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Flexural Testing of Weld Site and HVOF Coating Characteristics

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Preface

Defect sites can be created in solid substrates when subjected to a deformation or a thermal distortion and the property variation occurs in defected regions, such as elastic module, density, Poisson's ratio variations in the heat affected zone of the welded, or locally heat treated sites in the solid substrates. In addition, during thermal processing, the properties of the substrate material change because of the temperature dependence. Flexural motion of the substrate material, subjected to thermal processing, provides useful information about the level distortion in the substrate material, which is more pronounced as the temperature in the substrate material increases. However, nonlinear response of the elastic modulus, due to the temperature filed, changes amplitude and time shift of amplitudes due to residual thermal distortions. Determining magnitude and phase lagging of the maximum amplitude provides information about the heating rate, heat source speed, size of heat affected zone, etc. Therefore, flexural characteristics of the substrate material can be associated with the source of distortion, which in turn enables to control the process for improved performance and the end product quality. Nondestructive testing of coat layer thickness is very fruitful to assess characteristics of thermal spray coating process. Monitoring the flexural motion enables to correlate amplitude and frequency of the wave with geometric configuration of coating, such as coating thickness, coating uniformity, tapering, etc.

High velocity oxy-fuel (HVOF) spraying technique finds wide application in industry because of low cost and operational easiness. Although HVOF coating protects the substrate surfaces from high temperature and wear environments, presence of porous and oxide inclusions in the coating reduces mechanical strength of the resulting coating. Thermal integration of coating is possible by introducing control melting using high energy beams, in which case, pores and voids can be eliminated in the coating. Mechanical, metallurgical, and morphological characteristics of coating are highly affected by melting rates and cooling periods of the coating. In thermal processing, such as welding, property changes in the welded regions are important to secure sound and quality welds for the practical applications. Optimization of welding process, utilizing flexural characteristics of the welded material improves mechanical and metallurgical properties and assists to produce desirable welds for the particular applications.

In this book, thermal analysis and flexural motion of weld joins and thermally processed parts are presented. Analytical and numerical simulations of flexural characteristics of cantilever beams are provided in detail. Several heating situations and flexural behavior of the thermally treated beams are included. Various studies for high velocity oxy-fuel coating are introduced and coating characteristics prior and after controlled melting by a laser beam are discussed. Changes in metallurgical, mechanical, and morphological characteristics of coating are described for various types of coating powders. However, some cases related to modeling of flexural characteristics and coating properties are not presented in this book due to space limitations and, therefore, these cases are left for the future treatments.

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Chapter 1
Introduction

Abstract Flexural behavior of mechanical systems can be used to identify the system characteristics including parts and system failure. In this chapter, the basics of flexural characteristics of a simple system are presented and the formations associated with the flexural behavior of the system are presented. Heating effects and thermal deflection in terms of flexural motion are also included.

Keywords Flexural motion · Mechanical parts · Thermal analysis

The flexural motion of the mechanical parts provides physical insight into the structural disintegrations and enables to assess the presence of defect sites in the parts. On the other hand, thermal spray coating is a widely used in industry to protect the metallic surfaces from the high temperature, wear, and corrosive environments. The flexural characteristics of the beams, due to structural changes and heating, and mechanical, morphological and metallurgical properties of the high velocity oxy-fuel coatings are presented under the appropriate sub-headings in in line with the previous studies [1–57].

1.1 Flexural Characteristics

The flexural characteristics of a solid plate depend on the mechanical properties of the plate material, such as density, stiffness, and damping factor, external applied force, such as duration and amplitude, and geometric arrangement of the plate, such as tapered, parallel plate geometry with presence of holes, etc. Modification of any one of these properties results in alteration of the wave characteristics of the motion. This situation is often resulted when the flat solid is heated at one side during the welding or heat treatment process. In this case, the bar resembles the cantilever assembly with non-uniform mechanical properties due to heating. However, the size of the heat affected zone in the treated bar may be correlated with the wave characteristics of the flexural motion of the bar when the impact force is introduced from the free end of the bar. In addition, the characteristics of

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flexural motion for a cantilever plate can be used to identify the size of the heated zone of the cantilever plate subjected to the heating process. This is because of the temperature dependent properties of the substrate material such as temperature dependent modulus of elasticity. This situation is more pronounced as the temperature in the plate increases. The nonlinear response of the elastic modulus, due to the temperature filed, changes the amplitude of the oscillation during the flexural motion and the time shift in the amplitudes because of heating and no heating situations can provide useful information on the heating rate. Therefore, determining the amplitude difference and the time shift between the maximum amplitudes provide the data on the heating rate, particularly heat source speed and size of the heat affected zone when the source moves along one side of the plate.

The flexural characteristics including the time period of oscillation and the maximum amplitude of motion, can lead to improving quality of heated products, which are affected by the thermal distortion generated and the residual stresses. Investigating the oscillation characteristics of heated structures, and relating the resulting temperatures fields with the flexural characteristics, is an increasingly spreading approach by researchers to demonstrate the influence of heating parameters. The flexural wave generated in the substrate material is modified with the irregularities within the substrate material and the temperature dependent mechanical properties. Therefore, the local heating in the substrate material alters the mechanical properties as well as the resulting flexural characteristics when the substrate material is subjected to the flexural load. Consequently, measurement or prediction of the flexural characteristics enables to predict the heating location and the size of the heat affected zone in the substrate material. In addition, the structural irregularity generated at the workpiece surface through coating can be analyzed from the flexural characteristics of the coated substrate. In this case, coating thickness and its variation can be identified from the amplitude and frequency of the flexural motion of the coated specimen.

Considerable research studies were carried out to examine the flexural characteristics of the solid parts subjected to the various heating and process conditions. The propagation of flexural wave along a clamped supported beam was studied by Shim et al. [9]. They showed that the fringe pattern produced was the loci of constant out-of-plane displacement derivatives with respect to the direction of image shear, which were integrated to yield the instantaneous out-of-plane displacement induced by the flexural wave. The sub-ablation optical excitation of flexural vibration in cantilevers and suspended truck-wheel rim due the laser pulse were investigated by Philp et al. [10]. The results indicated that the technique employed could be used as non-destructive tool for mechanical testing of cantilever beams. The elastic properties of the solid substrates due to laser beam excitation were investigated by Bardenstein et al. [11]. They showed that the amplitude of thermoelastic flexural wave as well as the amplitude of longitudinal pulse was proportional to the energy absorbed by the beam, which in turn enabled to determine elastic constants of the beam. A theoretical study of laser generated transient Lamb waves in orthotropic plates was carried out by Cheng and Berthelot [12]. They introduced quantitative analysis for non-contact and non-destructive detection of the elastic stiffness properties of machine-made paper by a laser generated Lamb wave technique. Flexural waves transmitted by rectangular piezoceramic transducers were examined by Veidt et al. [13]. They showed that the assumption of a uniform contact pressure distribution between the transducer and the plate could accurately predict the frequency spectrum and time domain response signals of the propagating waves along the main axes of the transmitter element. The defective bonding areas within the brazed adjoining contact surfaces of composite ceramic-metal plates were studied by Conrod and Sayir [14] using a flexural waves and holography. They indicated that the experimental interferograms were matched precisely with analytic results derived from Midlin's plate equations. An investigation into flexural waves generated on silicon nitride thin films was carried out by Hang and Amit [15] using piezoelectric plates attached to silicon bulk substrate. They demonstrated that the predictions agreed well with the experimental results. Laser generated flexural acoustic waves traveling along the tip of a wedge were studied by Jia et al. [16]. They made the comparison between the laser-generated antisymmetrical flexural model and the flexural Lamb wave propagating along the edge of the plate. Yilbas et al. [17] studied the stress generation and flexural motion during the laser heating process. They showed that the stress level is higher once the displacement of the surface is high.

The flexural characteristics can be associated with the thermal process parameters of the solid parts when subjected to the heating such as laser heating. Considerable research studies were carried out on wave generation in solids due to laser irradiating pulse. A theoretical investigation of the dynamic thermoelastic response of various thickness workpieces due to Q-switched laser pulses was done by Cheng et al. [18] They showed that as the laser pulse duration decreased or workpiece thickness increased, the dynamic wave behavior gradually became apparent. Ultrasound radiation into water by a Lamb wave device using a piezoelectric ceramic with spatially varying thickness was investigated by Motegi [19] He indicated that there was a small velocity gap deriving from the inhomogeneity, but radiation efficiency was much higher than that in the case of a flat plate. A hybrid elastic wave method was employed by Wo and Lui [20] to determine the anisotropic constants of a thin fiber-reinforced composite plate. They noted that with the hybrid method, some of anisotropic constants could be measured accurately with well-developed bulk wave ultrasonics. The propagation velocities of laser generated ultrasonic plate waves were measured by Ridgway et al. [21]. They compared the laser-ultrasonic measurements of phase velocities for the fast-propagating vibration mode with contact transducer based measurements. The propagation of Lamb waves in multilayered plates was examined by Grondal et al. [22]. They indicated that the Lamb waves and particularly their velocities were very sensitive to defects in solids. Laser generation of Lamb waves in copy paper was investigated by Johnson et al. [23]. They observed the significant statistical variations of the wave forms and quantified the variations by means of cross-relation techniques. Propagation characteristics of plate-mode waves on a wedge shaped substrate were investigated by Wakatsuki et al. [24]. They indicated that the process in which two wave modes were reflected from the front end of the substrate was degenerated into the Rayleigh mode.

The thermal stresses generated during the laser quenching were investigated by Wang et al. [25]. They indicated that the residual stress developed in the surface vicinity resulted in the hardening of the surface. The experimental and numerical studies of fracture initiation in thin aluminum oxide ceramics during laser cutting were carried out by Li and Sheng [26]. They introduced plane stress model when predicting the stresses along the cut edges. They showed that low cutting speed reduces the stress level and minimizes the possible fracture initiation at the cut edges. An analytical solution of a two-dimensional thin coating attached to a substrate was obtained by Elperin and Rudin [27]. They indicated that the temperature difference across the coating generated excessive stress levels, which in turn caused inelastic deformation of the coating. Modest [28] investigated the elastic and viscoelastic stresses during laser drilling of ceramics. He indicated that the material softened in the region closer to the melt zone.

The laser-induced waves in solids received considerable attention, since it served as a nondestructive tool for materials evaluation. Several analytical methods were introduced when modelling the thermoelastic process that took place during the laser heating pulse. Some of these include Green function method, and Laplace and Henkel transformations. However, the thermal stresses developed in the substrate vary with time and it is, in general, difficult to present in a simple mathematical function. Therefore, the analytical solution to the transient problem with nonordered input load becomes very difficult to obtain. Consequently, the general trend is to solve the governing equations numerically. Dubois et al. [29] investigated numerically the laser thermoelastic generation of ultrasound in an orthotropic medium. They compared the predictions with the experimental results and indicated that their predictions agreed well with the experimental findings. They further argued that the model developed could be used to optimize the laser generation of ultrasound in various materials. Cheng et al. [30] studied the dynamic thermoelastic response of pulsed photothermal deformation deflection detection for Q-switched laser pulses. They indicated that when the laser pulse rise time decreases in the order of 10 ns or less, the pulsed photothermal deformation deflection signal reflected a totally dynamic wave behavior. The laser-induced waves such as Lamb waves can also be used to determine the properties of the substrate material. The Lamb wave is generated due to normal and shear loads applied to the surface of the half-space. Dewhurst et al. [31] introduced a Lamb wave technique to describe the measurement of the film thickness on the substrate material.

When a high-intensity laser beam interacts with a solid phase, change occurs in the surface region of the substrate material which, in turn, results in a vapour pressure acting on the surface. This generates flexural waves in the substrate material. Consequently, a stress field due to flexural motion of the workpiece is developed. Considerable research has been carried out to explore the flexural motion generated in the workpiece by laser pulse operation. The propagation of Lamb waves in a multilayer assembly was investigated by Sebastian et al. [32]. They showed that Lamb wave characteristics were very sensitive to any defects in the substrate material. Yilbas and Faisal [33] investigated laser-induced flexural waves in steel. They indicated that the flexural wave motion was affected by end reflections of the waves.

The change of direction of flexural ultrasonic waves, propagating due to modulating excitation frequency, was investigated by Loh and Ro [34]. They showed that modulation of excited frequency changed the direction of wave propagation. The wave decomposition technique was introduced by Szwerc et al. [35] to separate longitudinal and flexural wave intensities. They indicated that the method could be implemented to determine the power flow fields on structures, which were vibrating with these two wave types.

1.2 High Velocity Oxy-Fuel Coating and Characterization

Thermal barrier coating of parts subjected to the high-temperature is necessary for preventing the surface of the parts from excessive heating, erosion, and corrosion during the operation. The coating material selected for such an application must have the properties that resistant to harsh environments as well as the coating process must be efficient, low cost, and fast. One of the coating methods fulfilling these conditions is High Velocity Oxy-Fuel coating (HVOF). Although the process is fast, effective, and low cost, due to the irregularities associated within the resulting coating, investigation into the process is necessary for further improvements of the coating quality. The mechanical properties of the coating, particularly at coating-base material interface, are important to secure the sound coating. The powder emerging from the spray gun reaches almost the melting temperature of the constituting substrate material. Since the stand-off-distance between the gun exit and the base material surface is short, the splats changes from the round shape to the oval shape upon impacting onto the surface due to their elevated temperatures. Once the coating is formed on to the base material surface, the coating temperature remains high while the substrate bulk temperature is low. This, in turn, modifies the stress levels in the coating and at the coating-base material interface. However, depending on the coating thickness, the residual stresses can be compressive or tensile. The residual stress in the coating influences the mechanical performance of the coating.

Considerable research studies were carried out to examine the residual stress development in the coating. The residual stress measurement using the curvature method was carried out by Liao et al. [36]. They showed that temperature history of a part was of paramount importance in stress generation and distribution. The evolution of the residual stress in thermal barrier coatings using the modified layer removal method was carried out by Lima et al. [37]. They showed that the residual stresses were mainly influenced by the thermal history regarding the plastic deformation and the quenching of individual splats. The residual stress generation during thermal spraying was examined by Gill and Clyne [38]. They showed that the continuous curvature measurement technique was potentially a reliable and accurate method with a minimum measurement error. The measurement of the residual stress in plasma-sprayed coatings was carried out by Kesler et al. [39]. They indicated that the respective thermal expansion coefficients and the mechanical properties were the most important factors determining the stress level and the magnitude. The residual stress measurement using the curvature interferometer was carried out by Wang et al. [40]. They showed that the existence of sub-surface tensile layer was expected to play an important role in the wear process associated with the normal contact loading. The residual stress development in thermal spray coatings was investigated by Ghafouri-Azar et al. [41]. They indicated that the magnitude of residual stress increased significantly with increasing the coating thickness. The residual stress development in the HVOF carbide coatings was studied by Pejryd et al. [42]. They evaluated the coating crack resistance due to bending and low cycle fatigue loads. The residual stress distribution in thermally sprayed coatings was carried out by Otsubo et al. [43]. They indicated that the residual stress remained almost the same in the coating, except at interface, which reduced from the mean value. The residual stress in HVOF tungsten carbide coating and its effect on the fatigue life were studied by McGrann et al. [44]. They showed that the fatigue life of the tungsten carbide coating could be determined using the modified layer removal method. The fracture behavior of the brittle coatings was investigated by Bansal et al. [45]. They showed that the development of the finite element code made it possible to study the effect of various coating parameters in isolation, which was difficult to achieve in an experimental study.

The electron beam re-melting of HVOF coating was studied by Hamatani and Miyazaki [46]. They showed that low melting speed and homogeneous heating reduced the unevenness of the surface of the number of pores in the coating. The laser surface treatment of HVOF coating was carried out by Oksa et al. [47]. They successfully sealed the coating surface for corrosion prevention. HVOF coating and the laser treatment were investigated by Suutala et al. [48]. They observed that the cracks occurred in the coating after the laser treatment and the crack orientation was perpendicular to the laser processing direction. Adhesion testing of coatings produced via arc spray, HVOF, and laser cladding was carried out by Hjornhede and Nylund [49]. They indicated that the coatings deposited with the laser technique did not show the de-lamination. The effect of laser glazing on the microstructure of the HVOF coating was examined by Kumari et al. [50]. They showed that laser glazing of coatings affected the surface roughness in either way depending on the coating composition as well on the treatment parameters. Laser produced functionally graded coatings was examined by Riabkina-Fishman et al. [51]. They showed that the laser treatment resulted the coating, which had wear resistance five times higher than that of untreated coatings.

The thermal process pertinent to the formation of a WC-Ni coating on low alloy steel by HVOF spraying was studied by Sobolev et al. [52]. They indicated that the second and subsequent layers of the coating did not have practical influence on the thermal state of the substrate-coating interface. Moreover, the substrate coatings thermal interactions were completely determined by interactions between substrate and the first layer of the coatings. The influence WC-Cr was examined by Zhao et al. [53]. They indicated that the spray distance had less influence on the wear resistance of coating as compared to those corresponding to the total gas flow rate and the powder feed rate.

The microstructure of multilayer coatings produced by thermal spray process was examined by Fagoaga et al. [54]. They showed that the multilayer coatings presented a microstructure composed of alternate phases of chromium carbide cermet and oxides, which were resulted from preferential oxidation of chromium compounds. The mechanical properties of mono and multilayered coatings were examined by David et al. [55]. They observed that WC-CoCr deposited by HVOF spraying onto P91 steel presented on high fatigue resistance. The bond strength and thermal shock behavior of HVOF sprayed multi-layered coating was examined by Wilden et al. [56]. They showed that HVOF coatings had higher contact of oxygen and higher density than that produced by plasma sprayed coatings. The microstructure and wear resistance of WC-Co coatings thermally sprayed onto tool steel were studied by Pepe et al. [57]. They introduced the neural network to classify the wear performance of the coatings and indicated that the neural network would be used as an effective tool to classify the wear behavior of the resulting coatings.

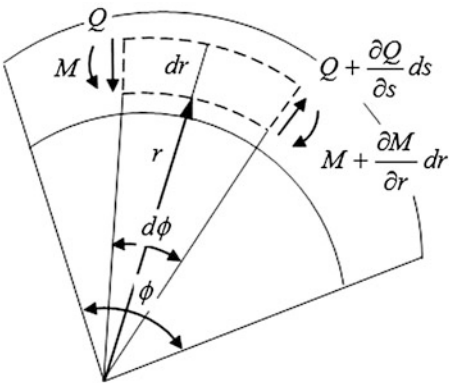
1.3 Analytical Formulation of a Beam Subjected to a Flexural Motion

The mathematical analysis associated with the flexural characteristics of a cantilever beam is given below in line with the previous study [33]. Therefore, the following assumptions are made in the analysis:

1. Beam is initially straight and unstressed.
2. The material of the beam is perfectly elastic, homogeneous and isotropic.
3. Every cross-section of the beam is symmetrical about the plane of bending i.e. about an axis perpendicular to the neutral axis.
4. There is no resultant force perpendicular to any cross-section.
5. Plane sections remain plain during bending, but no longer perpendicular to the centroidal plane. Accordingly, warping of the cross-section is no longer present. Thus shear deformation will be considered.
6. Relationship between moment and curvature as assumed in elementary theory still exists.
7. The effects of rotary inertia are included.
8. The deflections are small compared to the beam thickness.
9. Material properties are kept constant in the analysis.

Consider an element of beam subjected to shear force, bending moment and distributed load as can be seen in Fig. 1.1.

Fig. 1.1 Beam element subjected to bending moment and shear force



The transverse displacement is measured by $y \equiv y(x,t)$ and the slope of the centroidal axis is given by $\frac{\partial y}{\partial x}$. This slope is considered to be made of two contributions. The first is $\phi \equiv \phi(x,t)$ due to the effects of bending. An additional contribution is γ_0 which defines the shear strain due to shearing effects. Thus one has:

$$\frac{\partial y}{\partial x} = \phi + \gamma_0 \tag{1.1}$$

Now one can relate the above kinematic expression to the applied load using assumption 2 in terms of present parameters:

$$\frac{M}{EI} = \frac{-1}{R} \tag{1.2}$$

where R is the radius of curvature, M is the bending moment at the cross-section under consideration and E is the modulus of elasticity o the substrate material. For small curvature:

$$Rd\phi = dx \tag{1.3}$$

or

$$\frac{1}{R} = \frac{d\phi}{dx} \tag{1.4}$$

From Eq. (1.2) one can get:

$$\frac{M}{EI} = \frac{-\partial\phi}{\partial x} \tag{1.5}$$

The shear force Q as the cross-section is given in terms of the shear stress τ or shear strain γ as