

Dieter Schramm · Manfred Hiller  
Roberto Bardini

# Vehicle Dynamics

Modeling and Simulation

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ISBN 978-3-540-36044-5      ISBN 978-3-540-36045-2 (eBook)  
DOI 10.1007/978-3-540-36045-2  
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014942274

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# Preface

The main focus of this book is on the fundamentals of “Vehicle Dynamics” and the mathematical modeling and simulation of motor vehicles. The range of applications encompasses basic single track models as well as complex, spatial multibody systems. The reader will be enabled to develop own simulation models, supported to apply successfully commercial programs, to choose appropriate models and to understand and assess simulation results. The book describes in particular the modeling process from the real vehicle to the mathematical model as well as the validation of simulation results by means of selected applications.

The book is aimed at students and postgraduates in the field of engineering sciences who attend lectures or work on their thesis. To the same extent it addresses development engineers and researches working on vehicle dynamics or apply associated simulation programs.

The modeling of Vehicle Dynamics is primarily based on mathematical methods used throughout the book. The reader should therefore have a basic understanding of mathematics, e.g., from the first three semesters’ study course in engineering or natural sciences.

This edition of the book is the English version of the second German edition.

The authors thank all persons who contributed to this edition of the book. Amongst all persons who contributed by giving hints and sometimes simply asking the right questions we want to highlight in particular the indispensable contributions of Stephanie Meyer, Lawrence Louis and Michael Unterreiner who contributed with translation and proof reading of some chapters. We also thank Frederic Kracht for diligent proofreading and the solution of unsolvable problems incident to the secrets of contemporary word processor software.

Duisburg, May 2014

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# Nomenclature and Definitions

## Variables and Physical Quantities

The name of variables and physical quantities are in general written in italic letters. The notations of locations (points), components and names of coordinate systems, numbers as well as mathematical standard functions, such as e.g. “sin” or “cos” are not written in italic letters.

In addition, the following applies for vectors and tensors as well as matrices:

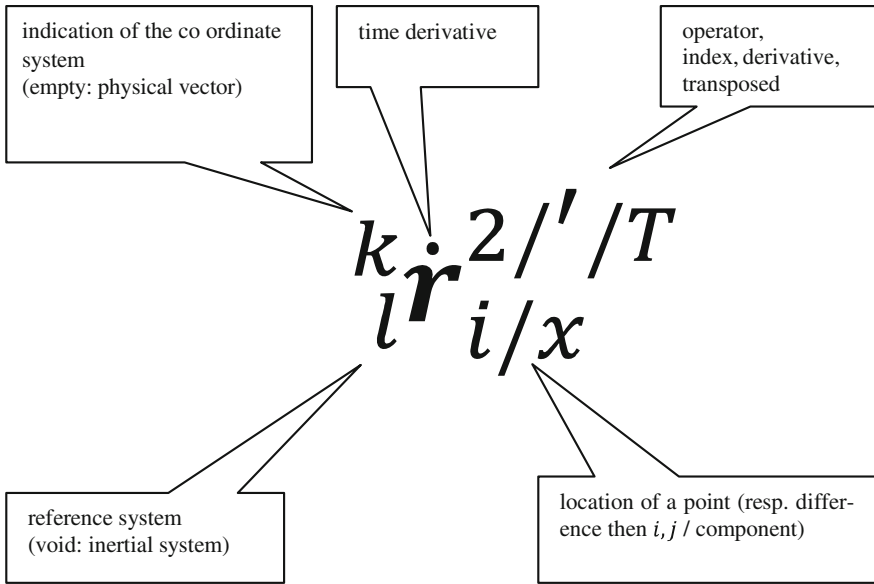
- Vectors are represented by bold lower case letters, tensors and matrices by bold upper case letters.
- Dots over the respective quantity indicate time derivatives.

## Special Notation for Physical Vectors

The subscription of vectors and tensors is made according to the following rules:

- An index on the lower right side represents a denotation and numbering. It denotes, e.g. the body or the coordinate system of the respective quantity.
- For quantities which are described with respect to other quantities a lower left index denotes the reference body or the reference coordinate system. A void index indicates the inertial system as reference system.
- In case that a physical vector is represented by coordinates, the coordinate system is indicated by a left upper index. If no index is present, a physical vector or tensor is given without indicating a specific coordinate system.
- Operators, like inversion, transposing and raising to power as well as differentiation with respect to other variables as time are indicated by a respective right upper index.

- Differentiation with respect to time is indicated by a dot over the respective variable. At this position also other indications like vinculi “-” or tildes “~” can be present.



### Examples for Subscriptions

- $\dot{r}_i$  Absolute velocity of point  $P_i$
- $\dot{r}_{i,j}$  Absolute velocity (absolute variation with time) of difference vector  $r_j - r_i$
- ${}_k \dot{r}_i$  Relative velocity of “ $P_i$ ” with respect to reference system “ $k$ ”
- ${}_k \dot{r}_{i,j}$  Relative velocity  ${}_k \dot{r}_j - {}_k \dot{r}_i$
- ${}_k^i v_j$  Coordinate representation of the absolute velocity of point  $P_j$  with respect to coordinate system “ $k$ ”, described in coordinates of coordinate system “ $i$ ”

${}^jT_i$  Rotation tensor, transforming the coordinate representation of vector “ $\mathbf{a}$ ” in coordinate system “ $i$ ” to coordinate system “ $j$ ”: “ ${}^j\mathbf{a} = {}^jT_i {}^i\mathbf{a}$ ”

Partial derivatives of a  $m$ -dimensional vectorial function

$$f(\mathbf{x}) = \begin{bmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_m(x_1, \dots, x_n) \end{bmatrix}$$

with respect to coordinates of a  $m$ -dimensional vector  $\mathbf{x}$  are arranged in a  $(m, n)$  - dimensional functional- or JACOBIAN-Matrix:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x} \\ \vdots \\ \frac{\partial f_m(\mathbf{x})}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \dots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(\mathbf{x})}{\partial x_1} & \dots & \frac{\partial f_m(\mathbf{x})}{\partial x_n} \end{bmatrix}.$$

### Examples for “Physical” Vectors and Their Representation

$\mathbf{e}_{x_i}, \mathbf{e}_{y_i}, \mathbf{e}_{z_i}$	Unity vectors for coordinate systems
$\mathbf{u}_i$	Normalized orientation vector (joint axes)
$\mathbf{r}_i$	Position vector to reference point $O_i$ of an “object” (body) “ $j$ ”
$\mathbf{r}_{\bar{i}}$	Position vector to predecessor of reference point $O_i$
$\mathbf{s}_i$	Position Vector to center of gravity $S_i$
$\mathbf{p}_i$	Position vector to “point of interest” $P_i$ (e.g. application point of a force)
$\mathbf{r}_{i,j} = \mathbf{r}_j - \mathbf{r}_i$	Vector difference between two reference points $P_i, P_j$
$\mathbf{v}_i, \dot{\mathbf{v}}_i, \mathbf{a}_i$	Velocities, accelerations
$\boldsymbol{\omega}_i, \dot{\boldsymbol{\omega}}_i, \boldsymbol{\alpha}_i$	Angular velocity, angular acceleration
$\mathbf{F}_i$	Force
$\mathbf{L}_i, \mathbf{T}_i$	Torque
$\boldsymbol{\Theta}_{S_i}, \theta_{S_i}$	Tensor of inertia, moment of inertia
$\mathbf{T}_i$	Rotation tensor
$(x, y, z)_i$	Coordinate system ( $K_i$ )
$\mathbf{K}_i = \{O_i; x_i, y_i, z_i\}$	Coordinate system ( $K_i$ ), alternative notation
$x_i, y_i, z_i$	Coordinate axes
$\xi_i, \eta_i, \zeta_i$	Coordinate axes

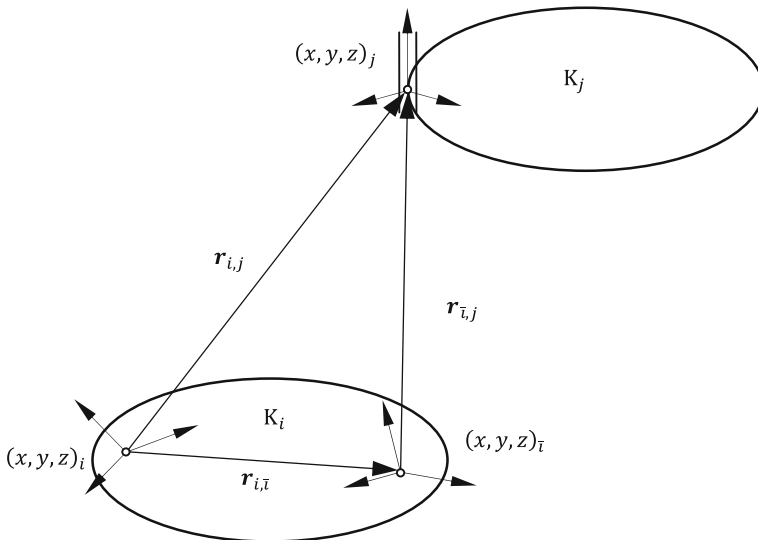
### Scalars

- $m_i$             Mass
- $i, j, k, \dots$     Indices
- $N, n$             Number of elements, components (e.g.  $n_\beta, N_\beta$ )
- $\alpha$             Angle
- $f$                 Number of degrees of freedom (DoF) (also  $f_i$ )

### Vectors and Matrices

- $E, I$     Unity matrix or unity tensor
- $\mathbf{g}$     “Vector” of implicit constraint equations
- $\mathbf{q}$     “Vector” of generalized coordinates
- $\mathbf{M}$     Mass matrix
- $\mathbf{b}$     “Vector” of generalized zentripetal- and CORIOLIS forces
- $\mathbf{Q}$     Generalized forces
- $\boldsymbol{\beta}$     Relative or natural joint coordinates
- $\mathbf{e}^{(i)}$      $\left[ \begin{array}{cccc} 0, & 0, & \dots, & 1, & \dots, & 0, & 0 \end{array} \right]$   

$i^{th} \text{ position}$
- $\mathbf{w}$     Position coordinates
- $\mathbf{Z}$     Reaction forces





$$\dot{\mathbf{r}}_j = \dot{\mathbf{r}}_i + \dot{\mathbf{r}}_{i,j}$$

$$\dot{\mathbf{r}}_{i,j} = \boldsymbol{\omega}_i \times \mathbf{r}_{i,j} + \dot{\mathbf{r}}_{i,j}$$

$${}_i\dot{\mathbf{r}}_{i,j} = \dot{\mathbf{r}}_j - \dot{\mathbf{r}}_i$$

(without components!)

## Note

$$\mathbf{r}_{i,j} = {}_i\mathbf{r}_j$$

$$\dot{\mathbf{r}}_{i,j} \neq {}_i\dot{\mathbf{r}}_j$$

## Trigonometric Functions

Due to space requirements “ $\cos \varphi$ ” and “ $\sin \varphi$ ” are, where appropriate, replaced by the short forms “ $c\varphi$ ” and “ $s\varphi$ ” respectively.

# Chapter 1

## Introduction

This book addresses the fundamentals, mathematical description and simulation of the dynamics of automobiles. In this context different levels of complexity will be presented, starting with basic single track models up to complex three-dimensional multibody models. A particular focus is the process of establishment of mathematical models from real cars and the validation of the simulation results. The methods presented will be explained in detail based on selected application scenarios.

The intention of this book is to enable the reader to develop his own simulation models and to use them for his daily work, to apply commercial simulation tools in an efficient and dedicated form. In particular the reader will be enabled to choose the appropriate model for a give technical task and to validate the results of simulations.

### 1.1 Problem Definition

Vehicle dynamics is a branch of vehicle mechanics that deals with the motional actions necessary for moving road vehicles and their resulting forces under consideration of the natural laws. Reference to vehicle dynamics is found in many areas of development of motor vehicles, vehicle systems and their components.

In this chapter an overview of the modeling methods, the fundamental definitions related to vehicle dynamics and the embedding of vehicle dynamics in the development of vehicles will be given.

The use of complex mathematical vehicle models to simulate and develop vehicle systems and their applications, such as in the development of vehicle dynamics control systems or braking systems, has gained significance especially over the last years. The reasons are, on the one hand, economical:

- The effort involved in vehicle testing and measurement has been increasing along with the complexity of the vehicle systems and the prescribed testing conditions. This has a corresponding influence on the development budget available.

- The increasingly competitive automotive market is forcing manufacturers and suppliers to also contain the costs in the development stage, by replacing prototypes and tests with simulations and virtual prototypes.

On the other hand, many reasons can be attributed to the technology of the new systems. The majority of these are mechatronic systems, ref. e.g. (Isermann 2008), whose typical increase in functionality and optimized product value are based on the function- and hardware oriented combination of mechanical, electrical and electronic components and subsystems, as well as their respective operating systems and functional software.

The interaction of these individual systems, which are derived from different technological domains, on the one hand results in never before seen functional range and product quality and on the other hand in cost efficient solutions, by integrating mechanical, electrical and electronic hardware into modules. The design and testing of such systems with their enormous functional diversity requires high standards in methods of design and testing programs and, as a result, modeling and simulation techniques:

- Vehicle models are the basis for the design and development of vehicle systems and components.
- Vehicle maneuvers can be simulated repeatedly under predefined parameters and conditions.
- Critical maneuvers can be replaced by safe simulations.
- The continuous shortening of product cycles for new models requires shorter developmental phases. This can only be achieved through the implementation of simulations and virtual prototypes.

Based on these requirements, the fields of application for the method of multibody systems in the development of vehicle systems, which is presented in this book, can be deduced:

- Kinematics and dynamics of the chassis and the steering.
- Vehicle dynamics of the entire vehicle.
- Ride comfort of the entire vehicle.
- Analysis of accidents.

The goal in each case is a mathematical description of the relevant areas and functions of the vehicle that can be variably applied for the design, development and evaluation of vehicle dynamics. The numerical simulation of vehicle handling, which is based on these mathematical models, has recently gained enormous significance. It allows the simple, quick and efficient investigation of maneuvers without the need for elaborate testing. The simulation allows for a variation of parameters and conditions in a way that is not possible in actual testing. Since, however, the results generated by numerical simulations are only approximations and their accuracy is dependent on the exactness of the models and the reliability of the system data, great care has to be put into the modeling of these systems.

The driving characteristics of passenger vehicles are influenced by several factors. The wheel locations, which are supposed to conduct predefined motions relative to the chassis, play an important role. By choosing beneficial geometrical parameters in the construction of a wheel suspension system, for example, the stability of the vehicle whilst cornering or changing lanes is guaranteed. Modern wheel suspensions are typically multibody systems with closed kinematic loops. In addition, the handling can be influenced through elastic bearings in the wheel suspensions. For example, the longitudinal flexibility of the wheel location can be achieved through a soft bearing of the transverse link.

The complicated systems and the wish for a reproduction of real events that is as accurate as possible make the development of simulation models a comprehensive and challenging task. Setting up equations efficiently is of vital importance in order to limit the modeling effort and minimize the computation time required for the simulation. Thus the goal of this book is to present an efficient way of creating realistic simulation models of a vehicle. To this end, an overview of the basic mechanical and mathematical processes will be provided, in which the topological structure of the vehicle will be described in detail using fundamentals such as the methodology of kinematic differentials and that of the characteristic joint pairs. Based on this, the modeling of the subsystems and components

- chassis, wheel suspensions,
- wheels and tires,
- force elements,
- drivetrain

and finally the entire system will be dealt with.

### ***1.1.1 Modeling Technical Systems***

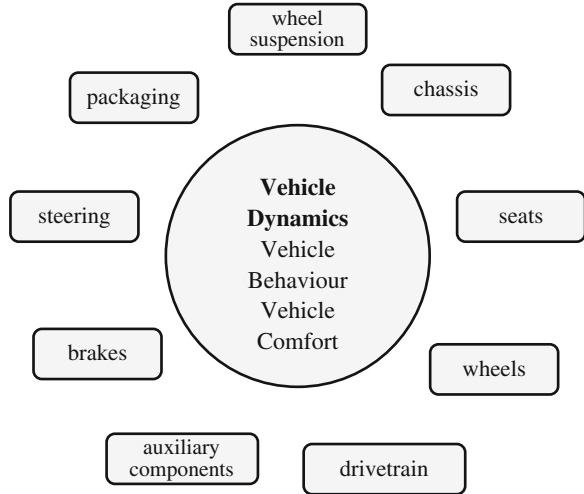
There are two fundamentally different methods to describe the dynamic behavior of a real process using mathematical models, appropriate to the task at hand:

- In theoretical modeling, the mathematical models are derived from physical laws.
- In experimental modeling a specific model structure, which in most cases is also mathematically formulated, is used as the foundation. On the basis of this model structure, the individual parameters are identified using input and output measurements. A special form of this method is called (model-) identification.

This book almost exclusively deals with theoretical modeling. Physical parameters will be assumed to be known or at least assessable. Typical sources of parameters in vehicle technology are:

- Computer-aided design models for measurements, masses and moments of inertia.

**Fig. 1.1** Vehicle Dynamics:  
Environment and related  
components



- Direct measurements of masses, moments of inertia, spring and damper characteristics, and, if possible, friction coefficients.
- Assumptions, estimations and, where applicable, identification methods for other, more difficult or vague characteristics such as friction effects, elasticity in bearings, etc.
- Identification of parameters and characteristic maps through other methods of calculation and simulation such as the finite-element method and of calculating electric and magnetic fields etc.

The identification methods are often used in this context to determine parameters of theoretical models that are either unknown or difficult to measure. Examples are tire models (Chap. 7) or characteristics of force elements (Chap. 9), such as rubber bearings or dampers.

The aim of modeling is to obtain a mathematical-analytical description of the respective system which allows for an investigation of the relevant aspects of the system behavior and the influence of the system components on it, (Fig. 1.1). It is possible to develop models of varying complexity and validity. On the one hand, the more complex a model is, the more accurate the simulation of the system behavior is. On the other hand, however, this will invariably result in complex and mostly nonlinear model equations as well as a need for better computing performance. Additionally, the number of model parameters that have to be determined increases along with the complexity of a model. Most of the time, the effort to procure the parameters required will outweigh the effort in creating the model equations by far. Therefore it is always necessary to critically evaluate whether an increase in model complexity is still adequate to its aims.

### ***1.1.2 Definition of a System***

The term system will often be used in this book. Therefore it is important to briefly define what is meant by a system in this book (Hiller 1983):

A system is identified as a set of elements (parts, components), that influence each other through internal functional relationships and physical laws (interaction), on which external influences act (inputs) and the effects of which are commuted to the outside (outputs). Thus, the most important properties of a system are its changeability (motion) and its controllability through a suitable choice of inputs (forces and applied motion). The relationship between external influences on the system and the resulting changes of the system state is defined as system dynamics.

A major part of this book deals with the investigation and analysis as well as the prediction of the dynamic behavior of the vehicle system and its subsystems and components. The following subtasks can be identified:

- Modeling: modeling always involves idealizations and abstractions (Sect. 1.1.4).
- Model investigation: deals with, primarily numeric, solutions to the equations of motion.
- Selection of controlling inputs: Examples in a motor vehicle are steering angle, accelerator and brake pedal position as well as the characteristics of the road surface, but also actuator forces, such as the active anti-roll bars which are examined in Chap. 14.
- Simulation of the system characteristics (Chap. 12).

### ***1.1.3 Simulation and Simulation Environment***

Every simulation aims at describing the observed system as accurate as possible in order to be able to deduce the behavior of the real system from the behavior of the model. In this book a vehicle or part of a vehicle as well as, if necessary, a part of its environment will be referred to as a system. Below, solely the simulation of mathematical models on one (or several connected) computers will be examined. The models will be purely mathematical in nature. The simulation is thus equivalent to the running of software, combined, if necessary, with hardware components which are connected via suitable interfaces. The latter are usually referred to as hardware-in-the-loop (HiL) simulations. It is necessary to run the simulation in real time in order to provide the hardware with data.

If one visualizes the vehicle as a mechatronic system, in which, for example, the aforementioned vehicle dynamics control systems and driver assistance systems play an ever increasing role, the simulation of the dynamics of the vehicle components or the entire vehicle as a tool in the process of mechatronic development (VDI-Guideline-2206 2004).

**Table 1.1** Modeling of motor vehicles

Model type	Degrees of freedom	Chapter
Single track model, linear	2	10
Single track model, nonlinear	3–7	10
Twin track model	14–30	11
Complex multibody system model	>20	12 and 13, subsystems: 5–9
Finite-Element-model	>500	Not dealt with
Hybrid model	>500	Not dealt with

### 1.1.4 Vehicle Models

The models described in this book are to make it possible to represent the dynamic behavior of real vehicles as realistically as possible. To accomplish this, the models have to meet at least the following criteria:

- Complete spatial kinematics and kinetics of the entire vehicle and, if required, also its subsystems.
- Nonlinear kinematics of the wheel suspension.
- Nonlinear and, where required, also dynamic representations of the force elements.
- Dynamic tire forces.

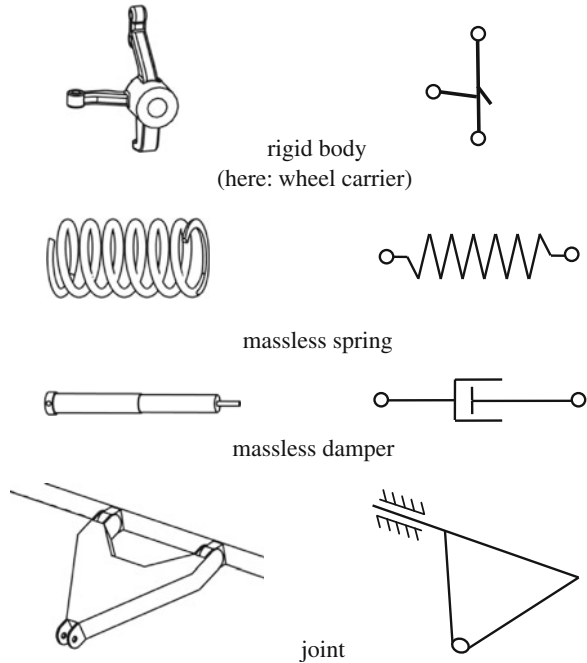
On the other hand the models have to remain manageable. This is especially important if the simulation models are to be implemented in a hardware-in-the-loop test rig or a drive simulator. In this case the computation time has to remain suitable. This also holds for the use of the models for optimization tasks.

To simulate the handling of a vehicle, different types of vehicle models are possible, depending on the desired level of detail and the task at hand, ref. Table 1.1.

If one assumes the vehicle chassis to be rigid, then the chassis has six degrees of freedom in space, which can, however, be reduced through further assumptions, such as those found in single track and twin track vehicle models. To simulate the vehicle longitudinal motion, it may be sufficient to define just one degree of freedom. Then the other degrees of freedom of the body have to be constrained by using so-called constraint or boundary conditions. Even for a simple model that describes the lateral dynamics, a minimum of two degrees of freedom, for the lateral motion and the yaw motion, is required.

In twin track and especially in complex multibody system models, further degrees of freedom to describe the motion of the components of the wheel suspensions and the drivetrain are required. Furthermore, the subsystems of the vehicle such as the drivetrain, brakes and steering have to be modeled. Systems such as the ABS, ESP, driver assistance systems and other mechatronic systems can also be integrated into the model.

**Fig. 1.2** Typical elements of a multibody system



The use of simulation programs for the development of technical systems in vehicles has gained significance in recent years and is currently state of the art. The prediction of the dynamic behavior of the vehicle allows for conclusions about driving stability, driving safety and comfort of new vehicle systems. Furthermore, such models can show the influence of control systems and actuators on the vehicle's handling and are often prerequisite for an efficient development of such systems. The mechanical components that occur in these systems can be modeled and simulated using the following approaches (Schiehlen and Eberhard 2004).

**Multibody Systems (MBS)** Multibody systems are suitable for the description of mechanical systems, which consist of bodies that are mostly rigid and are connected via bearings and joints. A multibody system usually consists of rigid bodies with mass, which are subject to concentrated forces and moments at discrete points (Schiehlen and Eberhard 2004). Some of the symbols, commonly used for a typical multibody system, are represented, along with a corresponding example from vehicle technology, in Fig. 1.2.

A rigid body of a multibody system is characterized by its mass and moment of inertia. Characteristic points of a rigid body are the center of gravity  $S$  as well as a finite number of node-points  $P_i$ , at which concentrated forces and moments act or other bodies are connected via corresponding joints. Elasticity and damping are represented as massless force elements and their typical symbols are shown in Fig. 1.2 along with the corresponding component typically found in a vehicle. Respectively, through depiction of force laws and constraint motions, drivetrains



and the actuators can be represented. Of major interest in modeling and simulation are the motion variables of the bodies and, sometimes the forces and moments acting on the bearings and drivetrains. The mathematical description of the kinematics and kinetics of the multibody system results, depending on the modeling and formalization, in ordinary differential or differential algebraic systems of equations with relatively small degrees of freedom and will be dealt with in Chaps. 3 and 4. Here, “kinematics” refers to the description of the possible motions of mechanical systems, while “kinetics” refers to the motion of mechanical systems under the influence of forces.

**Finite-Element-Method (FEM)** This method is primarily being used to give a mathematical description of the elastic and, where applicable, plastic characteristics of mechanical systems, in which mass and elasticity are distributed continuously throughout the body. The model consists of many finite elements with a simple geometry, whose principle deformation options are constrained by so-called elementary functions. The method is primarily used to examine the effect of external forces on the deformation and stress distribution of a body. The mathematical formulation of the finite-element-method leads to ordinary differential equations with many degrees of freedom.

**Continuous Systems (COS)** Continuous systems are used for the depiction of elastic characteristics of mechanical systems, in which mass and elasticity, as well as plasticity are distributed continuously throughout the body. The mathematical formulation of continuous systems leads to a description using partial differential equations with infinite number of degrees of freedom. The respective field of application of these structurally different substitute systems is mainly dependent on the geometry and the distribution of stiffness of the initial mechanical system, the goal of the investigation and, thus implicitly, the aspired area of validity of the simulation model. The method of finite elements and the continuous systems are primarily suitable for mechanical systems or bodies with evenly distributed elasticity.

Multibody systems are ideally suited for complex models that help describe vehicle dynamics. However it is also possible to create a vehicle model using the finite-elements-method. This has its advantages especially, when structural deformation and stress distribution have to be determined along with vehicle kinematics and kinetics.

Hybrid mechanical systems which require the modeling of both rigid and elastic bodies can be represented through a combination of multibody systems and the finite-elements-method for example. These are called hybrid systems (Louis and Schramm 2011).

When choosing a suitable method for the simulation of a vehicle, the following aspects have to be considered as well:

- For most of tasks in vehicle dynamics it is sufficient to examine a very limited frequency spectrum, Table 1.2, (Bürger and Dödelbacher 1988; Frik 1994). Hence, it is possible to limit the model to depict a spectrum between 0 and ca. 30 Hz.

**Table 1.2** Typical frequencies in vehicle subsystems

Oscillation mode	Frequency (Hz)
Body motion	1–2
Longitudinal vehicle oscillations	4–10
Motor jerk	10–13
Wheel suspensions, deflection	10–15
Steering oscillations	10–16
Body oscillations	30–40

- An exception from the limitation outlined above is for example the simulation of vehicle dynamics control systems. Because of the relatively short time constants of the hydraulics, higher frequency vibrations can occur. These oscillations occur within the region of the natural frequency of the wheel suspensions—due to the bearing elasticity in individual joints—between 15 and 30 Hz as well as the natural frequency of the radials at around 50 Hz. Since these oscillations influence the signals detected by the sensors of the vehicle control systems and lie within the sensor sampling rate, they must be included in the simulation of the regulated vehicle maneuvers.
- In every modeling approach it may be very difficult and in some cases even impossible to obtain the required model data. This is especially the case with friction and damping characteristics, bearing elasticity and tire parameters.

## 1.2 Complete Vehicle Model

Below, a complete vehicle model is considered to consist of the subsystems chassis, drivetrain, wheel suspensions, wheels, brakes and steering. Inputs to this model are the brake pedal and accelerator position, steering wheel angle, the engaged gear or the position of the automatic lever defined by the driver. The environment acts on the vehicle through the predefined environmental conditions, such as side and head wind, frictional connection coefficient of the road, road inclination and road bumps (Fig. 1.3).

An example of a complex vehicle model that has been modeled as a multibody system is shown in Fig. 1.4. It is not always necessary required to model an entire vehicle. It is possible to divide the vehicle model into its subsystems, which can be examined individually and, if necessary, combined to a complete model afterwards. In Fig. 1.4, this is shown using a complex vehicle model which has been split into its typical subsystems

- vehicle structure (chassis, underbody),
- drivetrain,
- wheel suspensions,
- wheels,
- brakes,
- steering system.

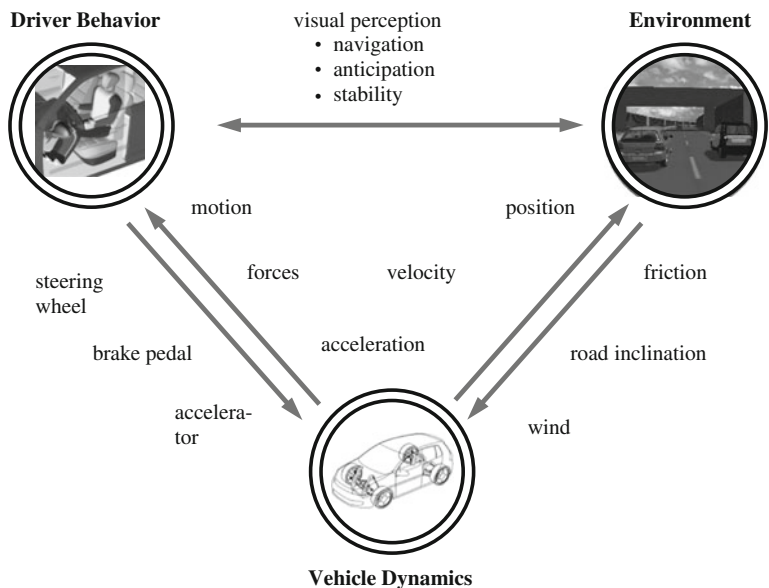


Fig. 1.3 Interaction of a vehicle with the driver and the environment

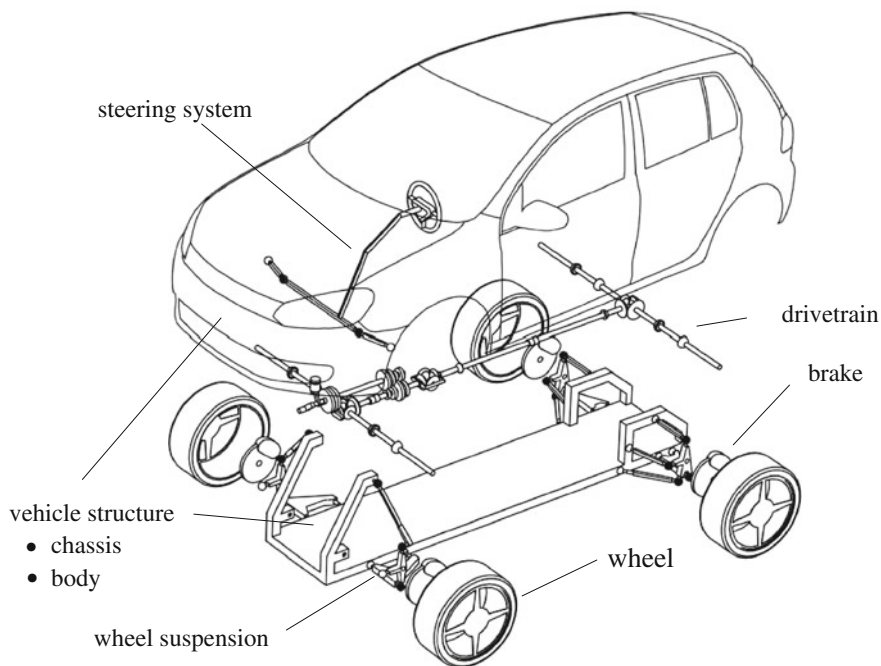
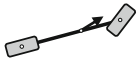





Fig. 1.4 Example of a complex vehicle model

**Table 1.3** Overview of vehicle models

	Single track model		Two track model		Multibody systems model		
	linear	nonlinear	without kinematics	with kinematics	reduced	complex	combined with finite-element method
							
Type of motion	planar translation, yaw						
	roll-, pitch-, vertical dynamics						
	Component motion						
	Special Applications, Rollover, Accident, Crash						
Degrees of freedom	2	3– 7	14– 25		20– ...		20– ...
Frequency spectrum	0–2 Hz		0–5 Hz		0–30 Hz		0–200 Hz

### 1.2.1 Vehicle Models and Application Areas

Depending on the required application, different vehicle models can be used. The fundamental models shown in Tables 1.3 and 1.4 will be discussed in detail in this book. Table 1.4 also indicates whether the respective model is suitable for offline (O) or real-time (E) applications.

### 1.2.2 Commercial Vehicle Simulation Systems

Nowadays, a multitude of simulation programs and even entire simulation environments with toolsets are available to simulate vehicle dynamics. In this section, a few of these commercial vehicle simulation software systems will be mentioned. The list is however neither complete, nor is the fact that a program has been mentioned or has been omitted an indication of its quality.

MBSExamples of frequently used software solutions for general multibody systems are listed in Table 1.5. They support the development of models through elementary libraries, which contain general as well as application specific elements and usually include graphic user interfaces for model creation (preprocessor) and evaluation (postprocessor). The systems mentioned are either useful to support

**Table 1.4** Simulation speed (O: offline, E: real-time)

	Single track		Twin track		MBS-Model	
	Linear	Nonlinear	W/o kinematics	With kinematics	Reduced	Complex
	O/E	O/E	O/E	O/E	O/E	O
Layout	O/E	O/E	O/E	O/E	O/E	O
Concept design and test	O/E	O/E	O/E	O/E	O/E	O
	O/E	O/E	O/E	O/E	O/E	O
Hardware	E	E	E			
Hardware-in-the loop (HiL)	E	E	E			
Software-in-the-loop (SiL)	E	E	E			
Human machine interface	E	E	E		E	
Investigation of algorithm and functions	E	E	E		E	
Acceptance	E	E	E		E	
Series application	E	E	E		E	

Latency speed (O: offline, E: real-time)

**Table 1.5** Programs for the simulation of multibody systems (MBS)

Program	Manufacturer	Type	Reference
Adams/Adams car	MSC. Software Corporation, Santa Ana, USA	MBS	MSC Software (2010)
Simpack	Simpack AG, Gilching, Deutschland	MBS	Simpack AG (2010)
Virtual. Lab vehicle motion	LMS International, Leuven, Belgium	MBS	LMS International (2010a)
Pro/ENGINEER Mechanica	PTC, Needham, USA	MBS	Parametric Technology Corporation (2010)
Simulia Abaqus	Simulia	Hybrid (FEM/MBS)	Simulia (2010)
Dymola	Dassault systemes, Vélizy- Villacoublay, France	MBS	Dassault Systems (2010)
DADS	LMS	MBS	LMS International (2010b)

**Table 1.6** Special programs for the simulation of vehicles

Program	Manufacturer	Topic	Literature reference
CarMaker	IPG	MBS, real-time, HiL	IPG (2010)
CarSim	Mechanical solution	MBS, real-time	Carsim (2010)
veDyna	TESIS Dynaware	MBS, real-time	Tesis (2003)
Proracingsim	ProRacing Sim	Motor simulation	Proracing Sim (2010)
AVL Advisor	AVL	MBS, real-time, drivetrain	AVL Advisor (2010)
AVL Cruise	AVL	MBS, real-time, drivetrain	AVL Cruise (2010)
ASM	dSPACE	HiL, embedded	dSpace (2010)
VDMS	Milliken research	MBS	Milliken Research (2010)
SwRi raptor	Southwest research		Southwest Research (2010)
Madymo	TASS	MBS, FEM	Tass (2010)

general mechanical applications or they are derived from the specialization of such systems. There are also simulation environments dedicated to the modelling of vehicle dynamics. These systems, generally, do not only allow the simulation of the vehicle, but also provide the simulation of road profiles and the consideration of the driver (driver model). Examples of such systems can be found in Table 1.6.

### 1.3 Outline of the Book

In the introduction (Chap. 1), an overview of the different modeling methods and simulation programs has been given. The method of multibody systems will be used in this book to describe the dynamics of vehicles, as it is particularly suited to describe important phenomena occurring in vehicle dynamics.