): filtered DNS, (c): Leray on 64^3 ; using a filterwidth $\Delta = \ell/16$. The light (dark) isosurfaces correspond to $u_2 = 0.3$ (-0.3).

A first introductory test of the Leray model is obtained by studying instantaneous solutions. As a typical illustration of the mixing layer the DNS prediction of the normal velocity u_2 , obtained at a spatial resolution of 256^3 , is shown in the turbulent regime in Fig. 2(a). We used $Re = 50$ based on the initial momentum thickness and free-stream flow properties. The filtered u_2 can be seen in Fig. 2(b) establishing a significant smoothing due to the ‘Simpson’ filter at $\Delta = \ell/16$. The Leray prediction (Fig. 2(c)) appears to capture the main ‘character’ as well as some of the details of the filtered DNS solution. A slight underprediction of the influence of the small scales is, however, apparent. Further visualization showed that the instantaneous Leray predictions display much better overall agreement with filtered DNS than the dynamic model, which relative to the Leray model significantly overpredicts the smoothing (Vreman, Geurts & Kuerten, 1997). Of course, assessing the quality of LES predictions in this way is difficult to quantify and we consider more specific measures next.

The evolution of a crucial mean-flow property such as the momentum thickness is shown in Fig. 3. The Leray results compare significantly better with filtered DNS results than those obtained with the dynamic model on 32^3 grid-cells. We observe

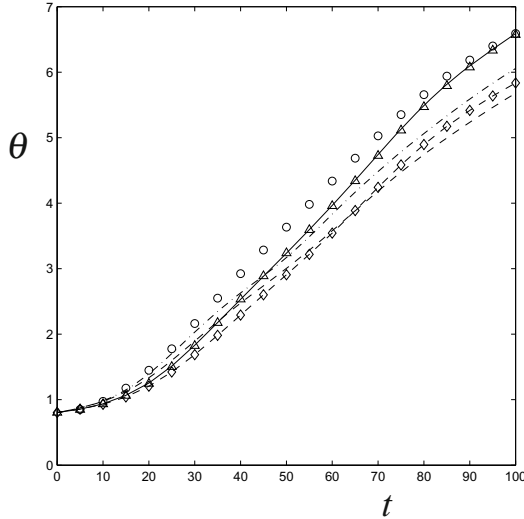


Fig. 3 Momentum thickness θ : filtered DNS (\circ), Leray-model (32^3 : dash-dotted, 64^3 : solid, 96^3 : \triangle), dynamic model (32^3 : dashed, 64^3 : dashed with \diamond). A fixed filterwidth of $\ell/16$ was used.

that some of the discrepancies between Leray and filtered DNS results are due to numerical contamination. By increasing the resolution at fixed Δ , a good impression of the grid-independent solution to the modeled equations can be inferred using $64^3 - 96^3$ grid-cells, i.e., $\Delta/h = 4$ to 6 (Geurts & Fröhlich, 2002). Numerical contamination also plays a role in the dynamic model. The grid-independent solution corresponding to the dynamic model appears less accurate than the corresponding Leray result.

A more detailed assessment is obtained from the streamwise kinetic energy spectrum shown in Fig. 4. The dynamic model yields a significant underprediction of the intermediate and smaller retained scales, particularly for the approximately grid-independent solution. The Leray predictions are much better. On coarse grids, an overprediction of the smaller scales is apparent due to interaction with the spatial discretization method (Vreman, Geurts & Kuerten, 1994). At proper numerical subgrid resolution the situation is considerably improved and the Leray model is seen to capture all scales with high accuracy. A slight, systematic underprediction of the smaller scales remains, consistent with the impression obtained from Figs. 2(b)-(c).

A particularly appealing property of Leray modeling is the robustness at very high Reynolds numbers, cf. Fig. 5. This is quite unique for a subgrid model without an explicit eddy-viscosity contribution. Although comparison with filtered DNS data is impossible here, we observe that the smoothed Leray dynamics is essentially captured as $r = \Delta/h \geq 4$ (Geurts & Fröhlich, 2002). The tail of the spectrum increases with Re , indicating a greater importance of small scale flow features.

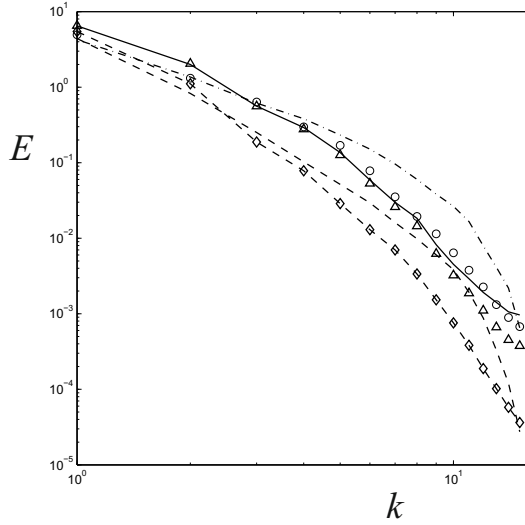


Fig. 4 Streamwise kinetic energy spectrum E at $t = 75$: filtered DNS (\circ), Leray-model (32^3 : dash-dotted, 64^3 : solid, 96^3 : \triangle), dynamic model (32^3 : dashed, 64^3 : dashed with \diamond). A fixed filterwidth of $\ell/16$ was used.

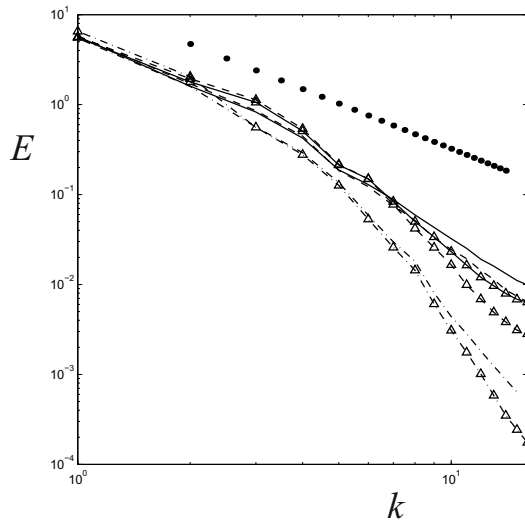


Fig. 5 Streamwise kinetic energy spectrum E at $t = 75$ predicted by the Leray model: $Re = 50$ (64^3 : dash-dotted, 96^3 : dash-dotted, \triangle), $Re = 500$ (64^3 : dashed, 96^3 : dashed, \triangle), $Re = 5000$ (64^3 : solid, 96^3 : solid, \triangle). A fixed filterwidth of $\ell/16$ was used. The dotted line represents $k^{-5/3}$.

Improved subgrid resolution shows a reduction of these smallest scales, consistent with the reduced numerical error. At high Re the spectrum corresponding to the Leray model tends to contain a region with approximately $k^{-5/3}$ behavior, which is absent at $Re = 50$. Further analysis showed that the solution develops self-similarly at high Re .

5 Concluding Remarks

The Leray model displays excellent robustness with increasing Reynolds number. This feature allows one to apply the Leray model accurately at reasonable computational costs and under flow-conditions that are well outside current DNS capabilities. However, the LANS- α model yields solutions with more realistic variability, corresponding better to the filtered DNS results than for the Leray model. Thus, a trade-off emerges between these two models. The solutions of the LANS- α model may more accurately represent the effects of intermittency in turbulence than the less-variable solutions of the Leray model. However, the LANS- α model is less robust and its application to flow at high Reynolds numbers is not as straightforward as with the Leray model. Further investigation of this trade-off may lead to interesting developments in the comparison of the time-dependent solutions of these two models.

A convenient benefit of the regularisation approach to turbulence modelling is that it enables one to derive the implied small-scale treatment from the underlying regularisation principle. This yields a systematic closure of the equations whose analysis allows an extension in which the filter width Δ is determined dynamically by the evolving flow. The evolving filter-width may even be anisotropic. The application of this self-adaptive modelling approach in a spatially developing mixing layer and, more importantly, in near wall turbulence is a topic of current research.

References

- Bardina, J., Ferziger, J.H., Reynolds, W.C.: Improved turbulence models based on LES of homogeneous incompressible turbulent flows. Department of Mechanical Engineering. Report No. TF-19, Stanford (1984)
- Clark, R.A., Ferziger, J.H., Reynolds, W.C.: Evaluation of subgrid-scale models using an accurately simulated turbulent flow. *J. Fluid Mech.* 91, 1–16 (1979)
- Foias, C., Holm, D.D., Titi, E.S.: The Navier-Stokes-alpha model of fluid turbulence. *Physica D* 152, 505–519 (2001)
- Germano, M., Piomelli, U., Moin, P., Cabot, W.H.: A dynamic subgrid-scale eddy viscosity model. *Phys. Fluids A* 3, 1760–1765 (1991)
- Germano, M.: Turbulence: the filtering approach. *J. Fluid Mech.* 238, 325–336 (1992)
- Geurts, B.J.: Inverse Modeling for Large-Eddy Simulation. *Phys. of Fluids* 9, 3585–3588 (1997)
- Geurts, B.J., Fröhlich, J.: A framework for predicting accuracy limitations in large-eddy simulation. *Phys. Fluids* 14, 17–22 (2002)