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Proceedings of the International Conference on Social Modeling and Simulation, plus Econophysics Colloquium 2014

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Preface

Big data analyses have uncovered many empirical laws hidden in our society and economy. Mathematical models have been introduced successfully explaining those empirical laws as typically seen in the new field of econophysics. One of the goals of this trend of research may be modeling and simulations of the whole society, which can directly contribute to the industry as well as help in decision making.

This book is the proceedings of the international conference, SMSEC2014, which was held on 4-6 November 2014 in Kobe, Japan, as a joint conference of the first "Social Modeling and Simulations" and the 10th "Econophysics Colloquium" (http://aph.t.u-tokyo.ac.jp/smsec2014/). It consisted of 21 invited talks, 77 oral and 53 poster presentations, with 174 participants. A variety of problems in wide fields, such as financial markets, traffic systems, epidemic contagion, and social media, were the subjects of intensive discussion. Data analysis, agent-based modeling, complex networks, and supercomputers were the examples of methods.

The conference was supported by many organizations: Tateishi Science and Technology Foundation, Kobe Convention and Visitors Association "MEET IN KOBE21", the Japanese Society for Artificial Intelligence, Society for Serviceology, the Japanese Association of Financial Econometrics and Engineering JAFEE, Center for Cooperative Work on Computational Science in University of Hyogo, the Physical Society of Japan, and RIKEN Advanced Institute for Computational Science. On behalf of all the participants, we would like to thank those supporters, as well as the following companies, without whose financial support the workshop would not have been possible: Hottolink, Sony CSL, and EBS.

As organizers, we are grateful for the cooperation of the steering committee: Kiyoshi Izumi (Univ. Tokyo), Yukie Sano (Univ. Tsukuba), Takahiro Sasaki (Sony CSL), Takashi Shimada (Univ. Tokyo), Kenta Yamada (Univ. Tokyo), and Naoki Yoshioka (Univ. Tokyo). Finally, we would like to thank all the authors for their contributions to this volume.

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April 2015

Contents

Part I
Financial Market

Chapter 1
Influence Networks in the Foreign Exchange Market

Arthur M.Y.R. Sousa, Hideki Takayasu, and Misako Takayasu

Abstract The Foreign Exchange Market is a market for the trade of currencies and it defines their relative values. The study of the interdependence and correlation between price fluctuations of currencies is important to understand this market. For this purpose, in this work we search for the dependence between the time series of prices for pairs of currencies using a mutual information approach. By applying time shifts we are able to detect time delay in the dependence, what enable us to construct a directed network showing the influence structure of the market. Finally, we obtain a dynamic description of this structure by analyzing the time evolution of the network. Since the period of analysis includes the great earthquake in Japan in 2011, we can observe how such big events affect the network.

1.1 Introduction

The Foreign Exchange Market is a market in which currencies are traded; it is continuously open during the weekdays and it has the largest transaction volume among the financial markets (average of \$5.3 trillion/day in April 2013 [1]). The importance of this market is that it defines the relative values of currencies and affects other markets, such as the stock markets [2].

In this market, traders can make orders for buying and selling which are organized in the order book according to their corresponding prices. The highest price of the buy orders in a given time is called best bid and the lowest price of the sell orders, best ask, and their average defines the mid-quote; a deal occurs when the best bid meets the best ask.

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Information about dependence between price fluctuations of currencies is important to understand the foreign exchange market. Several studies try to model this market and access those dependences [3–5]. However there are no studies on the influence structure in this market and the time evolution of the dependences. To contribute to fill this gap, we analyse the dependences in foreign exchange data during a period of 3 weeks using the mutual information, a non-linear dependence measure from the information theory [6, 7]. By doing a time shift analysis we can infer temporal dependence between markets making possible the construction of directed networks that show the influence structure of the foreign exchange market.

1.2 Data and Method

We analyze the foreign exchange data of the Electronic Broking Services (EBS) by ICAP. This data contains the orders for pairs of currencies in a resolution of 0.1 s. Here we use the 6 currencies with the largest transaction volume: USD (United States dollar), EUR (Euro), JPY (Japanese yen), GBP (Pound sterling), AUD (Australian dollar) and CHF (Swiss franc) in the period between 2011, March, 07th and 2011, March, 25th, each day from 22:00:00 to 21:59:59 GMT. The chosen period is a special one because it includes the great earthquake in Japan on 2011, March, 11th and the announcement of the intervention in the foreign exchange market as a response to the effects of the earthquake on 2011, March, 17th [8]. For this data we define the price *P(t)* as the last mid-quote, where t is the real time in intervals of 0.1 s. As an example of the data, Fig. 1.1 shows the price *P(t)* for the market USD/JPY on 2011, March, 09th, before the great earthquake in Japan.

We work with the sign of the difference of price *P(t)* [9]:

$$S(t) = sign[P(t) - P(t-1)], \tag{1.1}$$

so that we obtain a time series for each pair of currencies with the symbols $+$ (price increasing), $-$ (price decreasing) and 0 (price unchanged). By comparing two of these time series, we can identify 4 states not containing 0: $(+, +)$, $(+, -)$, $(-, +)$ and $(-, -)$. The removal of the states with 0, e.g. $(+, 0)$, is an important step because then we compare the series only when there is activity in both of them, avoiding issues regarding the volume difference and the time zone difference. Table 1.1 illustrates the number of occurrence of each state when comparing the EUR/USD with other markets on 2011, March, 07th (time series of each market with 863,999 points).

Studies in financial markets commonly use the Pearson correlation coefficient as a measure to infer dependence [5, 10]. But the correlation coefficient detects only linear correlation between two variables, not having information about the dependence. The mutual information on the other hand deals direct with the probability distributions being a measure not only for linear and non-linear correlations, but also for dependence. The mutual information is zero if and only if the random variables

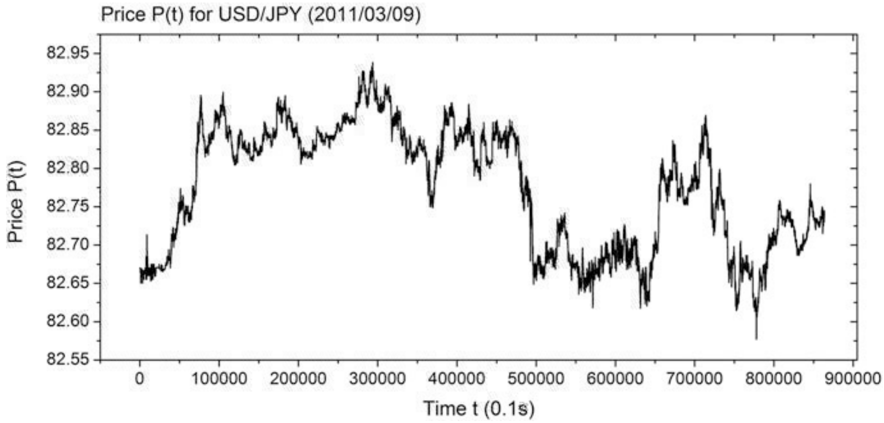

Fig. 1.1 Price P(t) for the market USD/JPY on 2011, March, 09th. Here we work with the sign of the difference of the price P(t)

Table 1.1 Number of states for EUR/USD and other markets on 2011, March, 07th (no time shift)

Market	(+, +)	(+, −)	(−, +)	(−, −)	0[a]
AUD/JPY	3256	2904	2941	3303	851,595
AUD/USD	2425	1707	1591	2332	855,944
CHF/JPY	125	129	184	184	863,377
EUR/AUD	55	59	66	48	863,771
EUR/CHF	3817	3061	3160	3895	850,066
EUR/GBP	3956	3305	3272	4086	849,380
EUR/JPY	5351	3918	3956	5202	845,572
GBP/AUD	53	47	45	53	863,801
GBP/CHF	43	47	56	52	863,801
GBP/JPY	4791	4431	4238	4807	845,732
GBP/USD	3088	2359	2533	3134	852,885
USD/CHF	2874	3656	3689	3032	850,748
USD/JPY	5822	7131	7081	5743	838,222

[a](+, 0), (−, 0), (0, 0), (0, −), (0, +)

are independent. There are evidences that mutual information can reveal aspects ignored by the correlation coefficient and studies comparing both measures [11–13]. Another reason for using mutual information in this work is that we are dealing with symbolic series, in which the numerical values that are taken in account for the correlation coefficient have no meaning.

The mutual information *I(X;Y)* between two random variables *X* and *Y*:

$$I(X;Y) = \sum_x \sum_y p(x,y) log \frac{p(x,y)}{p(x)p(y)}, \tag{1.2}$$

which can also be expressed in term of the entropies H:

$$I(X;Y) = H(X) - H(X \mid Y) \tag{1.3}$$

or

$$I(X;Y) = H(Y) - H(Y \mid X). \tag{1.4}$$

H(X) is the entropy of the random variable X and can be understood as a measure of its uncertainty. Similarly, *H(X | Y)* can be seen as the uncertainty of X given Y. Thus, one interpretation for the mutual information is the reduction in the uncertainty of a random variable given the knowledge of the other. If the variables are independent, the knowledge of one variable does not give information about the other and then the mutual information is zero.

The final dependence measure we use is the global coefficient:

$$\lambda(X;Y) = \sqrt{1 - e^{-2I(X;Y)}}, \tag{1.5}$$

This quantity has desired characteristics for a dependence measure, as taking value zero for independent variables and being in the range [0;1] [14], and has been used in financial data [12].

In order to compute the global coefficient of the financial series, we estimate the probability of each state using the relative frequency in a time window of 1 day. We also determine a significance level to decide if the computed coefficient is significantly different from the one of a random series; we randomize the analysed series and calculate the global coefficient until it reaches a stationary value which corresponds to the coefficient for the corresponding random series and we take this value as the significance level.

1.3 Results and Discussion

For each two pairs of currencies we compute the global coefficient for their sign time series as function of the time shift between them. For this data, we find four general types of structures according to the presence of peaks that represent dependence between the markets, as illustrated in Fig. 1.2.

- No peak: no dependence between markets.
- Peak at time shift zero: both markets are synchronized. External influences (e.g. economic news) make the markets to have similar behaviour, the change in the price occurs simultaneously in both markets.

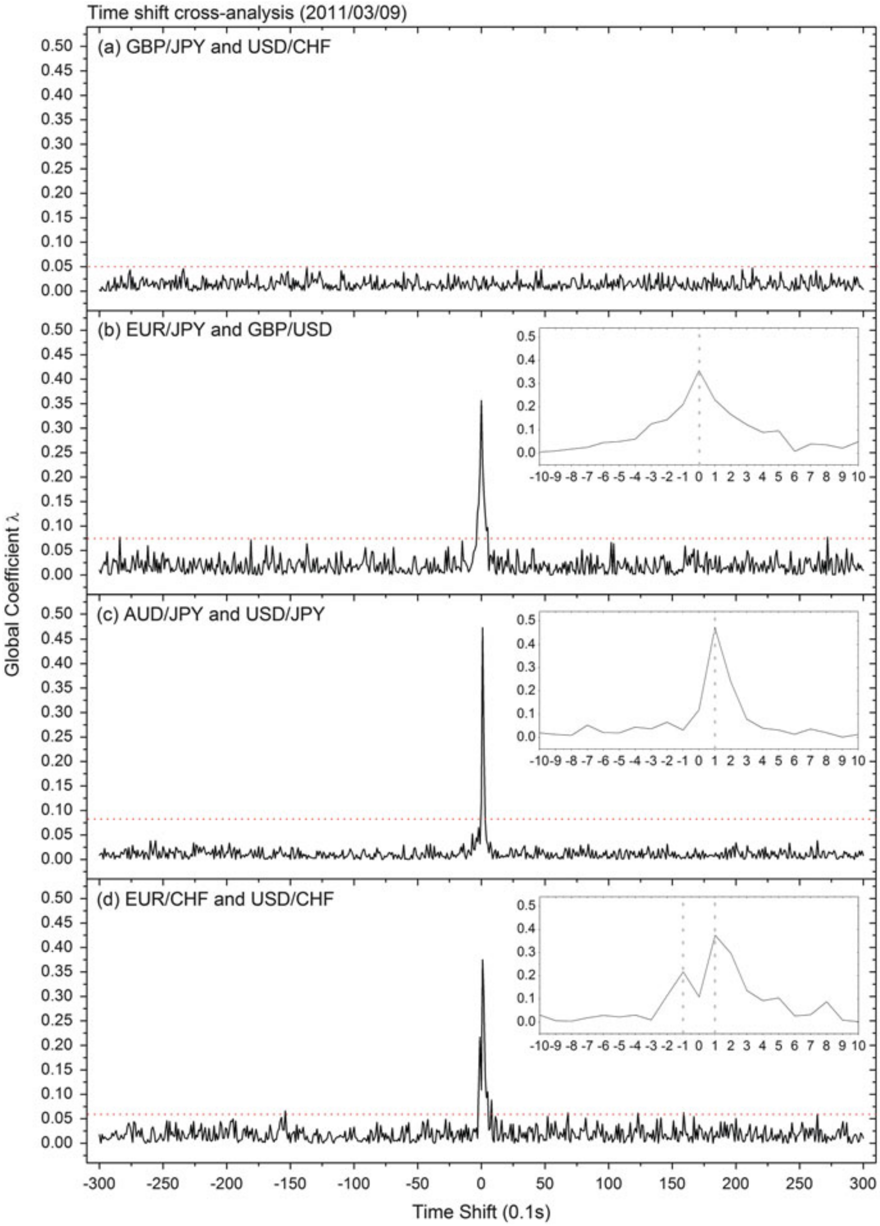

Fig. 1.2 Examples of results for the time shift cross-analysis. (**a**) GBP/JPY and USD/CHF on 2011, March, 09th: no dependence between the markets, same result for random time series. (**b**) EUR/JPY and GBP/USD on 2011, March, 09th: dependence at time shift 0. (**c**) AUD/JPY and USD/JPY on 2011, March, 09th: dependence when the USD/JPY series is shifted 0.1 s forward in relation to the AUD/JPY series. (**d**) EUR/CHF and USD/CHF on 2011, March, 09th: dependence at time shift 0.1 s in both directions. *Dotted lines* indicate the significance level

- Peak at a time shift different of zero: one market influences the other, i.e., there is an internal influence. This means that the past of one market affects the present of the other market, which could be interpreted as an information flow.
- Two peaks at time shifts in both directions: there are also internal influences, but in this case both markets affect each other during the analysed period.

We can build an influence network defining the pairs of currencies as nodes and adding the links according to the time shift cross-analysis between the markets that correspond to the nodes: (a) no peak: no link; (b) peak at time shift zero: undirected link; (c) peak at a time shift different from zero: directed link from the market that influences the other one, i.e., the market that goes ahead, whose past values affects the present values of the other market; (d) two peaks at time shifts in both directions: extraverted link.

We proceed with this analysis for all weekdays from 2011, March, 07 to 2011, March, 25. In this period two important events took place: the great earthquake in Japan on March, 11 and the announcement intervention in the foreign exchange market on March, 17. Figures 1.3, 1.4 and 1.5 show the time evolution of the influence network with day resolution during those 3 weeks. Figure 1.6 shows the time evolution of the different types of links in the influence network.

We observe that the structure does not present major changes within the first week from March, 07th to March, 10th, before the earthquake in Japan. Some characteristic features are: (a) EUR/USD and USD/JPY are the nodes with higher out-degree, meaning those are the markets that always go ahead being followed by the others, and (b) almost no extraverted links (with exception of link between USD/CHF and EUR/CHF, which is always present), i.e., information flows only in one direction, creating a hierarchy of importance between the markets.

From March, 11th (first week) to March, 17th (second week), which corresponds to the period between the earthquake in Japan and the intervention, we notice that the influence network changes compared to the structure in the first week. An important change is the increase in the number of directed and extraverted links, suggesting the interdependence between markets becomes stronger (not only due external influences, but internal ones). The new extraverted links that appeared involve the nodes EUR/USD and USD/JPY, that continue being the most important nodes (highest out degree), but now they are also influenced by other markets. One possible interpretation is that the players of these important markets are now being more careful, waiting for the information of other markets to decide to change the price.

After the announcement of the intervention on March, 17th, we observe another change in the structure, specially the disappearance of the extraverted link between EUR/USD and USD/JPY. Gradually the influence network returns to a structure similar to the one of the first week (before the earthquake).

Those results suggest that the event of the earthquake affected the dependence between markets and the event of the announcement of the intervention contributed for the return of the market to a state previous the earthquake, i.e., it was efficient in the sense of reversing the changes caused by the earthquake in the foreign exchange

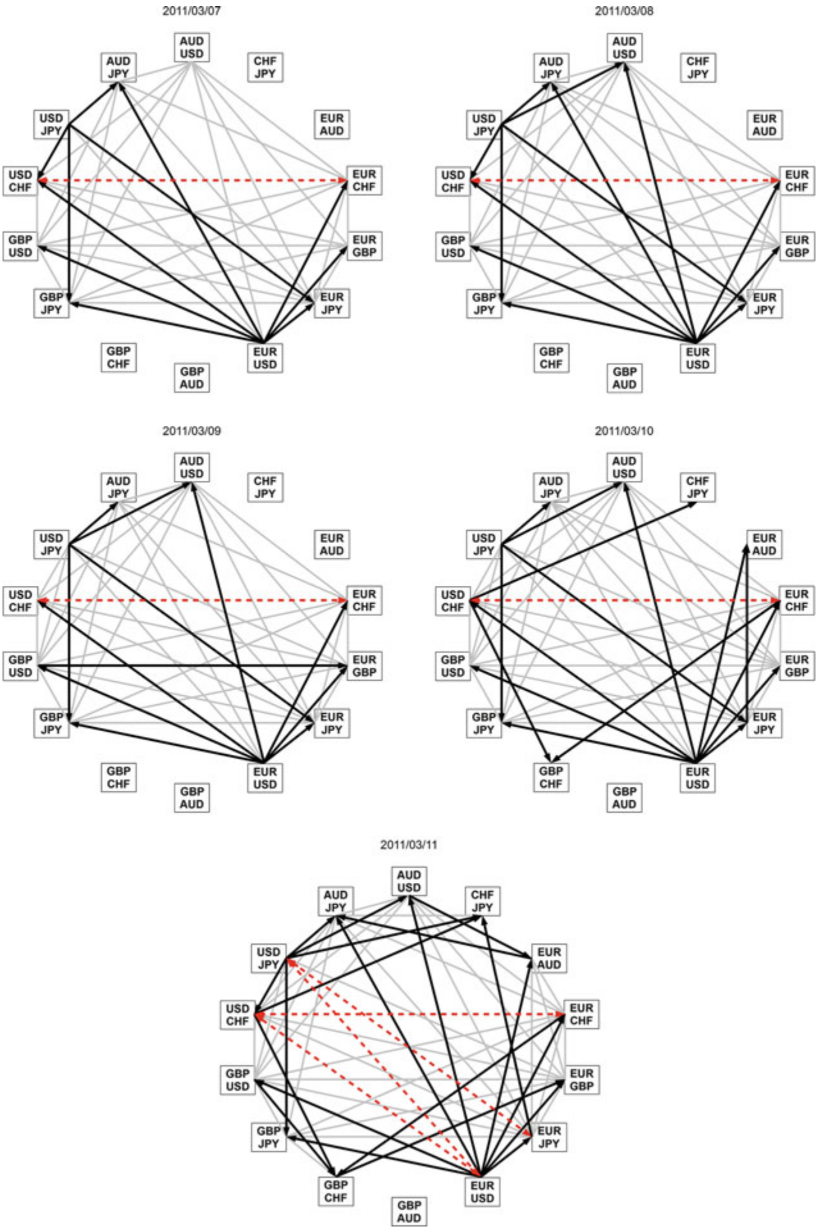

Fig. 1.3 Influence Networks of the Foreign Exchange Market for the currencies USD, EUR, JPY, GBP, AUD and CHF from 2011, March, 07th to 2011, March, 11th. The Great Earthquake in Japan took place on 2011, March, 11th. In this network nodes represent the pairs of currencies and there are three types of links according to the time shift cross-analysis: (i) undirected link (*gray*) corresponding to peak at time shift zero; (ii) directed link (*black*), peak at a time shift different from zero, in this case 0.1 s, from the market that influences the other one; (iii) extraverted link (*red*), two peaks at time shifts, also 0.1 s, in both directions

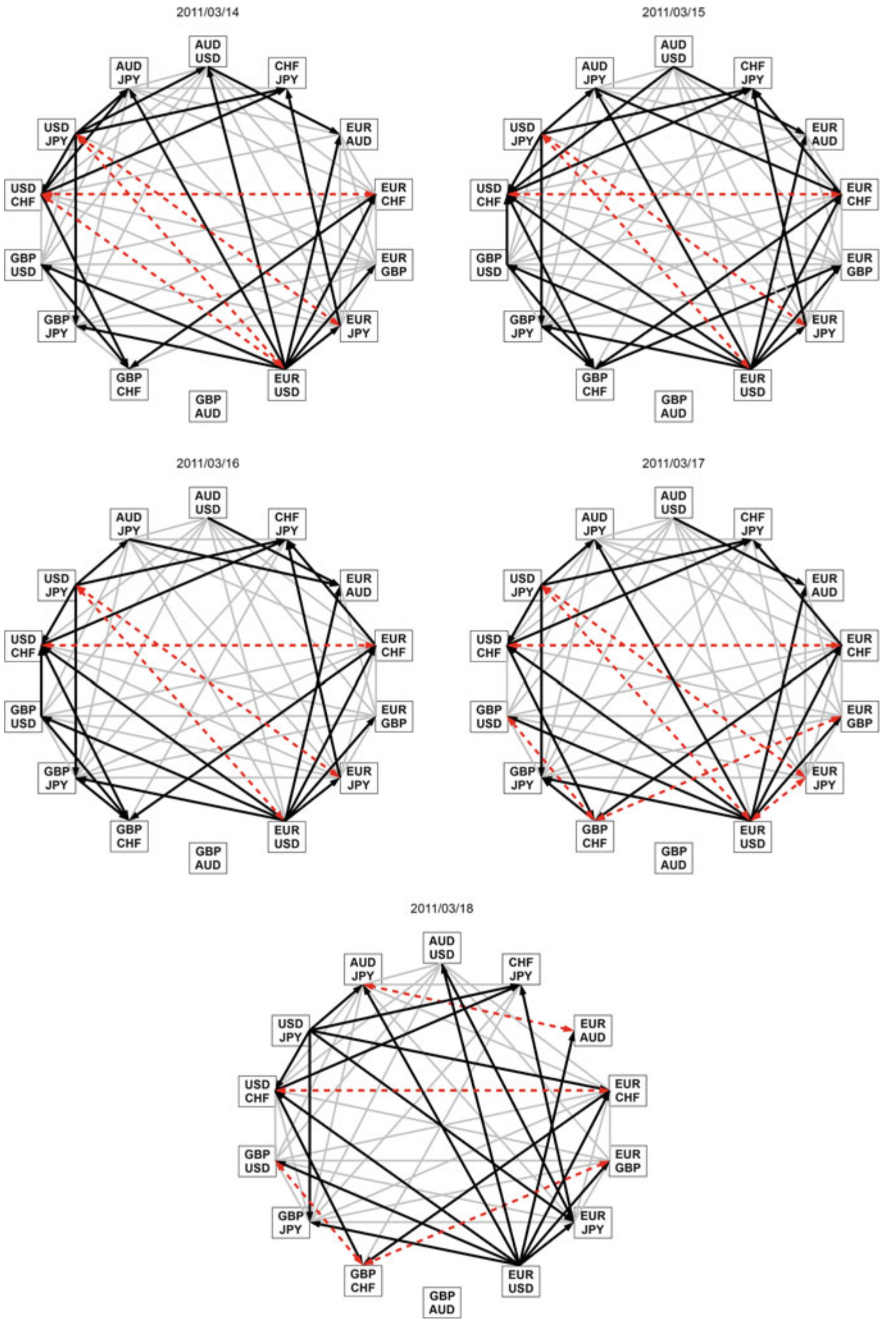

Fig. 1.4 Influence Networks of the Foreign Exchange Market for the currencies USD, EUR, JPY, GBP, AUD and CHF from 2011, March, 14th to 2011, March, 18th. The Intervation in the Foreign Exchange Market was announced in the end of 2011, March, 17th

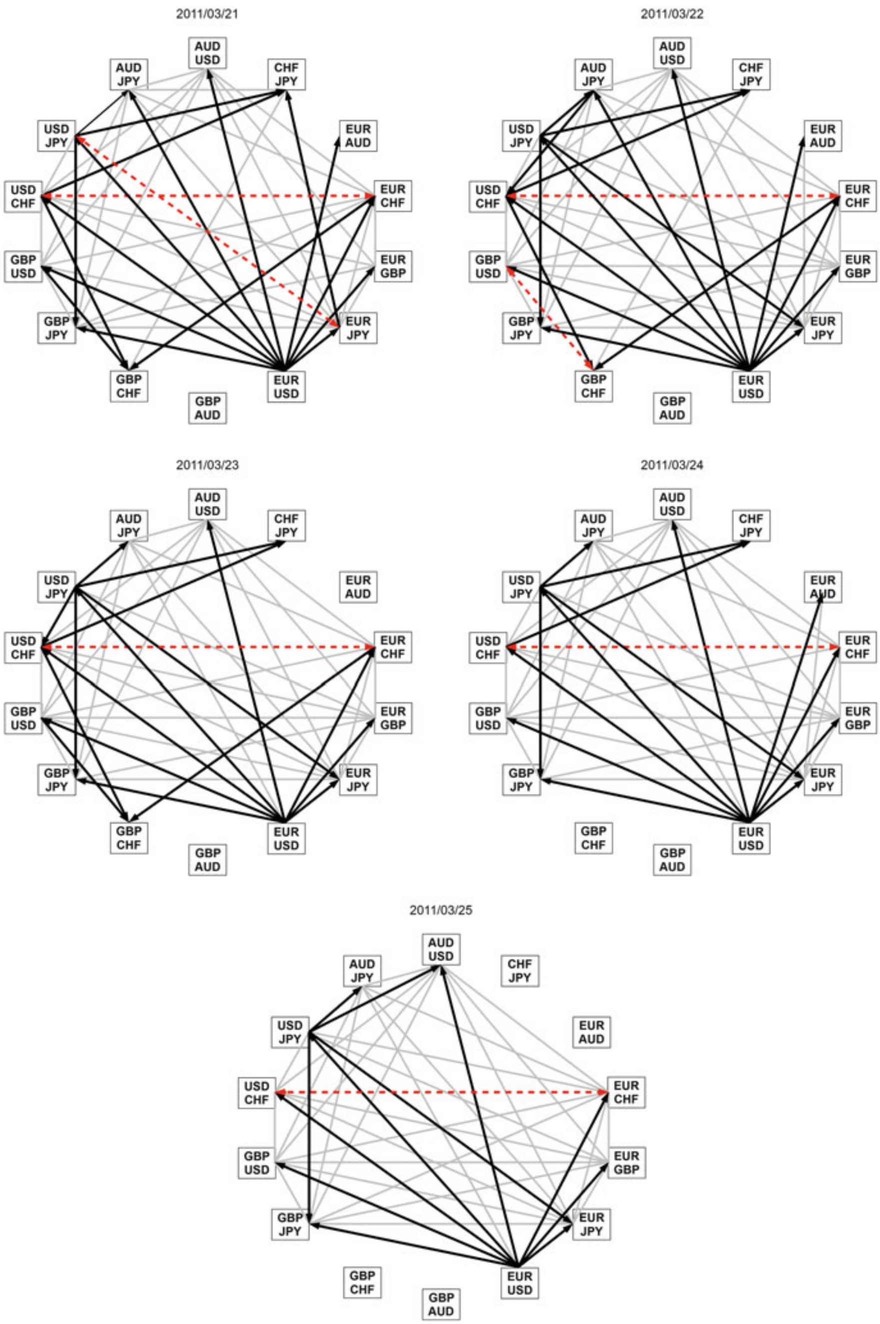

Fig. 1.5 Influence Networks of the Foreign Exchange Market for the currencies USD, EUR, JPY, GBP, AUD and CHF from 2011, March, 21st to 2011, March, 25th

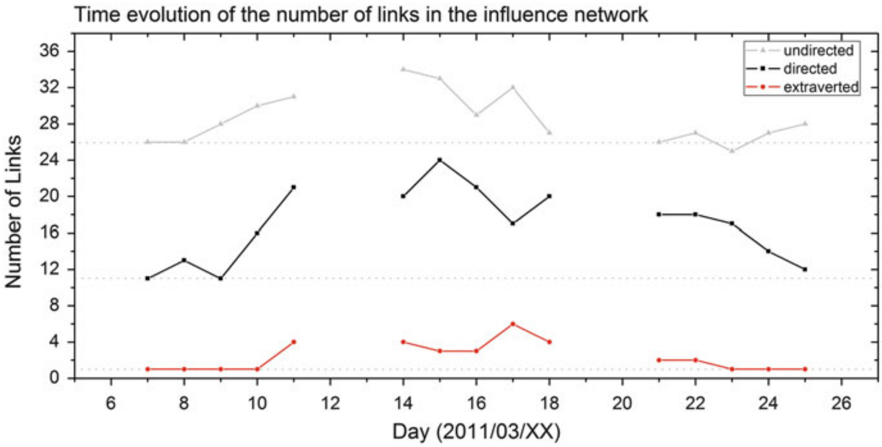

Fig. 1.6 Time evolution of the number of the different types of links in the influence network from 2011, March, 07th to 2011, March, 25th. *Dotted lines* indicate the number of links on 2011, March, 07th

market. It is possible that other factors besides the intervention contributed to the stabilization of the market; to discuss this aspect, it would be necessary the analysis of other periods where stability was reached with no intervention.

1.4 Final Remarks

In this paper we used a non-linear dependence measure based on the mutual information to access the dependence between pairs of currencies of the foreign exchange market. We analysed the sign of price difference of these markets from 2011, March, 07th to 2011, March, 25th, a period that includes the great earthquake in Japan and the intervention. By applying a time shift between the sign series we obtained different dependence structures between markets and then constructed an influence network based on them. The analysis of the influence network and its time evolution showed that the markets EUR/USD and USD/JPY are the most important nodes, with the information flowing from them to the other markets. It also suggested that the event of the earthquake changed the influence structure of the network, intensifying the interdependence between markets and changing the dynamics of the markets EUR/USD and USD/JPY; and the announcement of the intervention was effective in reverting the effects of the earthquake: changes could be observed in the day right after the announcement and the network totally returned to the state previous the earthquake in less than 1 week. The results represent a contribution to understand how the foreign exchange market reacts to big events and thus what can be done in periods of crisis. The analysis can also be useful to predict the behavior of one market based on the past behavior of another, if there is an influence relationship between them.

One important observation is that in the time shift cross-analysis the typical time shift is 0.1 s, i.e., when we have a market influencing another the time delay is 0.1 s. This fact is possibly related to the resolution of the data, also 0.1 s. We analysed the same data but with resolution 1s and could not detect time delay between markets as we found for resolution 0.1 s. We still need to study if we can detect the directionality between markets in other time resolution data or if the resolution 0.1 s is essential to detect such feature. Further researches also should include other currencies, a larger period of analysis and the possibility of time windows smaller than 1 day.

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Chapter 2
Entropy and Transfer Entropy: The Dow Jones and the Build Up to the 1997 Asian Crisis

Michael Harré

Abstract Entropy measures in their various incarnations play an important role in the study of stochastic time series providing important insights into both the *correlative* and the *causative* structure of the stochastic relationships between the individual components of a system. Recent applications of entropic techniques and their linear progenitors such as Pearson correlations and Granger causality have included both *normal* as well as *critical* periods in a system's dynamical evolution. Here I measure the entropy, Pearson correlation and transfer entropy of the intra-day price changes of the Dow Jones Industrial Average (DJIA) in the period immediately leading up to and including the Asian financial crisis and subsequent mini-crash of the DJIA on the 27th October 1997. I use a novel variation of transfer entropy that dynamically adjusts to the arrival rate of individual prices and does not require the binning of data to show that quite different relationships emerge from those given by the conventional Pearson correlations between equities. These preliminary results illustrate how this modified form of the TE compares to results using Pearson correlation.

2.1 Introduction

One of the most pressing needs in modern financial theory is for more accurate information on the structure and drivers of market dynamics. Previous work on correlations [1] has lead to a better understanding of the topological structure of market correlations and mutual information [2] has been used to extend an earlier notion [3, 4] of a market crash as analogous to the phase-transitions studied in physics. These studies are restricted to static market properties in so far as there is no attempt to consider any form of causation. However, one of the goals of econophysics is to gain a better understanding of market dynamics and the drivers of these dynamics need to be extended to trying to measure causation. This is

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extremely difficult, strongly non-linear systems such as financial markets have feedback loops where the most recent change in price of equity a influences the price of b which in turn influences the price of a. This can make extracting causation relationships exceptionally difficult: the empirical distributions need to accurately reflect the temporal order in which price changes in the equities occur, and the time between these changes is itself a stochastic process. The goal of this paper is to introduce a (non-rigourous) heuristic that addresses these concerns using a modification to the conventional definition of the Transfer Entropy (TE) applied to the intraday tick data of the equities that make up the Dow Jones Industrial Average (DJIA) in the tumultuous build up of the Asian Financial Crisis (AFC) that culminated in the crash of the DJIA on the 27th October 1997. This article is arranged in the following way: Sect. 2.2 introduces the linear Pearson correlations I use as a comparison to the TE introduced in Sect. 2.3 in order to make comparisons and then discuss the results in Sect. 2.4.

2.2 Correlations

A statistical process generates a temporal sequence of data: $\mathbf{X}_t = \{\ldots, x_{t-1}, x_t\}$, X_t is a random variable taking possible states S_X at time t, $x_t \in S_X$ and $\mathbf{X}_t^k = \{x_{t-k}, \ldots, x_{t-1}\} \in \{S_X\}^{k-1}$ is a random variable called the k-lagged history of X_t. The marginal probability is $p(X_t)$, the conditional probability of X_t given its k-lagged history is $p(X_t|\mathbf{X}_t^k)$ and further conditioned upon the second process $\mathbf{Y}_t^k$ is $p(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k)$. The Pearson correlation coefficient r between such time series is:

$$r_t^k = \frac{\mathrm{cov}(\mathbf{X}_t^k, \mathbf{Y}_t^k)}{\sigma_X \sigma_Y} \tag{2.1}$$

where $\mathrm{cov}(\cdot, \cdot)$ is the covariance, σ_X and σ_Y are standard deviations and r_t^k is calculated over a finite historical window of length k where in order to calculate the dynamics of r_t^k this window is allowed to slide over the data, updating r_t^k as t progresses. A key issue with data that arrives at irregular or stochastic time intervals and r_t^k is desired is what counts as a co-occurrence at time t of new data. The most common method is to bin the data into equally separated time intervals of length δ_t and if two observations x_t and y_t occur in the interval $[t - \delta_t, t]$ then x_t and y_t are said to co-occur at time t, this approach is used for the correlations calculated in this article. Throughout the change in the log price is the stochastic event of interest: if at time t the price is p_t and at time t' it changes to $p_{t'}$ then the stochastic observable is $x_{t'} = \log(p_{t'}) - \log(p_t)$ [5], the increment $t' - t$ may be fixed in which case it is labelled δt or may dynamically vary, more on this below.

2.3 Transfer Entropy

Transfer Entropy was developed by Schreiber [6] as a rigorous way of measuring the directed transfer of information from one stochastic process to another after accounting for the history of the primary process (see below) for arbitrary distributions. This is a natural extension of Granger Causality, based on covariances rather than information measures, first introduced by Granger [7] in econometrics and in the case of Gaussian processes Granger causality and Transfer Entropy are equivalent [8]. Specifically, the entropic measures we are interested in are:

$$\mathbf{H}(X_t) = -\mathbf{E}_{p(X_t)}[\log p(X_t)], \tag{2.2}$$

$$\mathbf{H}(X_t, Y_t) = -\mathbf{E}_{p(X_t,Y_t)}[\log p(X_t, Y_t)], \tag{2.3}$$

$$\mathbf{H}(X_t|\mathbf{X}_t^k) = -\mathbf{E}_{p(X_t)}[\log p(X_t|\mathbf{X}_t^k)], \tag{2.4}$$

$$\mathbf{H}(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k) = -\mathbf{E}_{p(X_t)}[\log p(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k)], \tag{2.5}$$

where $\mathbf{E}_{p(\cdot)}[\cdot]$ is the expectation with respect to distribution $p(\cdot)$. The mutual information between two stochastic time series $\mathbf{X}_t$ and $\mathbf{Y}_t$ is:

$$\mathbf{I}(\mathbf{X}_t; \mathbf{Y}_t) \equiv \mathbf{H}(\mathbf{X}_t) - \mathbf{H}(\mathbf{X}_t|\mathbf{Y}_t) \;=\; \mathbf{H}(\mathbf{Y}_t) - \mathbf{H}(\mathbf{Y}_t|\mathbf{X}_t) \tag{2.6}$$

with a finite data window of length k this is the information theoretical analogue of r_t^k and the k-lagged *transfer entropy* (TE) from the *source* $\mathbf{Y}$ to the *target* $\mathbf{X}$ is:

$$\mathbf{T}^k_{Y \to X} \equiv \mathbf{H}(X_t|\mathbf{X}_t^k) - \mathbf{H}(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k). \tag{2.7}$$

$\mathbf{T}^k_{Y \to X}$ measures the degree to which X_t is disambiguated by the k-lagged history of Y_t beyond that to which X_t is already disambiguated by its own k-lagged history. This work presents recent developments in TE [9], information theory and the 'critical phenomena' of markets [2], and adds new results for real systems to the recent success in using it as a predictive measure of the phase transition in the 2-D Ising model [10]. The implementation of TE used in this work was done in Matlab using [11].

2.3.1 Transfer Entropy Without Binning

The most common and direct method of calculating any of r_t^k, $\mathbf{I}(\mathbf{X}_t; \mathbf{Y}_t)$ or $\mathbf{T}^k_{Y \to X}$ is to use discrete time series data. This is made possible either by the nature of the study itself where discrete time steps are inherent or through post-processing of the data by binning it into a discrete ordered sequence. However, a lot of interesting data, including intra-day financial markets data, is inherently unstructured and binning the data loses some of the temporal resolution and obfuscates the relationship between past and future events making causal relationships difficult to establish, so an alternative is proposed that addresses these issues.

I define a modified form of $\mathbf{T}^k_{Y\to X}$ by first redefining the stochastic time series in order to capture the continuous nature of the price arrival process. With t and $t' \in \mathbb{R} > 0$ where 0 is taken as the start of trading on any given trading day and $\{t_i\}$ and $\{t'_j\}$ are the finite sequence of times at which the (log) price changes for two different equities during that day. Define the arrival indices of time series of length I and J as $\{i \leq I\} \in \mathbb{N}$ and $\{j \leq J\} \in \mathbb{N}$. Now there are two finite sequences of price changes on a single trading day d: $\{X^d(t_i)\}$ and $\{Y^d(t'_j)\}$. The entropy of $\{X^d(t_i)\}$ conditioned on its most recent past value is:

$$\mathbf{H}(X^d(t_i)|X^d(t_{i-1})) = -\mathbf{E}_{p(X^d)}\big[\log(p(X^d(t_i)|X^d(t_{i-1}))\big], \quad i > 1. \tag{2.8}$$

An equivalent definition for the entropy conditioned on the most recent past of both $\{X^d(t_i)\}$ and $\{Y^d(t'_j)\}$ is:

$$\mathbf{H}(X^d(t_i)|X^d(t_{i-1}), Y^d(t'_{j-1})) = -\mathbf{E}_{p(X^d)}\big[\log(p(X^d(t_i)|X^d(t_{i-1}), Y^d(t'_{j-1}))\big] \tag{2.9}$$

where $i, j > 1$ and t'_{j-1} is the minimum value such that, for a given t_i: $(t_i - t'_{j-1}) > 0$. This modified definition of the TE (for the rest of this article this is simply referred to as *the* TE) is:

$$\overline{\mathbf{T}}_{Y^d\to X^d} \equiv \mathbf{H}(X^d(t_i)|X^d(t_{i-1})) - \mathbf{H}(X^d(t_i)|X^d(t_{i-1}), Y^d(t'_{j-1})). \tag{2.10}$$

The relationship between this and other measures is illustrated in Fig. 2.1. The first row shows the log price changes for two equities (Alcoa and Boeing) as a stochastic time series with an irregular arrival rate. The black arrows indicate the direction and magnitude of the log price changes. The second row shows the changes in prices binned into time intervals of width δt so that changes that occur in the same time interval are considered co-occurring. In the third row is the lag-1 Pearson correlations or lag-1 mutual information, the causal direction of correlations is implicit in the time ordering of the bins, hence the arrows point forward in time. This does not account for the shared signal between x_{t-1} and y_{t-1}. The fourth row shows the lag-1 Granger causality or transfer entropy, the signal driving y_t is x_{t-1} after excluding the common driving factor of y's past: y_{t-1}. Red arrows indicate the measured signal from the source (Alcoa) to the target (Boeing) and blue arrows indicate y's signal that is being removed. Fifth row (fewer price changes shown for clarity): An alternative way to calculate the TE. Choose the target time series (in this case Boeing) and condition out the most recent previous price change in Boeing and then use only the most recent change in Alcoa as the source signal. Note that some Alcoa price signals are missed and some are used more than once and that price changes will rarely co-occur.

The definition of Eq. (2.10) has a number of appealing properties:

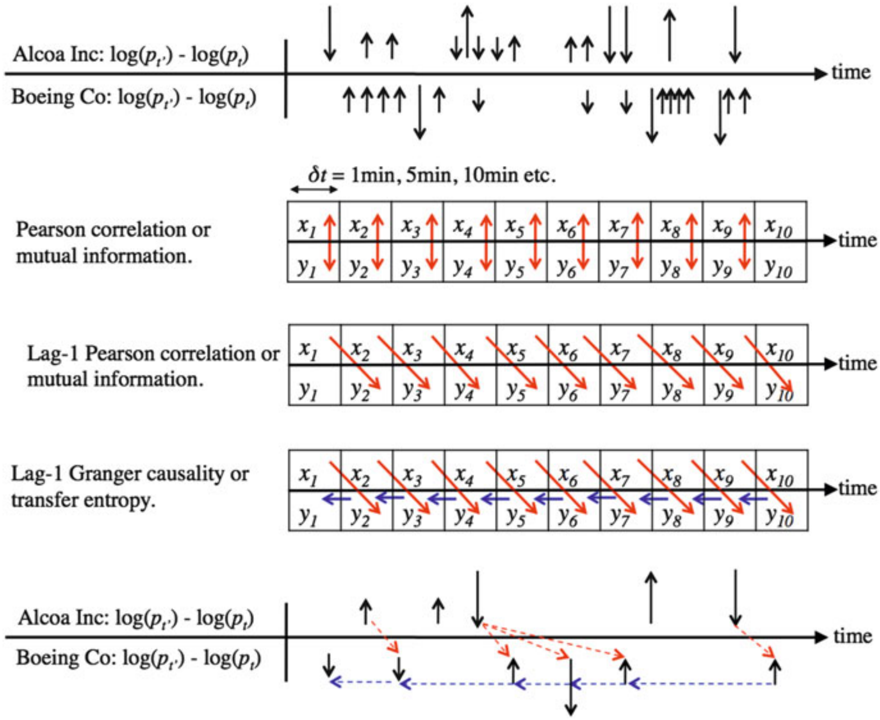

Fig. 2.1 A representation of different measures of 'instantaneous' and 'lagged' relationships between stochastic time series data

- Using a fixed interval in which the price at the beginning is compared with the price at the end of the interval conflates signals that may occur before or after another signal but arrives during the same binning interval, thereby mixing future and past events in the measured relationships between bins.
- Similarly, multiple price changes within δt may net to zero change and so some price signals are missed.
- As bin sizes get smaller they are less statistically reliable as fewer events occur within each bin, equally as bin sizes get larger there are fewer bins per day, thereby also reducing the statistical reliability.
- Over the period of a single day, for each bin size the number of total bins is: $\delta t = 30\,\text{min}$: 13 bins/day, $\delta t = 1\,\text{min}$: 390 bins/day, whereas the raw data may have 50–5000+ price changes in a day.

The proposed heuristic for the TE introduced above addresses some of these shortcomings but not without introducing some other issues. First, it will always condition out the most recent price change information in the target equity (Boeing in Fig. 2.1) and so uses every bit of relevant information in the target time series. It also uses the most recent price change from the source time series, however it will sometimes miss some price changes or repeatedly count the same price changes (see bottom of Fig. 2.1). This is good if we are interested in the most recent price signals and in financial markets this is the case. It also reflects the dynamical nature of the time series, as the inter-arrival times may vary from day to day or between equities no new δt needs to be defined, it will always use only the most recent information in both the source and the target time series. The most significant shortcoming is that this TE assumes there is no information being carried by the inter-arrival time interval and it is not clear that some of the theoretical foundations on which the original TE is based necessarily hold, from this point of view this method of calculating the TE is currently only a heuristic and the results presented here are for the moment qualitative in nature.

2.4 Empirical Results

The AFC began in Thailand in July 1997 with the devaluation of the Thai currency (the Bhat) and the crisis rapidly spread throughout South East Asia, ultimately resulting in the October 27 "mini-crash" of the DJIA, losing around 7 % on the day which was at the time the largest single day points drop on record for the DJIA, for a review of the crisis see [12] and the top plot of Fig. 2.2. Note that the entropy measurements shown illustrate that some care needs to be taken when comparing simple systems with data from real 'complex systems': the increase in the entropy of the DJIA on the 24th of June looks like what might be described as a 'first order' phase transition as studied in complex systems [13], but it is almost certainly caused by the rescaling of price increments on the New York Stock Exchange.[1]

This rescaling did have an interesting impact on the TE though, as can be seen in Fig. 2.3. Prior to the 24th of June there is considerable structure in the TE measure (warm colours denote high TE values, cooler colours denote lower TE values), however all signals drop off significantly immediately after this date although much of the structured signal eventually returns (not shown). The most notable signals are equities that act as targets of TE for multiple other equities, seen as yellow vertical strips indicating that many equities act as relatively strong sources of TE for a single equity: AT&T (equity 26), Wall Mart (equity 30) and McDonalds (equity 31) stand out in this respect. Notable single sources of TE are less obvious but Cocoa Cola and AT&T (equities 19 and 26) show some coherent signals indicated by multiple red points loosely forming a horizontal line. It is intriguing to note that the Pearson correlations showed no similar shift on the 24th of June (not shown) while conversely in Fig. 2.4 the mini-crash on the 27th October 1997 (day 64) there is a clear signal that the DJIA equities are significantly more correlated with no corresponding increase in the TE on that day (not shown) despite the general turmoil of the markets, as seen by significant fluctuations in the correlations on nearby days.

[1] For details see: http://www1.nyse.com/nysenotices/nyse/rule-changes/detail?memo_id=97-33.

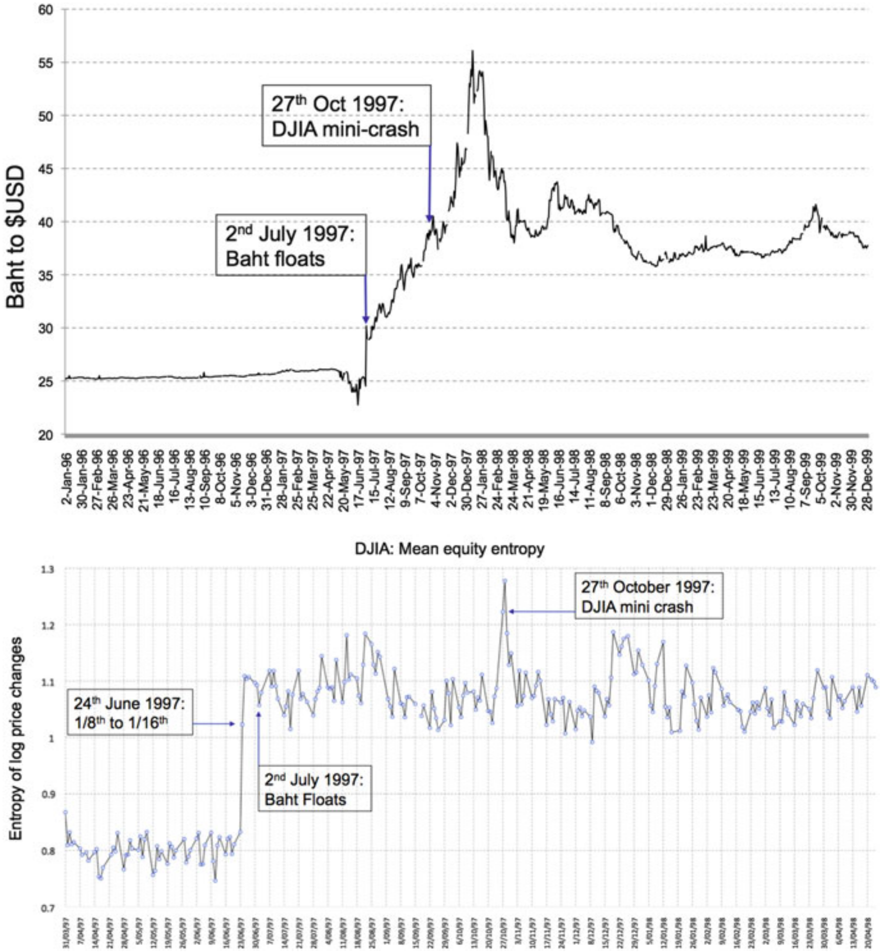

Fig. 2.2 The AFC and its key components. *Top plot*: the AFC is thought to have begun as the Baht was allowed to float against the US dollar on the 2nd of July 1997. The crisis contagion spread through the asian markets ultimately leading to the mini-crash of the DJIA on the 27th October 1997. *Bottom plot*: on the 24th of June 1997 the New York Stock Exchange changed its minimum incremental buy/sell price from 1/8th of a dollar to 1/16th of a dollar, causing the entropy of the price changes to shift suddenly and permanently, but not influencing the DJIA index itself. The crash on the 27th October 1997 is seen as the second largest peak in the entropy, the largest being the 28th of October

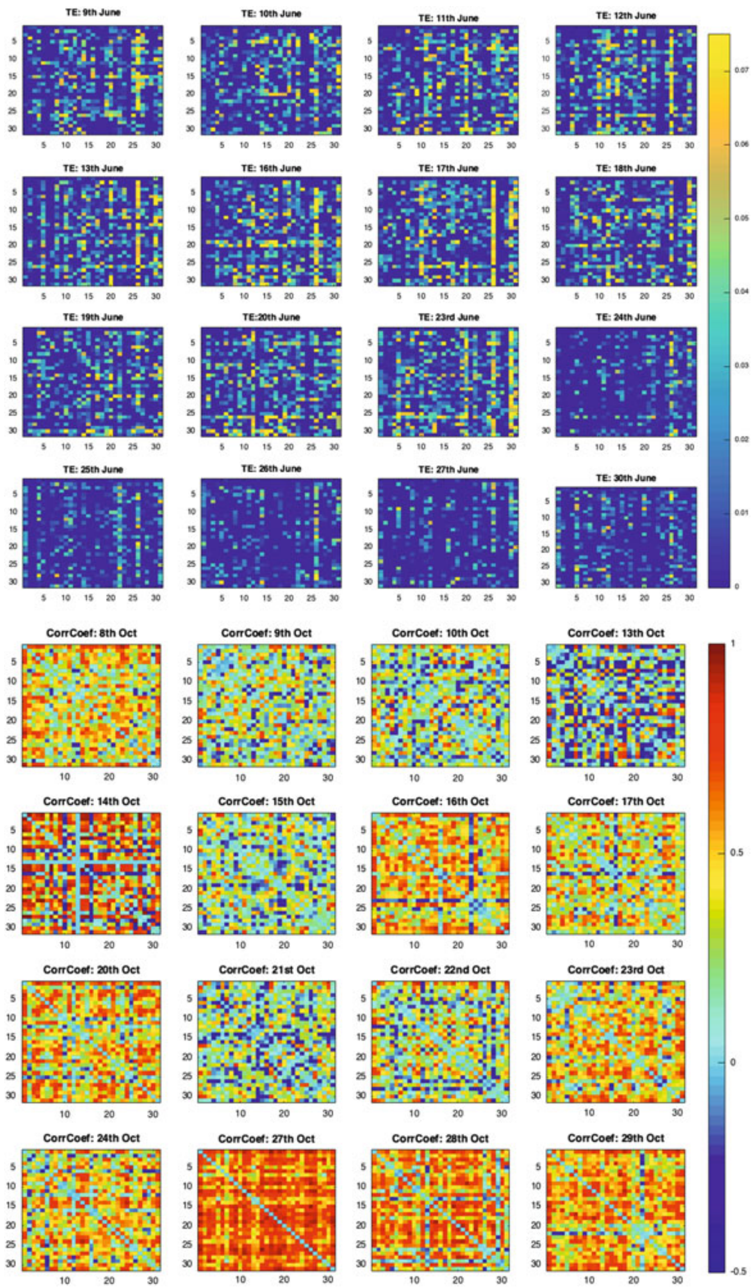

Fig. 2.3 *Top*: the TE from one DJIA equity to another equity indexed from 1 to 31. Index 1 = the DJIA, vertical axis is the *source* equity, horizontal axis is the *target* equity. The 24th of June 1997 clearly stands out as the first day of a substantive reduction in the TE between equities. *Bottom*: the Pearson correlation for the DJIA data binned using $\delta t = 30$ min. The market crash on the 27th October stands out during a turbulent time in the market's dynamics