

Klaus Feyrer

Wire Ropes

Tension, Endurance, Reliability

Second Edition

Wire Ropes

Klaus Feyrer

Wire Ropes

Tension, Endurance, Reliability

With 178 Figures and 51 Tables

Second Edition

 Springer

Klaus Feyrer
Institute for Materials Handling
and Logistics
University of Stuttgart
Stuttgart
Germany

ISBN 978-3-642-54995-3 ISBN 978-3-642-54996-0 (eBook)
DOI 10.1007/978-3-642-54996-0
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014939938

© Springer-Verlag Berlin Heidelberg 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

The present book *Wire Ropes* is again dedicated mainly to all users of wire ropes—construction engineers, operators and supervisors of machines and installations with wire ropes. It has been the main concern of this book to present the methods used to calculate the most important rope quantities (rope geometry, wire stresses in the rope under tension, bending and twist, rope elasticity module, torque, rope efficiency, the bearable number of load cycles or bending cycles and the discard number of wire breaks, etc.) as well as to explain how they are applied by means of a large number of example calculations.

Since 2007, after the first edition of the book *Wire Ropes* was presented, a row of research works in the field of wire ropes has been conducted. Important examples are the wire rope twist and the size effect. The results of these works are introduced in the book by discussing their influence on the existing knowledge. The practical calculation work will be assisted by mentioning the right Excel program.

It would not have been possible to revise the book after my retirement so simply without the support and encouragement of Professor Dr.-Ing., Dr. hc. K.-H. Wehking, the head of the Institut für Fördertechnik und Logistik, Universität Stuttgart. I am extremely grateful to Professor Wehking for his advice and support and for being able to use the infrastructure of the institute. There have also been many enlightening discussions held with Prof. em. Dr. techn. Prof. E.h. Franz Beisteiner, the former head of the institute. I would like to thank him very much indeed for his constant readiness to have a discussion and for his sound advice which helped to clarify many a point in question. In the same way, I would also like to thank all members of staff at the institute for their readiness to discuss details and especially for their willingness to help promptly in solving any computer problems that arose.

From the first edition remains the polishing of my English by Mrs. Meryll Zepf with a great deal of understanding for the project. I am extremely grateful for all her efforts. The English polishing of my corrections to the second edition comes from Springer Verlag. For that and the pleasant cooperation all my thanks go to Dr. Christoph Baumann, Almas Schimmel, and the staff of Springer Verlag.

Putting together the compilation of what we know about wire ropes today—even though there are probably some gaps—was certainly made easier by the Organisation pour L'Étude de L'Endurance des Cables (OIEPEC). During the past

few decades, the OIPEEC has developed into the most important forum for discussing questions in connection with wire ropes. I am very grateful to my colleagues at the OIPEEC for their very stimulating discussions. The same is true for the members of the DRAHTSEIL-VEREINIGUNG e.V. (Wire Rope Association, Germany). Furthermore, I would also like to thank the wire rope manufacturers who have been interested in and supportive of the wire rope research from the very beginning and have helped enormously by donating nearly all the wire ropes the Institut für Fördertechnik Stuttgart ever tested.

Even though extreme care is always taken, it is hardly possible to print a book that has absolutely no errors. This is true for this book as well. Because of this a list has been created where any printing errors or inaccuracies can be entered. The latest version of this list of corrections can be found in the Internet under: <http://www.uni-stuttgart.de/ift/update.rope>

For complicated calculations there are again Excel programs that can be downloaded free of charge from the address:

<http://www.uni-stuttgart.de/ift/forschung/berechnungsprogramme>

To make the list of corrections as comprehensive as possible, I would like to ask all readers for their assistance to report any mistakes found to the following address:

K. Feyrer, Institut für Fördertechnik
Holzgartenstrasse 15B
70174 Stuttgart
or Fax: +49 (0) 6858 3769
or E-mail: feyrer@ift.uni-stuttgart.de

Stuttgart

Klaus Feyrer

Contents

1	Wire Ropes, Elements and Definitions	1
1.1	Steel Wire	1
1.1.1	Non-alloy Steel	1
1.1.2	Wire Manufacturing	3
1.1.3	Metallic Coating	4
1.1.4	Corrosion Resistant Wires	5
1.1.5	Wire Tensile Test	5
1.1.6	Wire Endurance and Fatigue Strength	8
1.2	Strands	23
1.2.1	Round Strands	23
1.2.2	Shaped Strands	26
1.2.3	Compacted Strands	27
1.3	Rope Cores	29
1.4	Lubrication	31
1.4.1	Lubricant	31
1.4.2	Lubricant Consumption	32
1.4.3	Rope Endurance	33
1.5	Wire Ropes	33
1.5.1	The Classification of Ropes According to Usage	33
1.5.2	Wire Rope Constructions	34
1.5.3	Designation of Wire Ropes	38
1.5.4	Symbols and Definitions	40
1.6	The Geometry of Wire Ropes	45
1.6.1	Round Strand with Round Wires	45
1.6.2	Round Strand with Any Kind of Profiled Wires	47
1.6.3	Fibre Core	52
1.6.4	Steel Core	54
	References	55
2	Wire Ropes Under Tensile Load	59
2.1	Stresses in Straight Wire Ropes	59
2.1.1	Global Tensile Stresses	59
2.1.2	Real Stresses	60

2.1.3	Basic Relation for the Wire Tensile Force in a Strand. . .	60
2.1.4	Wire Tensile Stress in the Strand or Wire Rope	63
2.1.5	Additional Wire Stresses in the Straight Spiral Rope. . .	70
2.1.6	Additional Wire Stresses in Straight Stranded Ropes. . .	73
2.2	Wire Rope Elasticity Module	79
2.2.1	Definition.	79
2.2.2	Rope Elasticity Module of Strands and Spiral Ropes, Calculation.	80
2.2.3	Rope Elasticity Module of Stranded Wire Ropes	81
2.2.4	Waves and Vibrations	94
2.3	Reduction of the Rope Diameter Due to Rope Tensile Force . .	104
2.4	Torque and Torsional Stiffness	105
2.4.1	Rope Torque from Geometric Data	105
2.4.2	Torque of Twisted Round Strand Ropes	108
2.4.3	Rotary Angle of a Load Hanging on Two or More Wire Rope Traces	115
2.4.4	Rope Twist Caused by the Height-Stress	119
2.4.5	Change of the Rope Length by Twisting the Rope	125
2.4.6	Wire Stresses Caused by Twisting the Rope.	129
2.4.7	Rope Endurance Under Twist.	135
2.5	Wire Rope Breaking Force	139
2.5.1	Measured Breaking Force	139
2.5.2	Minimum Breaking Force	140
2.5.3	Wire Rope Breaking Force with Different Terminations	140
2.6	Wire Ropes Under Fluctuating Tension	141
2.6.1	Conditions of Tension–Tension Tests	141
2.6.2	Evaluating Methods.	142
2.6.3	Results of Tension Fatigue Test-Series	147
2.6.4	Further Results of Tension Fatigue Tests	157
2.6.5	Calculation of the Number of Load Cycles	162
2.7	Dimensioning Stay Wire Ropes.	167
2.7.1	Extreme Forces.	168
2.7.2	Fluctuating Forces.	169
2.7.3	Discard Criteria	172
	References	172
3	Wire Ropes Under Bending and Tensile Stresses.	179
3.1	Stresses in Running Wire Ropes	179
3.1.1	Bending and Torsion Stress	179
3.1.2	Secondary Tensile Stress	186
3.1.3	Stresses from the Rope Ovalisation.	191
3.1.4	Secondary Bending Stress	193
3.1.5	Sum of the Stresses.	194

- 3.1.6 Force Between Rope and Sheave (Line Pressure) 196
- 3.1.7 Pressure Between Rope and Sheave 205
- 3.1.8 Force on the Outer Arcs of the Rope Wires 210
- 3.2 Rope Bending Tests 213
 - 3.2.1 Bending-Fatigue-Machines, Test Procedures 213
 - 3.2.2 Number of Bending Cycles 220
 - 3.2.3 Further Influences on the Number of Bending Cycles 233
 - 3.2.4 Reverse Bending 248
 - 3.2.5 Fluctuating Tension and Bending 249
 - 3.2.6 Palmgren–Miner Rule 253
 - 3.2.7 Limiting Factors 253
 - 3.2.8 Ropes During Bendings 256
 - 3.2.9 Number of Wire Breaks 260
- 3.3 Requirements on Rope Drives 271
 - 3.3.1 General Requirements 271
 - 3.3.2 Lifting Installations for Passengers 274
 - 3.3.3 Cranes and Lifting Appliances 276
- 3.4 General Calculation Method for Rope Drives 278
 - 3.4.1 Analysis of Rope Drives 280
 - 3.4.2 Tensile Rope Force 285
 - 3.4.3 Number of Bending Cycles 287
 - 3.4.4 Palmgren–Miner Rule 292
 - 3.4.5 Limits 295
 - 3.4.6 Rope Drive Calculations, Examples 300
- 3.5 Rope Efficiency 313
 - 3.5.1 Single Sheave, Efficiency 313
 - 3.5.2 Rope Drive, Efficiency 317
 - 3.5.3 Lowering an Empty Hook Block 320
- References 323
- Index 331**

Chapter 1

Wire Ropes, Elements and Definitions

1.1 Steel Wire

The very high strength of the rope wires enables wire ropes to support large tensile forces and to run over sheaves with relative small diameters. Very high-strength steel wires had already been existence for more than a hundred years when patenting—a special heating process—was introduced and the drawing process perfected. Since then further improvements have only occurred in relatively small steps.

There are a number of books about the history of wire ropes and wire rope production beginning with its invention by Oberbergrat Wilhelm August Julius Albert in 1834 and one of these is by Benoit (1935). Newer interesting contributions on the history of wire ropes have been written by Verreet (1988) and Sayenga (1997, 2003).

A voluminous literature exists dealing with the manufacture, material and properties of rope wires. In the following, only the important facts will be presented, especially those that are important for using the wires in wire ropes.

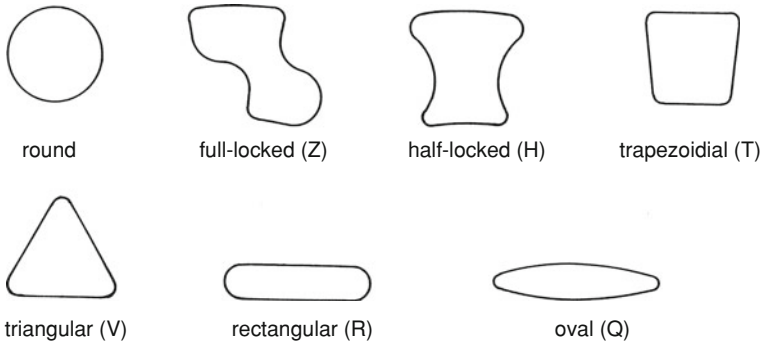
1.1.1 Non-alloy Steel

Steel wires for wire ropes are normally made of high-strength non-alloy carbon steel. The steel rods from which the wires are drawn or cold-rolled are listed in Table 1.1 as an excerpt of a great number of different steels from the European Standard EN 10016-2. The rods for rope wires have a high carbon content of 0.4–0.95 %.

The number in the name of the steel gives the mean content of carbon in weight percent multiplied with the factor 100. For example, the steel name C 82 D means that the steel has a mean carbon content of 0.82 %. Steels with high carbon content close to 0.86 % with eutectoid fine perlite—a mix of cementite (Fe_3C) and ferrite—are preferred for rope wires.

Table 1.1 Non-alloy steel rod for drawing (excerpt of EN 10 016-2)

Steel name	Steel number	Heat analysis carbon content (%)
C 42 D	1.0541	0.40–0.45
C 48 D	1.0517	0.45–0.50
C 50 D	1.0586	0.48–0.53
.....
C 82 D	1.0626	0.80–0.85
C 86 D	1.0616	0.83–0.88
C 88 D	1.0628	0.85–0.90
C 92 D	1.0618	0.90–0.95

**Fig. 1.1** Wire cross-sections for wire ropes

Carbon steels only contain small quantities of other elements. EN 10016-2 gives the following limits for the chemical ingredients of carbon steel rods used for rope wires: Si 0.1–0.3 %, Mn 0.5–0.8 %, P and S < 0.035 %, Cr < 0.15 %, Ni < 0.20 %, Mo < 0.05 %, Cu < 0.25 % and Al < 0.01 %. The strength increases with an increasing carbon content and the breaking extension decreases if all other influences are constant. Higher contents of sulphur S, phosphorus P, chrome Cr and copper Cu reduce the steel's ductility, Schneider and Lang (1973).

Usually, wires for wire ropes have a round cross-section. In special cases, however, wires with other cross-sections—called profile wires—are used. The different cross-sections are to be seen in Fig. 1.1. The profile wires in the upper row are inserted in locked coil ropes. The wires below are used for triangular and oval strands.

In wires with a high carbon content which had been aged artificially, Unterberg (1967) and Apel and Nünninghoff (1983) found a distinct decrease in the breaking extension and the number of turns from the torsion test. The number of test bendings is slightly reduced and the strength slightly increased. The finite life fatigue strength is partly increased or decreased.

Bending tests were repeated with three wire ropes after they had been in storage for 22 years. The original tests were well documented and the new tests were done

in the same way with the usual lubrication. There was virtually no difference in the rope bending endurance documented for the original tests and the new tests. For two of these wire ropes, the mean strength of the wires was reduced during the long period of storage by a maximum of 3 %. For one rope, the mean strength of the wires increased by 2.7 %.

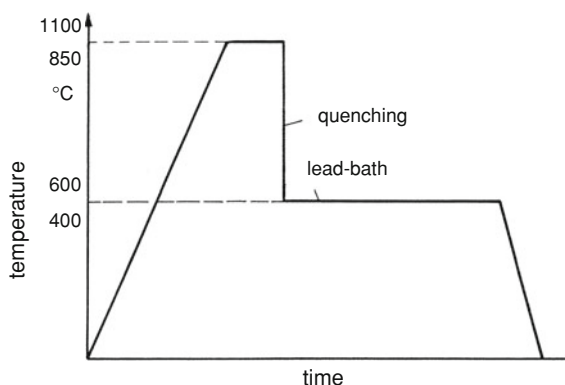
1.1.2 Wire Manufacturing

After the rod has been patented in a continuous system, the wire diameter is reduced in stages by cold drawing or cold rolling, rolling especially for profile wires. Patenting is a heating process. First the wire is heated in an austenising furnace at about 900 °C. Then the temperature is abruptly reduced to about 500 °C when the wire is put through a lead bath. After remaining there for a while, the wire then leaves the bath and enters the normal temperature of the surroundings. Figure 1.2 shows the course of the temperature during the patenting process. In recent times, the patenting process has partly been replaced by cooling in several stages while drawing or rolling the rod, Marcol (1986).

By patenting, the steel rod gets a sorbite structure (fine stripes of cementite and ferrite) which is very suitable for drawing. In the following drawing process, the wire cross-section is reduced in stages, for example in seven stages from 6 to 2 mm in diameter. After the wires have been patented, they can be drawn again. The quality of the wire surface can be improved by draw-peeling the wire rod, Kieselstein and Wißuwa (2005).

The principle of the wire drawing was described at an early date by Siebel (1959). The strength increases with the growing decrease of the cross-section by drawing and at the same time the breaking extension also decreases. The higher the carbon content of the wires, the stronger they are. For wires with small diameters below 0.8 mm, the strength can reach about 4,000 N/mm², for thicker wires about 2,500 N/mm², and in all cases the remaining ductility is low. The standardised nominal strengths of rope wires are

Fig. 1.2 The course of the temperature in the patenting process



- $R_0 = 1,370 \text{ N/mm}^2$ (in special cases)
- $R_0 = 1,570 \text{ N/mm}^2$
- $R_0 = 1,770 \text{ N/mm}^2$
- $R_0 = 1,960 \text{ N/mm}^2$
- $R_0 = 2,160 \text{ N/mm}^2$ (with a smaller wire diameter)
- $R_0 = 2,450 \text{ N/mm}^2$ (with a smaller wire diameter).

The nominal strength is the minimum strength. The deviation allowed above the nominal strength is about 300 N/mm^2 . However, the real deviation is usually much smaller.

1.1.3 Metallic Coating

Rope wires needing to be protected against corrosion are normally zinc coated. Zinc coating provides reliable protection against corrosion. Even if the zinc layer is partly damaged, the steel remains protected as the electro-chemical process results in the zinc corroding first. With zinc, the wires can be coated by hot zincing or a galvanizing process. With hot zincing, the outer layer consists of pure zinc. Between this layer and the steel wire there is a boundary layer of steel and zinc compounds. With zinc galvanized wires, the whole layer of the coating, which can be relatively thick, consists of pure zinc and has a smooth surface.

In most cases the wires are covered by hot zincing. The layer of FeZn-compounds should be avoided or at least kept thin as they are relatively brittle which can lead to cracks when the wire is bent. To keep the FeZn layer thin, the wires should only be left in the zinc bath (with a temperature $440\text{--}460 \text{ }^\circ\text{C}$) for a short time.

During the hot zincing, the strength of the wires is somewhat reduced Wyss et al. (1956). Because of this, and also because of the rough surface resulting from the zincing, the wires are often drawn again. This process increases the strength of the wire again and the zinc surface is smoothed. Before drawing, the zinc layer should be thicker than required as part of the zinc layer will be lost during the drawing process.

Blanpain (1964) found that during the re-drawing the brittle Fe–Zn layer may tear especially if the Fe–Zn layer is relatively thick. The resulting gaps will be entered from inside by a steel arch and are not visible from outside as they are closed with zinc. The fatigue strength of these wires is reduced due to the sharp edges of the gaps.

As an alternative to zinc, the wires can be coated with galfan, an eutectoid zinc–aluminium alloy Zn95Al5 (95 % zinc, 5 % aluminium). Nünninghoff and Szczepanski (1987) and Nünninghoff (2003) found that this Zn–Al alloy offers better protection against corrosion than pure zinc. The Zn95Al5 coating also has the further advantage that the brittle Fe–Zn-layer is avoided. However, the Zn95Al5-layer is not as resistant to wear as the pure zinc layer which means that Zn95Al5-coated wires are not as suitable for running ropes.

In Table 1.2, the surface-related mass of zinc coating is listed as an excerpt of Table 1.1 of EN 10244-2 in different classes. For a very thick coating, a multiple

Table 1.2 Surface-related mass of zinc coating (excerpt of EN 10 244-2)

Wire diameter (mm)	Class				
	A (g/m ²)	AB (g/m ²)	B (g/m ²)	C (g/m ²)	D (g/m ²)
$0.20 \leq \delta < 0.25$	30	20	20	20	15
$0.50 \leq \delta < 0.60$	100	70	50	35	20
$1.00 \leq \delta < 1.20$	165	115	80	60	25
$1.85 \leq \delta < 2.15$	215	155	115	80	40
$2.8 \leq \delta < 3.2$	255	195	135	100	50
$4.4 \leq \delta < 5.2$	280	220	150	110	70
$5.2 \leq \delta < 8.2$	290	–	–	110	80

of class A can be used, as for example $A \times 3$. A surface-related zinc mass of 100 g/m^2 means that the thickness of the zinc layer is about 0.015 mm. For the Zn95Al5 coating, EN 10244-2 provides nearly the same surface-related mass for the classes A, B and AB. Unlike EN 10244-2, in Table 1.2 and in the following the symbol δ is used for the diameter of the wire.

1.1.4 Corrosion Resistant Wires

In exceptional cases corrosion resistant wires (stainless steel) have been used as rope wires. Some corrosion resistant steels for wires are listed in Table 1.3 from prEN 10088-3:2001. The steel names of these high alloy steels begin with the capital letter X. The following number gives the carbon content in % multiplied with the factor 100. Then the symbols and the contents in % of the alloy elements are given. For example, for the steel X5CrNiMo17-12-2 the contents are 0.05 % carbon, 17 % chromium, 12 % nickel and 2 % molybdenum.

Corrosion resistant wires for ropes have an austenite structure. Because of this structure they cannot be magnetized which means that the highly effective magnetic method of testing cannot be used to inspect the ropes. It should also be taken into consideration that these steels are not corrosion resistant in all environments.

Like non-alloy carbon steel wires, corrosion resistant steel wires are produced by drawing. The strength range stated in Table 1.3 is valid for drawn wires with a diameter ≥ 0.05 mm. From the different corrosion resistant steels available usually those of medium strength are used. Corrosion resistant wire ropes running over sheaves are not usually as durable as those made of non-alloy carbon steel.

1.1.5 Wire Tensile Test

The tensile test is standardised according to EN 10002-1. The main results of the tensile test provide the measured tensile strength R_m and the total extension ϵ_t . It is not possible to detect precisely the limit where the yielding of the wire begins.

Table 1.3 Strength of drawn wires out of corrosion resistant steel (excerpt of prEN 10 088-3:2001, Table 1.8)

Steel name	Steel number	Strength range (N/mm ²)
X10CrNi18-8	1.4310	600–800
X5CrNiMo17-12-2	1.4401	900–1,100
X3CrNiMo17-13-3	1.4436	1,000–1,250
X1CrNiMoCuN20-18-7	1.4547	1,400–1,700
X1CrNi25-21	1.4335	1,600–1,900

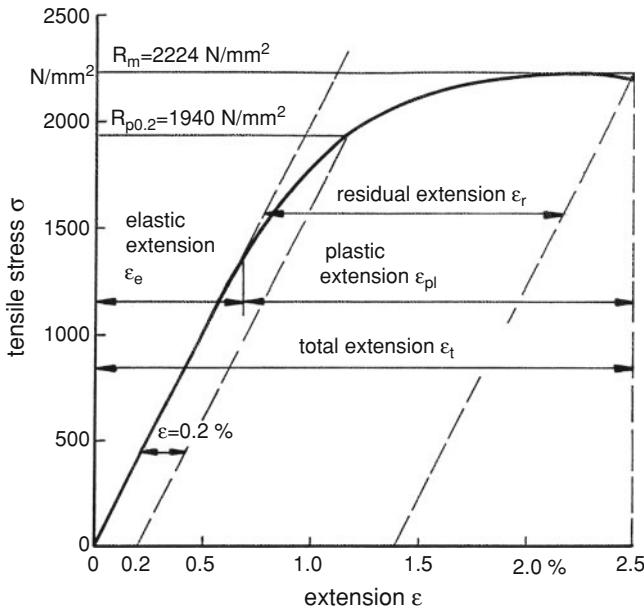


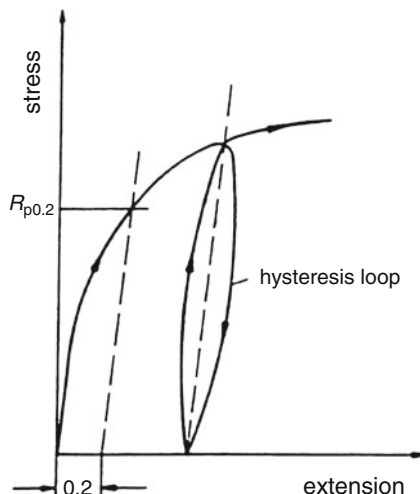
Fig. 1.3 Stress extension diagram of a straightened wire, $\delta = 1.06$ mm

However, the yield strength is defined for a small residual extension. Here, the most frequently used extension is $\epsilon = 0.2\%$ and the stress at this point is the yield strength $R_{p0.2}$. The elasticity module can be evaluated with a special tensile test.

If only the tensile strength R_m has to be evaluated, it can be done without straightening the wire. However, if the different extensions and the yield strength have to be evaluated too, the wire has to be straightened prior to testing. The measurement starts at a stress of about 10% of the tensile strength R_m . Under this stress, the height of the wire bow at a distance measured of 100 mm should be smaller than 0.5 mm.

A typical stress–extension diagram of a straightened wire is shown in Fig. 1.3. It is possible to take the tensile strength R_m , the total extension ϵ_t and the residual extension ϵ_r directly from this figure. To determine the elasticity module E and the yield strength $R_{p0.2}$, the following method has to be used. After a certain yielding,

Fig. 1.4 Evaluation of the yield strength $R_{p0.2}$ according to EN 10002-1



the wire has to be unloaded and loaded again. As a result, a hysteresis loop occurs as seen in Fig. 1.4. A middle line of this hysteresis defines the elasticity module $E = \Delta\sigma/\Delta\varepsilon$. To evaluate the yield strength $R_{p0.2}$, a parallel to the middle line of the hysteresis has to be drawn through the residual extension $\varepsilon_r = 0.2\%$ on the abscissa. Then the yield strength $R_{p0.2}$ is found as an ordinate where the parallel meets the stress extension line.

To determine stresses, strengths and elasticity modules, the cross-section A of the unloaded wire has to be measured very precisely. (Unlike EN 10002-1, the symbol A is used here for the cross-section.) The error in measurement of the cross-section should be 1% at the most. For round wires the cross-section has to be calculated from two wire diameters δ measured perpendicular to each other.

To fulfil this accuracy requirement for the cross-section, the wire diameter δ should be measured with a maximum deviation of 0.5%. With commonly used measuring instruments this accuracy requirement can only be achieved for thicker wires. For thin wires and for profile wires, the cross-section can be evaluated by weighing. With the wire weight m in g, the wire length l in mm and the density ρ , the cross-section is then

$$A = \frac{m}{l \cdot \rho}. \quad (1.1)$$

The density for steel is normally $\rho = 0.00785 \text{ g/mm}^3$. However, because of the great carbon content of wires used for wire ropes it is to use $\rho = 0.00780 \text{ g/mm}^3$.

The total extension of steel wires for ropes amounts to about $\varepsilon_t = 1.5\text{--}4\%$ and the yield strength $R_{p0.2}$ is about 75–95% of the measured tensile strength R_m . For wires taken out of ropes and straightened, the total extension is about $\varepsilon_t = 1.4\text{--}2.9\%$ and the yield strength $R_{p0.2}$ is about 85–99% of the tensile strength R_m , Schneider and Lang (1973).

Because of early yielding in parts of the cross-section of the non-straightened wires and even if the wires are straightened in the normal way, the elasticity module can only be evaluated precisely enough if the wire has yielded before over the whole cross-section. However, that does not mean that the two parts of a broken wire resulting from a tensile test can be used for the evaluation of the elasticity module. These wire parts cannot be used because they may have new inherent stresses due to buckling from the wire breaking impact, Unterberg (1967).

For straightened wires from wire ropes, Wolf (1987) evaluated a mean elasticity module $E = 199,000 \text{ N/mm}^2$. For new wires, Häberle (1995) found the mean elasticity module $E = 195,000 \text{ N/mm}^2$. Together with other measurements—after loading the wires close to the breaking point—a mean elasticity module has been evaluated for the stress field of practical usage. This mean elasticity module of rope wires made of carbon steel in the following is $E = 196,000 \text{ N/mm}^2$.

The elasticity module decreases a little with larger upper stresses. For drawn corrosion resistant wires with the steel number 1.4310 and 1.4401, Schmidt and Dietrich (1982) evaluated the elasticity module $E = 160,000 \text{ N/mm}^2$, respectively, $E = 150,000 \text{ N/mm}^2$.

1.1.6 Wire Endurance and Fatigue Strength

1.1.6.1 Test Methods, Definitions

The wires in wire ropes are stressed by fluctuating tension, bending, pressure and torsion. For a long time wires have been tested in different testing machines under one or a combination of these fluctuating stresses. The tests with combined stresses, especially bending and pressure, have been done with the aim to imitate the stresses in a wire rope. Such tests have been done by Pfister (1964), Lutz (1972), Pantucek (1977) and Haid (1983). However the test results do not come up to expectations, or only imperfectly. The wire endurance for example has even been increased when the wires—loaded by fluctuating bending—are loaded in addition by fluctuating pressure. This effect can probably be attributed to a strain hardening of the wire surface. An overview of the test methods with single or combined stresses has been described by Wolf (1987).

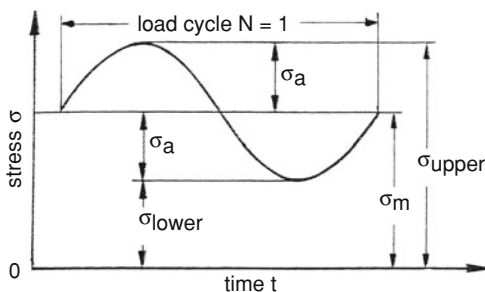
Nowadays, wire fatigue tests are normally tests with only one fluctuating stress—mostly a longitudinal stress. The test methods with fluctuating longitudinal stresses are:

- Tensile fatigue test (wire under fluctuating tensile force)
- Simple bending test (fluctuated bending of the wire over one sheave)
- Reverse bending test (fluctuating bending of the wire over two sheaves or sheave segments)
- Rotary bending test (wire bending by rotating the bent wire).

	tension	simple bending	reverse bending	rotary bending
wire arrangements for testing				
zone of maximum fluctuating stress				
stresses				
stress amplitude	$\sigma_a = \sigma_{t,a}$	$\sigma_a = \sigma_b / 2$	$\sigma_a = \sigma_b$	$\sigma_a = \sigma_b$
middle stress	$\sigma_m = \sigma_{t,m}$	$\sigma_m = \sigma_{t,m} + \sigma_b / 2$	$\sigma_m = \sigma_{t,m}$	$\sigma_m \approx 0$

Fig. 1.5 Wire arrangement for the fatigue tests, zones of maximum stress amplitude in the wire cross-section, stress amplitudes and middle stresses

Fig. 1.6 Stress course during a load cycle

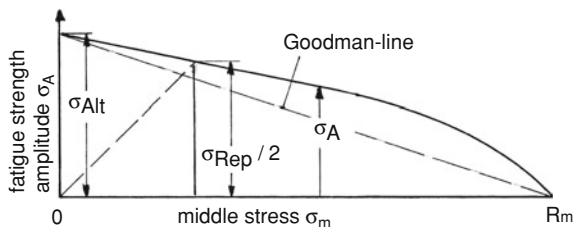


For these test methods, the principle wire arrangement in the test machines is shown in Fig. 1.5. The fluctuating longitudinal stress affects different zones of the wire cross-sections. The wire cross-sections with the zones of the highest fluctuating longitudinal stress are shown in Fig. 1.5 below the wire arrangements. The highest stressed zones are shaded. The highest fluctuating longitudinal stress is taken as the nominal fluctuating stress.

For fatigue strength (infinite life), instead of the stress the symbols σ are written with indices as capital letters.

In Fig. 1.5, the stress amplitude σ_a and the middle stress σ_m are listed for general cases in fatigue tests. Figure 1.6 shows the stress course over one load

Fig. 1.7 Haigh-diagram with some definitions



cycle with the stress amplitude σ_a and the middle stress σ_m for general cases. In a fatigue test, the endurance of the wire is counted by the number of load cycles N it takes.

Figure 1.7 shows a Haigh-diagram with the abscissa for the constant middle stress σ_m and the ordinate for the fluctuating strength σ_A as amplitude around the middle stress σ_m . The special cases alternate and repetitive stresses are inserted. The alternate strength σ_{Alt} is the amplitude for a middle stress $\sigma_m = 0$. The repetitive strength $\sigma_{Rep} = 2\sigma_A$ is the strength range for a middle stress $\sigma_m = \sigma_A$. That means for the repetitive strength σ_{Rep} the lower stress is $\sigma_{lower} = 0$.

The two basic stresses are tensile stress

$$\sigma_t = S/A \quad (1.1a)$$

and bending stress according Reuleaux (1861)

$$\sigma_b = \frac{\delta}{D} \cdot E. \quad (1.1b)$$

In these equations S is the tensile force, A the wire cross-section and δ the wire diameter. D is the curvature diameter of the wire centre on the sheave, which means $D = D_0 + \delta$, with the contact diameter D_0 between wire and sheave. E is the elasticity module.

1.1.6.2 Testing Machines

Tensile fatigue test. Testing methods with fluctuating tensile forces for the testing of materials and components are very commonly used. For rope wires, such tests were started as early as those from Pomp and Hempel (1937). The wire terminations are the main problem in carrying out these tests. If a normal press clamp is used, the wire would mostly break in the clamp.

In order to find out the real endurance or the real tensile fatigue strength of the wire, the wire has to be fastened in such a way that the wire breaks in the free length. To do this, a lamella clamp is used where the tensile force is gradually transferred from lamella to lamella. In addition the wire ends—which are fastened in the clamps—are strain-hardened by a rolling process. During this process, the

fatigue tensile strength of the wire ends is increased slightly over that of the wire in the free length. This hardening of the wire ends together with the lamella clamp provides a high probability that the wire breaks in the free length.

In the fluctuating tensile tests the stress is constant over the whole cross-section as shown in Fig. 1.5. The total stress is composed of a constant middle stress σ_m and a stress amplitude σ_a ,

$$\sigma = \sigma_m \pm \sigma_a.$$

Because of the risk of buckling, the stress amplitude σ_a normally should be smaller than the middle stress σ_m . Tensile fatigue tests with a compressive section can only be done with very short wires.

Simple bending test, one sheave. In this method the wire moves over one sheave forwards and backwards, Woernle (1929), Donandt (1950) Müller (1961) etc. The wire is loaded by a constant tensile force S and a fluctuating bending. For this test,

$$\begin{array}{ll} \text{the middle stress is} & \sigma_m = \sigma_{t,m} + \sigma_b/2 \\ \text{and the stress amplitude is} & \sigma_a = \sigma_b/2. \end{array}$$

The fluctuating bending stress σ_b exists only in one small segment of the wire cross-section in the outside wire bow, as shown in Fig. 1.5. In the other small wire segment lying on the sheave, the bending stress is compressive. There is no wire breakage to be expected from this stress, especially if this stress is reduced—as is normal—by a tensile stress $\sigma_{t,m}$.

Reverse bending test, two sheaves. In this method the wire first moves over one sheave and then over a second sheave with reverse bending forwards and backwards, Schmidt (1964). In another method the wire is bent and reverse bent over sheave segments, Unterberg (1967). For both methods, the wire is loaded by a constant tensile force S and a fluctuating bending. For the wire reverse bending test

$$\begin{array}{ll} \text{the middle stress is} & \sigma_m = \sigma_{t,m} \\ \text{and the stress amplitude is} & \sigma_a = \sigma_b. \end{array}$$

The fluctuating bending stress σ_b exists only in two small segments of the wire cross-section, as shown in Fig. 1.5.

The pressure between the wire and the single sheave or two sheaves is small and can be neglected. The advantage of this bending test method is that the tensile stress σ_m , and the bending stress can be chosen quite freely. There should be only a tensile force chosen that is large enough to ensure that the wire lies securely on the sheave in contact with the bow.

Rotary bending test. In a rotary bending machine, the wire is bent in a free bow around its own axis. By turning the wire, the stress in an outer fibre of the bent wire changes from compressive to tensile stress and back again. In one turn of the wire around the wire axis, each of the outside fibres of the wire is stressed by a complete cycle of longitudinal stress. The stress amplitude $\pm\sigma_a$ decreases linearly from the

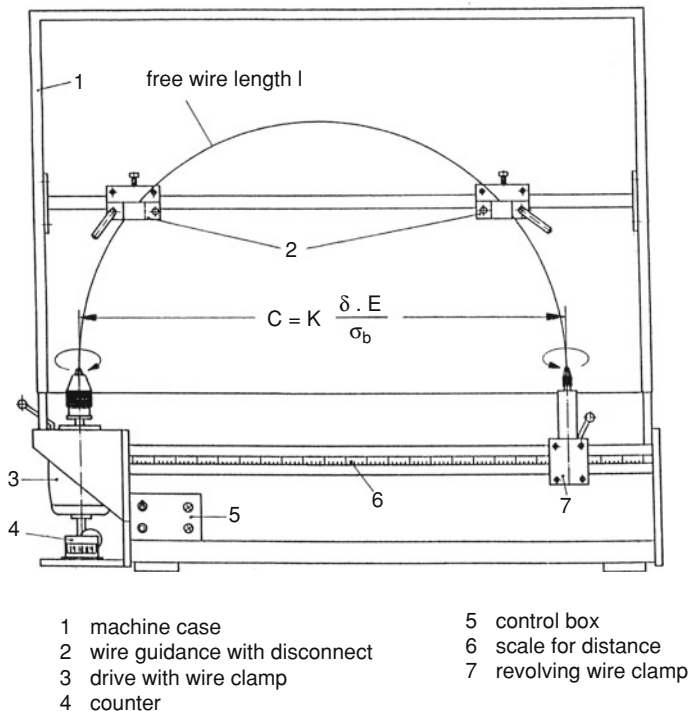


Fig. 1.8 Stuttgart rotary bending machine, Wolf (1987)

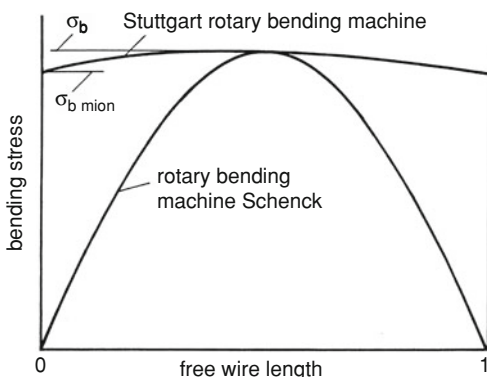
outside of the wire to the wire axis. Therefore—as shown in Fig. 1.5—the maximum (nominal) stress exists only in a small ring zone. The advantage of rotary bending tests is that it can be done very quickly with a frequency of 50 and more turns/second.

Older bending machines which rotate the wire are the Haigh/Robertson machine, NN (1933), the Schenck machine, Erlinger (1942) and the Hunter testing machine, Votta (1948). These machines have the disadvantage that of the whole wire length only a small part is bent with the maximal (nominal) bending stress σ_a . The newer Stuttgart rotary bending machine, Fig. 1.8, avoids this disadvantage, Wolf (1987). In this machine the wire has almost the same bending stress for the whole bending length. The wire bow between the two parallel axes of the rotating wire terminations with the distance C is nearly a circular arc. One of the two wire terminations is driven.

One slight disadvantage of both the Hunter testing machine and the Stuttgart testing machine is that the bending length is determined by the chosen bending stress. The great advantage of these two machines is, however, that the bending stress is determined by geometric dimensions only and these can be measured very simply.

For the Stuttgart rotary bending machine, the bending length (free wire length between the terminations) l is only slightly larger than the circle bow length $C\pi/2$.

Fig. 1.9 Bending stress along the wire bending length, Wolf (1987)



Therefore the bending stress on the terminations is only slightly smaller than in the middle of the bending length. This means that wire breakage in or close to the terminations is almost certainly avoided and the bending stress is nearly constant over the whole of the bending length. Figure 1.9 shows the bending stress along the bending length in the Schenck machine and the Stuttgart machine.

The bending stress amplitude in the middle of the bending length l (free wire length) is

$$\sigma_a = \sigma_b = \frac{k \cdot \delta \cdot E}{C} \tag{1.1c}$$

This bending stress, the maximum stress, is taken as the nominal bending stress of the wire in the Stuttgart rotary bending machine. The minimum bending stress on both of the wire terminations is

$$\sigma_{b,min} = \frac{k_0 \cdot \delta \cdot E}{C} \tag{1.1d}$$

For both equations: k and k_0 are constants in Table 1.4, δ is the wire diameter, E is the elasticity module and C is the distance between the parallel axes of the wire terminations.

Furthermore, in Table 1.4, the ratio of the minimum and the nominal bending stress $\sigma_{b,min}/\sigma_b$ is listed. For his tests Wolf (1987) used the ratio $l/C = 1.6$ instead of $\pi/2$ with the minimum stress $\sigma_{b,min} = 0.883 \cdot \sigma_b$ or a 11.7 % smaller stress at

Table 1.4 Constants k , k_0 and the ratio of wire bending stresses $\sigma_{b,min}/\sigma_b$ in the Stuttgart rotary bending machine

l/C	$\pi/2$	1.58	1.59	1.60
k	1.0	1.008	1.017	1.026
k_0	1.0	0.969	0.936	0.906
$\sigma_{b,min}/\sigma_b$	1.0	0.961	0.921	0.883

the wire terminations. Later on, it was shown in a great number of tests that in practically all cases where the ratio is $l/C = 1.58$ the wires break in the free wire length. Since 1990, therefore, all the tests with the Stuttgart rotary bending machine have been done with the ratio $l/C = 1.58$. Because of the very small stress reduction of only 4 % at the wire terminations, the maximum bending stress amplitude can be considered as stress over the whole bending length.

The middle stress is practically $\sigma_m \approx 0$. As an example for the ratio $l/C = 1.58$ and the bending stress $\sigma_b = 600 \text{ N/mm}^2$, the compressive stress is only $\sigma_m = 965 \cdot (\delta/C)^2 = 0.0089 \text{ N/mm}^2$.

1.1.6.3 Wöhler Diagram

Wolf (1987) did a great number of fatigue tests with the simple Stuttgart rotary bending machine using wires taken from wire ropes. He straightened all the wires in the same way with a special device before conducting the fatigue tests. Figure 1.10 shows the numbers of bending cycles N resulting from a series of tests with wires of 1 mm diameter taken from Seale ropes for varying amplitude of the rotary bending stress (alternate bending stress on the whole circumference) $\sigma_{rot} \approx \sigma_{b,alt}$. For the logarithm normal distribution of the bending cycles N , the standard deviation increases in the usual way with decreasing bending stress amplitude σ_{rot} .

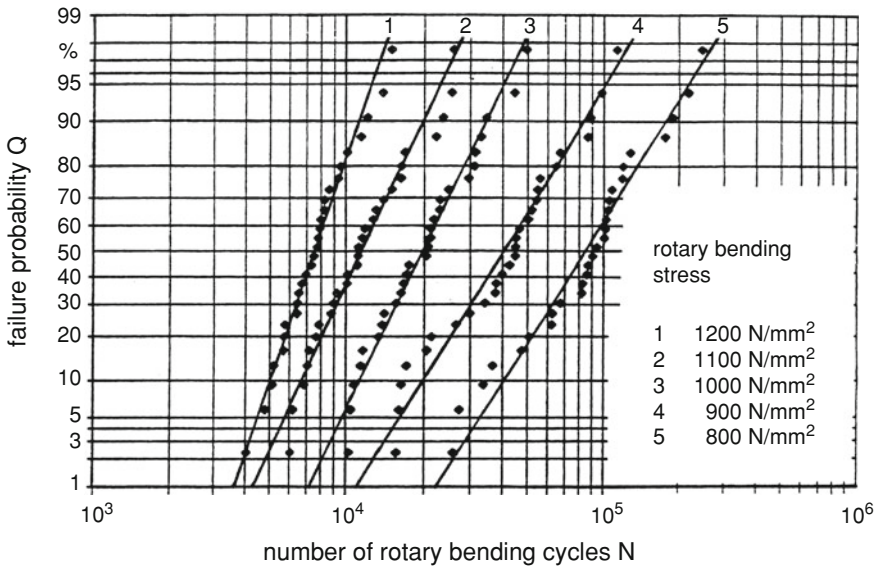


Fig. 1.10 Number of bending cycles for straightened wires, diameter $\delta = 1 \text{ mm}$ from Seale ropes, Wolf (1987)

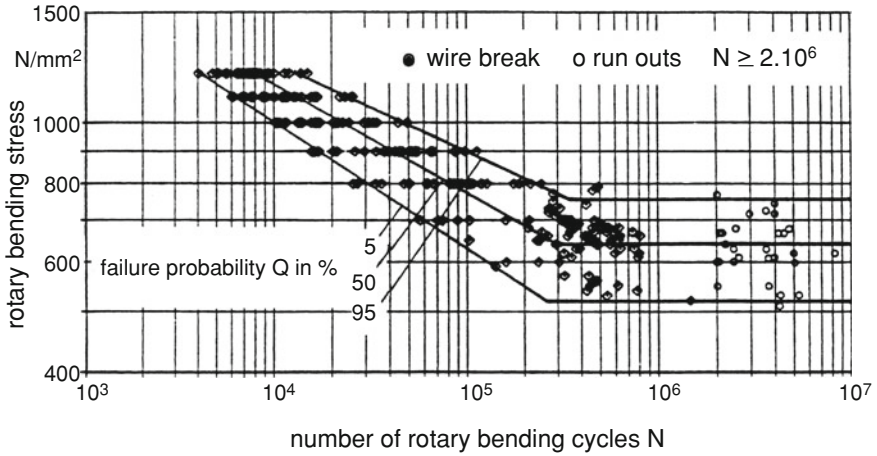


Fig. 1.11 Wöhler-diagram for wires, diameter $\delta = 1$ mm, from Seale ropes, Wolf (1987)

Wolf (1987) transferred this number of bending cycles to a Wöhler diagram as shown in Fig. 1.11. In the Wöhler diagram, he drew a line for the mean number of bending cycles and lines for 5 and 95 % of the breaking probability. The mean rotary bending strength (infinite life fatigue strength) for wires in 12 Seale ropes is $\bar{\sigma}_{Rot} = \pm 640 \text{ N/mm}^2$. In the Wöhler diagram shown in Fig. 1.12, the number of rotary bending cycles for 0.95 mm diameter wires taken from 20 Warrington ropes has been transferred in the same way. The mean rotary bending strength is $\bar{\sigma}_{Rot} = \pm 640 \text{ N/mm}^2$. The deviation for the number of rotary bending cycles N and

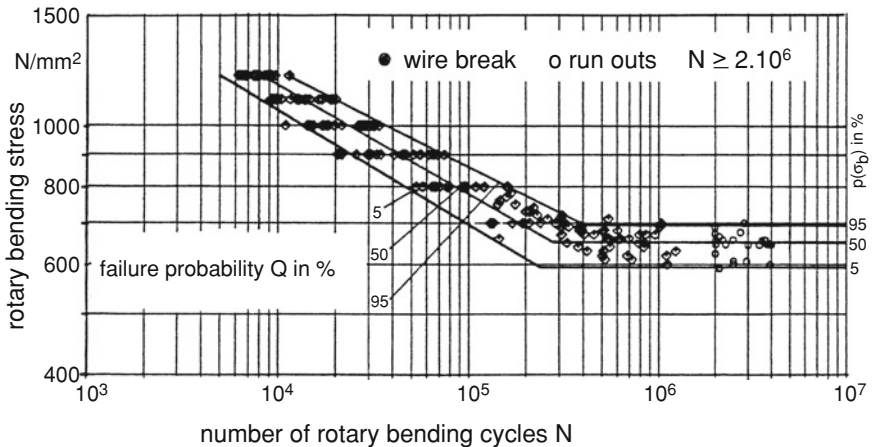


Fig. 1.12 Wöhler-diagram for wires, diameter $\delta = 0.95$ mm, from Warrington ropes, Wolf (1987)

for the rotary bending strength σ_{Rot} is much smaller than in the Wöhler-diagram in Fig. 1.11.

For both wires, the transition from the finite to the infinite life strength lies at the number of bending cycles of about $N = 300,000$. This is situated in the range between $N = 150,000$ and $N = 500,000$ that Hempel (1957) and Unterberg (1967) previously found in rotary bending and fluctuating tensile tests.

1.1.6.4 Finite Wire Endurance

For straightened wires taken from wire ropes, Wolf (1987) evaluated a mean number of rotary bending cycles for wires with diameter $\delta = 0.8\text{--}1.0$ mm

$$\lg \bar{N} = 21.708 - 5.813 \cdot \lg \sigma_{\text{rot}}. \quad (1.2a)$$

Briem (2000) and Ziegler et al. (2005) have also done a great number of fatigue tests with a Stuttgart rotary bending machine. In both series of fatigue tests the wires were new (not taken from a rope). They were only straightened before the tests. The following endurance equations were found by regression calculation using the test results in the finite life region Briem (2000):

$$\lg \bar{N} = 13.74 - 3.243 \cdot \lg \sigma_{\text{rot}} - 0.30 \cdot \lg \delta - 0.74 \cdot \lg \frac{R_0}{1,770} \quad (1.2b)$$

for wire diameters $\delta = 0.8 - 2.2$ mm and

for nominal strength $R_0 = 1,770; 1,960; \text{ and } 2,160$ N/mm².

Ziegler et al. (2005) found

$$\lg \bar{N} = 12.577 - 3.542 \cdot \lg \sigma_{\text{rot}} - 0.072 \cdot \lg \delta + 0.612 \cdot \lg R_m \quad (1.2c)$$

for wire diameters $\delta = 0.8 - 1.8$ mm and

for nominal strength $R_0 = 1,370 - 2,160$ N/mm².

The influence of the diameter of the wire and its tensile strength is different in the two equations. Briem (2000) even found that the number of rotary bending cycles is reduced when the tensile strength is increased. Only the bending stress as a main influence may be considered as a common result because of the relatively small range of wire diameters and tensile strength tested. Thus, the mean number of rotary bending cycles for new wires with a diameter $\delta = 1$ mm and tensile strength $R_0 = 1,770$ N/mm² is

$$\lg \bar{N} = 14.152 - 3.393 \cdot \lg \sigma_{\text{rot}}. \quad (1.2d)$$

According to these equations with rotary bending stress $\sigma_{\text{rot}} = \pm 900 \text{ N/mm}^2$ as an example, the mean number of bending cycles for new wires is $\bar{N} = 13,000$ (1.2d) and for wires taken from ropes $\bar{N} = 34,000$ (1.2a). As an additional test result Briem (2000) found a 19 % smaller endurance for zinc-coated wires than for bright wires.

The endurance of the wires depends on the size effects of the two parameters, the wire diameter and the wire length where the wire is stressed (bending length or stressing length). The wire diameter cannot change in isolation. That means, with the wire diameter, the other parameters which influence endurance will always also be changed. Therefore, to find out the influence of the wire diameter, the other parameters which influence wire endurance should be kept as similar as possible and there should be a wide range of different wire diameters. As already mentioned, the influence of the wire diameter on the wire fatigue endurance is only known in a first form using the given results represented by Eqs. (1.2b) and (1.2c).

The influence on wire endurance of the wire length, which is the other parameter affecting the size, can be evaluated reliably by conducting tests with parts of one and the same wire and theoretically with the help of the reliability theory. A series of wire fatigue tests done by bending over one sheave have been used to evaluate the influence of the bending length, Feyrer (1981). The wire diameter is $\delta = 0.75 \text{ mm}$, the measured tensile strength is $R_m = 1,701 \text{ N/mm}^2$. The wire bending diameter over the sheave is 115.75 mm ; with these conditions the wire bending stress is $\sigma_b = 1,270 \text{ N/mm}^2$. The constant tensile stress from a loaded weight is $\sigma_{t,m} = 400 \text{ N/mm}^2$. For the test bending over one sheave, the wire is loaded by

$$\begin{aligned} \text{the middle stress } \sigma_m &= \sigma_{t,m} + \sigma_b/2 = 1,035 \text{ N/mm}^2 \\ \text{and the stress amplitude } \sigma_a &= \sigma_b/2 = 635 \text{ N/mm}^2. \end{aligned}$$

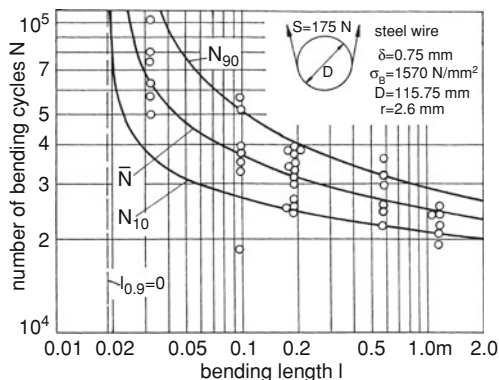
The test results are shown in Fig. 1.13. Together with the points taken from the test results, the figure shows the curves calculated for the mean number of bending cycles \bar{N} and the limiting number of bending cycles for 10 and 90 % probability. The calculation of these curves is based on the reliability theory.

The survival probability is the smaller the larger the bending length l (as a string of bending lengths l_0) of the wire being considered is. For a given survival probability P_0 of the wire bending length l_0 , the survival probability $P(l)$ of the wire with the bending length l is

$$P(l) = P_0^{(l-\Delta l)/(l_0-\Delta l)}. \quad (1.2e)$$

The bending lengths l and l_0 are the theoretical lengths without considering the bending stiffness of the wire. These lengths would occur for bending limp yarn. For the wire near the sheave, the fluctuating bending stress is small. The short bending length Δl is introduced to take this into account. Δl is the shorter part of the bending length having the smaller radius difference of the rope curvature than

Fig. 1.13 Number of bending cycles of a wire with different bending lengths, Feyrer (1981)



90 % of the total one. The curves in Fig. 1.13 are calculated using standard deviation $\lg s = 0.086$ of the logarithm normal distribution derived from the 13 bending cycles found for the wire bending length $l_0 = 2 \times 96$ mm. More information on this method of calculation is presented for wire ropes in Sect. 3.2.2 where the influence of the bending length is of practical interest.

In principle, the findings of Luo's (2002) tensile fatigue tests produced the same result. For his tests with different stressing lengths, he used a wire with diameter $\delta = 2$ mm, made of material X5CrNi18-10, No. 4301 and having tensile strength $R_m = 840$ N/mm². He did 60 tensile fatigue tests for each of the wire lengths $l = 25, 125$ and 250 mm with the middle stress $\sigma_{t,m} = 356.5$ N/mm² and the stress amplitude $\sigma_{t,a} = \pm 290$ N/mm² only a little above the infinite tensile fatigue strength. For the short wire length of 25 mm there are five run-outs with more than $N = 2 \times 10^6$. For the wire length $l = 250$ mm, the parameters for the logarithm normal distribution are the mean number of tensile cycles $\bar{N} = 238,000$ and the standard deviation $\lg s = 0.136$.

1.1.6.5 Infinite Wire Endurance

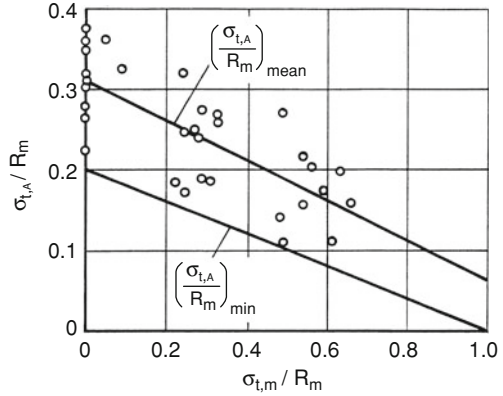
The fatigue strengths (infinite life fatigue strengths) have been evaluated using Wöhler-diagrams. As before, the fatigue strengths are characterized by indices in capital letters and the stresses by indices with small letters.

For his tensile fatigue tests, Unterberg (1967) used short wire pieces with a length between 15 and 35 mm so that he could start—without the risk of buckling—with the middle stress $\sigma_m = 0$. The test results are shown in Fig. 1.14. The mean relative tensile strength amplitude is

$$\bar{\sigma}_{t,A}(\bar{R}_0 = 1,770; \bar{\delta} = 2.7) = 0.313 \cdot R_m - 0.249 \cdot \sigma_{t,m} \quad (1.3)$$

and related to the lower stress $\sigma_{t,lower}$

Fig. 1.14 Tensile strength amplitude for wires with diameters $\delta = 1.17\text{--}4.2$ mm and nominal tensile strength $R_0 = 1,570\text{--}1,960$ N/mm², Unterberg (1967)



$$\bar{\sigma}_{t,A}(\bar{R}_0 = 1,770; \bar{\delta} = 2.7) = 0.251 \cdot R_m - 0.199 \cdot \sigma_{t,lower}.$$

The standard deviation is large. As a lower limit for this tensile strength amplitude Unterberg gave the Goodman-line

$$\sigma_{t,A,min}(\bar{R}_0 = 1,770; \delta = 2.7) = 0.2 \cdot R_m - 0.2 \cdot \sigma_{t,m}.$$

From his reverse bending tests, Unterberg (1967) found the mean relative bending strength amplitude

$$\bar{\sigma}_{b,A}(\bar{R}_0 = 1,770; \delta = 2.7) = 0.271 \cdot R_m - 0.170 \cdot \sigma_{t,m}. \quad (1.3a)$$

The bending strength amplitude (1.3a), is a little smaller than the tensile strength amplitude (1.3a). The deviation of the bending strength is also a little smaller than that of the tension strength to be seen in Fig. 1.14. The bending length $l = 80$ mm in relation to the smaller tensile stressing length $l = 15\text{--}35$ mm in case of tensile fatigue tests may be the reason for that. For the middle stress $\sigma_m = 0$ and the tensile strength $R_m = 1,770$ N/mm², the mean bending strength amplitude (alternate bending amplitude) is $\bar{\sigma}_{b,Alt} = 0.271 \cdot R_m = 480$ N/mm² and the mean tensile strength amplitude is $\bar{\sigma}_{t,Alt} = 0.313 \cdot R_m = 554$ N/mm².

In any case, if the theory of stress gradient effect is valid, there should be an advantage for the bending strength. However there is no such advantage to be found in the test results. According to the theory of stress gradient effect, Faulhaber (1933), Hempel (1957) and Siebel (1959), the fatigue strength of the wire should be the greater, the greater the stress gradient in the wire cross-section is. The theory of stress gradient means that if the stress gradient is large, the outer highly stressed lay can be supported by the less stressed layer below. However the bending strength amplitude according to (1.3a) is not at all greater than the tensile

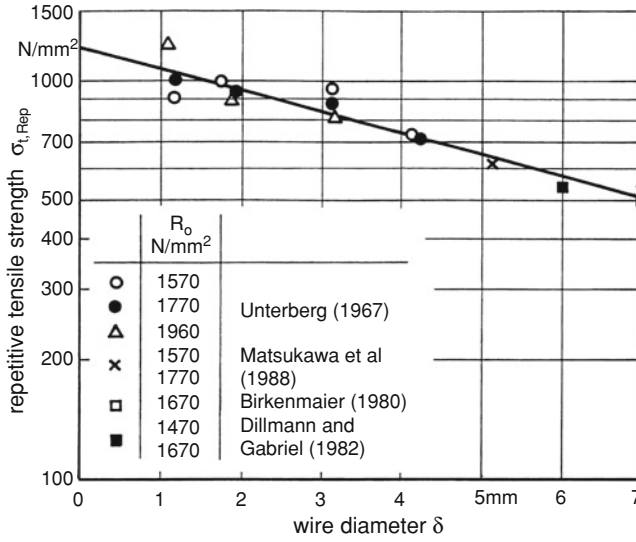


Fig. 1.15 Repetitive tensile strength for different wire diameters δ for a mean nominal tensile strength $\bar{R}_0 = 1,720 \text{ N/mm}^2$

strength amplitude according to (1.3a) although there the stress gradient is 0. Because of this and also as a result of other observations, Unterberg (1967) stated that the theory of stress gradient does not exist for rope wires.

In Fig. 1.15 the influence of the wire diameter δ is shown using the results from different authors. In this figure the mean repetitive fatigue strength $\bar{\sigma}_{t,Rep} = 2 \cdot \bar{\sigma}_{t,A}$ is used. As a reminder, repetitive strength means that the middle stress is $\sigma_{t,m} = \sigma_{t,A}$ and the lower stress is 0. As an equation using the results in Fig. 1.15, the repetitive strength is expressed as

$$\sigma_{t,Rep} = 2 \cdot \sigma_{t,A} (\sigma_{lower} = 0) = 1,200 \cdot e^{-0.122 \cdot \delta} \tag{1.3b}$$

The influence of the other size parameter, the stressed wire length l , can be evaluated for the fatigue strength amplitude in the same way as for a number of load cycles N if the standard deviation of the fatigue strength amplitude for one and the same wire were known.

The influence of the tensile strength R_m on the rotary bending strength σ_{Rot} is shown in Fig. 1.16 from Wolf (1987). In this diagram, Wolf put in the results gained by Buchholz (1965) wires with lower tensile strength to get an overview for a greater strength range. For small tensile strengths, the rotary bending strength increases almost proportionally with tensile strength R_m . The rotary bending strength does not increase as much for rope wires (wires with tensile strength between 1,300 and 2,200 N/mm²). According to Wolf (1987), it is

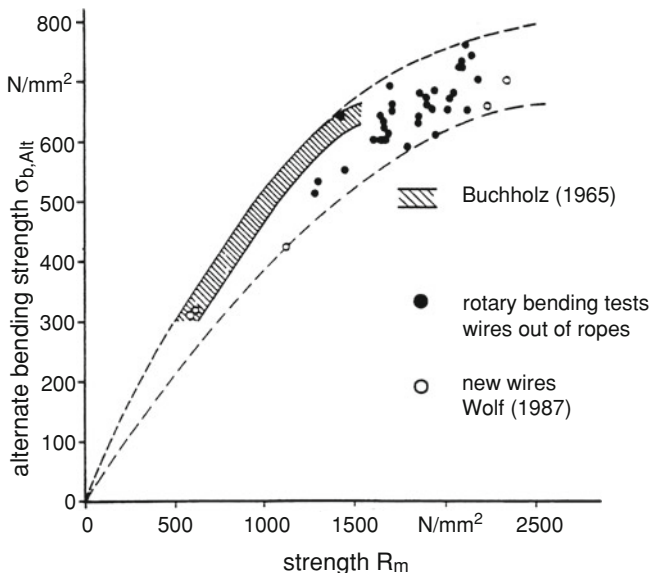
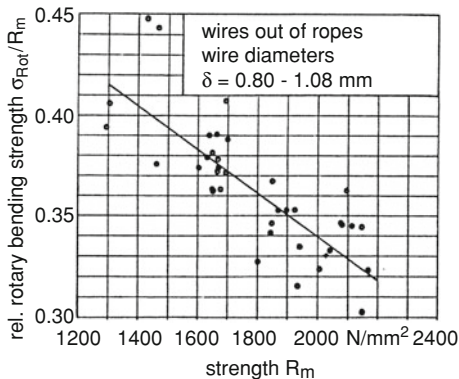


Fig. 1.16 Alternate strength σ_{Alt} for wires with a great range of measured tensile strength R_m , Wolf (1987)

Fig. 1.17 Relative rotary bending strength σ_{Rot}/R_m for rope wires, Wolf (1987)



$$\sigma_{Rot} = 0.334 + 0.173 \cdot R_m. \tag{1.3c}$$

That means that with increasing tensile strength R_m , the relative rotary bending strength σ_{Rot}/R_m will be reduced as seen in Fig. 1.17, Wolf (1987).

Ziegler et al. (2005) and Wehking (2005) evaluated the rotary bending strength from tests with the Stuttgart rotary bending machine to be

$$\lg \sigma_{Rot} = 1.411 + 0.396 \cdot \lg R_m - 0.128 \cdot \lg \delta. \tag{1.3d}$$